

# Mathematica 11.3 Integration Test Results

on the problems in "4 Trig functions\4.7 Miscellaneous"

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Test results for the 254 problems in "4.7.1 (c trig)^m (d trig)^n.m"

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \csc[2a + 2bx] \sin[a + bx] dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\sin[a + bx]]}{2b}$$

Result (type 3, 72 leaves):

$$\frac{1}{2} \left( -\frac{\text{Log}\left[\cos\left[\frac{a}{2} + \frac{bx}{2}\right] - \sin\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{\text{Log}\left[\cos\left[\frac{a}{2} + \frac{bx}{2}\right] + \sin\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} \right)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \csc[2a + 2bx]^3 \sin[a + bx] dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$\frac{3 \text{ArcTanh}[\sin[a + bx]]}{16b} - \frac{3 \csc[a + bx]}{16b} + \frac{\csc[a + bx] \sec[a + bx]^2}{16b}$$

Result (type 3, 132 leaves):

$$-\frac{1}{32b} \left( 2 \operatorname{Cot} \left[ \frac{1}{2} (a+bx) \right] + 6 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (a+bx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (a+bx) \right] \right] - 6 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (a+bx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (a+bx) \right] \right] - \right. \\ \left. \frac{1}{\left( \operatorname{Cos} \left[ \frac{1}{2} (a+bx) \right] - \operatorname{Sin} \left[ \frac{1}{2} (a+bx) \right] \right)^2} + \frac{1}{\left( \operatorname{Cos} \left[ \frac{1}{2} (a+bx) \right] + \operatorname{Sin} \left[ \frac{1}{2} (a+bx) \right] \right)^2} + 2 \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)$$

**Problem 11: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc} [2a + 2bx]^4 \operatorname{Sin} [a + bx] \, dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$-\frac{5 \operatorname{ArcTanh} [\operatorname{Cos} [a + bx]]}{32b} + \frac{5 \operatorname{Sec} [a + bx]}{32b} + \frac{5 \operatorname{Sec} [a + bx]^3}{96b} - \frac{\operatorname{Csc} [a + bx]^2 \operatorname{Sec} [a + bx]^3}{32b}$$

Result (type 3, 205 leaves):

$$\frac{1}{24b \left( \operatorname{Csc} \left[ \frac{1}{2} (a+bx) \right]^2 - \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right)^3} \\ \operatorname{Csc} [a + bx]^8 \left( 22 - 40 \operatorname{Cos} [2(a+bx)] + 13 \operatorname{Cos} [3(a+bx)] - 30 \operatorname{Cos} [4(a+bx)] + 13 \operatorname{Cos} [5(a+bx)] + 15 \operatorname{Cos} [3(a+bx)] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (a+bx) \right] \right] + \right. \\ \left. 15 \operatorname{Cos} [5(a+bx)] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (a+bx) \right] \right] - 15 \operatorname{Cos} [3(a+bx)] \operatorname{Log} \left[ \operatorname{Sin} \left[ \frac{1}{2} (a+bx) \right] \right] - \right. \\ \left. 15 \operatorname{Cos} [5(a+bx)] \operatorname{Log} \left[ \operatorname{Sin} \left[ \frac{1}{2} (a+bx) \right] \right] + \operatorname{Cos} [a + bx] \left( -26 - 30 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (a+bx) \right] \right] + 30 \operatorname{Log} \left[ \operatorname{Sin} \left[ \frac{1}{2} (a+bx) \right] \right] \right) \right)$$

**Problem 12: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc} [2a + 2bx]^5 \operatorname{Sin} [a + bx] \, dx$$

Optimal (type 3, 89 leaves, 7 steps):

$$\frac{35 \operatorname{ArcTanh} [\operatorname{Sin} [a + bx]]}{256b} - \frac{35 \operatorname{Csc} [a + bx]}{256b} - \frac{35 \operatorname{Csc} [a + bx]^3}{768b} + \frac{7 \operatorname{Csc} [a + bx]^3 \operatorname{Sec} [a + bx]^2}{256b} + \frac{\operatorname{Csc} [a + bx]^3 \operatorname{Sec} [a + bx]^4}{128b}$$

Result (type 3, 277 leaves):

$$\begin{aligned}
& - \frac{19 \cot\left[\frac{1}{2}(a+bx)\right]}{384b} - \frac{\cot\left[\frac{1}{2}(a+bx)\right] \csc\left[\frac{1}{2}(a+bx)\right]^2}{768b} - \frac{35 \log\left[\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right]}{256b} + \\
& \frac{35 \log\left[\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right]}{256b} + \frac{1}{512b \left(\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right)^4} + \\
& \frac{11}{512b \left(\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right)^2} - \frac{1}{512b \left(\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right)^4} - \\
& \frac{11}{512b \left(\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right)^2} - \frac{19 \tan\left[\frac{1}{2}(a+bx)\right]}{384b} - \frac{\sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]}{768b}
\end{aligned}$$

**Problem 28: Result more than twice size of optimal antiderivative.**

$$\int \csc[2a+2bx] \sin[a+bx]^3 dx$$

Optimal (type 3, 28 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\sin[a+bx]]}{2b} - \frac{\sin[a+bx]}{2b}$$

Result (type 3, 71 leaves):

$$\frac{1}{2} \left( - \frac{\log\left[\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right]}{b} + \frac{\log\left[\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right]}{b} - \frac{\sin[a+bx]}{b} \right)$$

**Problem 30: Result more than twice size of optimal antiderivative.**

$$\int \csc[2a+2bx]^3 \sin[a+bx]^3 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{\text{ArcTanh}[\sin[a+bx]]}{16b} + \frac{\sec[a+bx] \tan[a+bx]}{16b}$$

Result (type 3, 69 leaves):

$$\frac{1}{16b} \left( - \log\left[\cos\left[\frac{1}{2}(a+bx)\right] - \sin\left[\frac{1}{2}(a+bx)\right]\right] + \log\left[\cos\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{1}{2}(a+bx)\right]\right] + \sec[a+bx] \tan[a+bx] \right)$$

### Problem 32: Result more than twice size of optimal antiderivative.

$$\int \csc[2a + 2bx]^5 \sin[a + bx]^3 dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{15 \operatorname{ArcTanh}[\sin[a + bx]]}{256b} - \frac{15 \csc[a + bx]}{256b} + \frac{5 \csc[a + bx] \sec[a + bx]^2}{256b} + \frac{\csc[a + bx] \sec[a + bx]^4}{128b}$$

Result (type 3, 219 leaves):

$$\begin{aligned} & -\frac{\cot\left[\frac{1}{2}(a + bx)\right]}{64b} - \frac{15 \operatorname{Log}\left[\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]\right]}{256b} + \frac{15 \operatorname{Log}\left[\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]\right]}{256b} + \\ & \frac{1}{512b \left(\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]\right)^4} + \frac{7}{512b \left(\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]\right)^2} - \\ & \frac{1}{512b \left(\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]\right)^4} - \frac{7}{512b \left(\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]\right)^2} - \frac{\tan\left[\frac{1}{2}(a + bx)\right]}{64b} \end{aligned}$$

### Problem 40: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx] \sin[2a + 2bx] dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{2 \sin[a + bx]}{b}$$

Result (type 3, 23 leaves):

$$2 \left( \frac{\cos[bx] \sin[a]}{b} + \frac{\cos[a] \sin[bx]}{b} \right)$$

### Problem 41: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx] \csc[2a + 2bx] dx$$

Optimal (type 3, 28 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin[a + bx]]}{2b} - \frac{\csc[a + bx]}{2b}$$

Result (type 3, 95 leaves):

$$-\frac{\operatorname{Cot}\left[\frac{1}{2}(a+bx)\right]}{4b} - \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} - \frac{\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{4b}$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[a+bx] \operatorname{Csc}[2a+2bx]^2 dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[a+bx]]}{8b} + \frac{3 \operatorname{Sec}[a+bx]}{8b} - \frac{\operatorname{Csc}[a+bx]^2 \operatorname{Sec}[a+bx]}{8b}$$

Result (type 3, 143 leaves):

$$\left( \operatorname{Csc}[a+bx]^4 \left( 2 - 6 \operatorname{Cos}[2(a+bx)] + 2 \operatorname{Cos}[3(a+bx)] + 3 \operatorname{Cos}[3(a+bx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right] - 3 \operatorname{Cos}[3(a+bx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right] \right) + \operatorname{Cos}[a+bx] \left( -2 - 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right] + 3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right] \right) \right) / \left( 8b \left( \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2 - \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right) \right)$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[a+bx] \operatorname{Csc}[2a+2bx]^3 dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$\frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[a+bx]]}{16b} - \frac{5 \operatorname{Csc}[a+bx]}{16b} - \frac{5 \operatorname{Csc}[a+bx]^3}{48b} + \frac{\operatorname{Csc}[a+bx]^3 \operatorname{Sec}[a+bx]^2}{16b}$$

Result (type 3, 215 leaves):

$$-\frac{13 \operatorname{Cot}\left[\frac{1}{2}(a+bx)\right]}{96b} - \frac{\operatorname{Cot}\left[\frac{1}{2}(a+bx)\right] \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{192b} - \frac{5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{16b} + \frac{5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{16b} + \frac{1}{32b \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)^2} - \frac{1}{32b \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)^2} - \frac{13 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{96b} - \frac{\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{192b}$$

### Problem 44: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[a + b x] \operatorname{Csc}[2 a + 2 b x]^4 dx$$

Optimal (type 3, 89 leaves, 7 steps):

$$-\frac{35 \operatorname{ArcTanh}[\operatorname{Cos}[a + b x]]}{128 b} + \frac{35 \operatorname{Sec}[a + b x]}{128 b} + \frac{35 \operatorname{Sec}[a + b x]^3}{384 b} - \frac{7 \operatorname{Csc}[a + b x]^2 \operatorname{Sec}[a + b x]^3}{128 b} - \frac{\operatorname{Csc}[a + b x]^4 \operatorname{Sec}[a + b x]^3}{64 b}$$

Result (type 3, 268 leaves):

$$-\frac{1}{384 b \left( \operatorname{Csc}\left[\frac{1}{2}(a + b x)\right]^2 - \operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 \right)^3} \\ \operatorname{Csc}[a + b x]^{10} \left( -204 + 658 \operatorname{Cos}[2(a + b x)] - 228 \operatorname{Cos}[3(a + b x)] + 140 \operatorname{Cos}[4(a + b x)] - 76 \operatorname{Cos}[5(a + b x)] - 210 \operatorname{Cos}[6(a + b x)] + \right. \\ \left. 76 \operatorname{Cos}[7(a + b x)] - 315 \operatorname{Cos}[3(a + b x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + b x)\right]\right] - 105 \operatorname{Cos}[5(a + b x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + b x)\right]\right] + \right. \\ \left. 105 \operatorname{Cos}[7(a + b x)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + b x)\right]\right] + 3 \operatorname{Cos}[a + b x] \left( 76 + 105 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + b x)\right]\right] - 105 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]\right] \right) + \right. \\ \left. 315 \operatorname{Cos}[3(a + b x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]\right] + 105 \operatorname{Cos}[5(a + b x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]\right] - 105 \operatorname{Cos}[7(a + b x)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]\right] \right)$$

### Problem 46: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[a + b x]^2 \operatorname{Sin}[2 a + 2 b x]^7 dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$-\frac{16 \operatorname{Cos}[a + b x]^8}{b} + \frac{128 \operatorname{Cos}[a + b x]^{10}}{5 b} - \frac{32 \operatorname{Cos}[a + b x]^{12}}{3 b}$$

Result (type 3, 91 leaves):

$$-\frac{5 \operatorname{Cos}[2(a + b x)]}{8 b} - \frac{5 \operatorname{Cos}[4(a + b x)]}{64 b} + \frac{5 \operatorname{Cos}[6(a + b x)]}{48 b} + \frac{\operatorname{Cos}[8(a + b x)]}{32 b} - \frac{\operatorname{Cos}[10(a + b x)]}{80 b} - \frac{\operatorname{Cos}[12(a + b x)]}{192 b}$$

### Problem 61: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[a + b x]^3 \operatorname{Sin}[2 a + 2 b x]^8 dx$$

Optimal (type 3, 46 leaves, 4 steps):

$$-\frac{256 \cos [a + b x]^9}{9 b} + \frac{512 \cos [a + b x]^{11}}{11 b} - \frac{256 \cos [a + b x]^{13}}{13 b}$$

Result (type 3, 104 leaves):

$$-\frac{5 \cos [a + b x]}{4 b} - \frac{25 \cos [3 (a + b x)]}{48 b} + \frac{\cos [5 (a + b x)]}{16 b} + \frac{\cos [7 (a + b x)]}{8 b} + \frac{\cos [9 (a + b x)]}{72 b} - \frac{3 \cos [11 (a + b x)]}{176 b} - \frac{\cos [13 (a + b x)]}{208 b}$$

**Problem 69: Result more than twice size of optimal antiderivative.**

$$\int \csc [a + b x]^3 \csc [2 a + 2 b x] dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\text{ArcTanh}[\sin [a + b x]]}{2 b} - \frac{\csc [a + b x]}{2 b} - \frac{\csc [a + b x]^3}{6 b}$$

Result (type 3, 153 leaves):

$$-\frac{7 \cot \left[ \frac{1}{2} (a + b x) \right]}{24 b} - \frac{\cot \left[ \frac{1}{2} (a + b x) \right] \csc \left[ \frac{1}{2} (a + b x) \right]^2}{48 b} - \frac{\log \left[ \cos \left[ \frac{1}{2} (a + b x) \right] - \sin \left[ \frac{1}{2} (a + b x) \right] \right]}{2 b} +$$

$$\frac{\log \left[ \cos \left[ \frac{1}{2} (a + b x) \right] + \sin \left[ \frac{1}{2} (a + b x) \right] \right]}{2 b} - \frac{7 \tan \left[ \frac{1}{2} (a + b x) \right]}{24 b} - \frac{\sec \left[ \frac{1}{2} (a + b x) \right]^2 \tan \left[ \frac{1}{2} (a + b x) \right]}{48 b}$$

**Problem 70: Result more than twice size of optimal antiderivative.**

$$\int \csc [a + b x]^3 \csc [2 a + 2 b x]^2 dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$-\frac{15 \text{ArcTanh}[\cos [a + b x]]}{32 b} + \frac{15 \sec [a + b x]}{32 b} - \frac{5 \csc [a + b x]^2 \sec [a + b x]}{32 b} - \frac{\csc [a + b x]^4 \sec [a + b x]}{16 b}$$

Result (type 3, 195 leaves):

$$-\frac{7 \csc \left[ \frac{1}{2} (a + b x) \right]^2}{128 b} - \frac{\csc \left[ \frac{1}{2} (a + b x) \right]^4}{256 b} - \frac{15 \log \left[ \cos \left[ \frac{1}{2} (a + b x) \right] \right]}{32 b} + \frac{15 \log \left[ \sin \left[ \frac{1}{2} (a + b x) \right] \right]}{32 b} +$$

$$\frac{7 \sec \left[ \frac{1}{2} (a + b x) \right]^2}{128 b} + \frac{\sec \left[ \frac{1}{2} (a + b x) \right]^4}{256 b} + \frac{\sin \left[ \frac{1}{2} (a + b x) \right]}{4 b \left( \cos \left[ \frac{1}{2} (a + b x) \right] - \sin \left[ \frac{1}{2} (a + b x) \right] \right)} - \frac{\sin \left[ \frac{1}{2} (a + b x) \right]}{4 b \left( \cos \left[ \frac{1}{2} (a + b x) \right] + \sin \left[ \frac{1}{2} (a + b x) \right] \right)}$$

### Problem 71: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[a + b x]^3 \operatorname{Csc}[2 a + 2 b x]^3 dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$\frac{7 \operatorname{ArcTanh}[\operatorname{Sin}[a + b x]]}{16 b} - \frac{7 \operatorname{Csc}[a + b x]}{16 b} - \frac{7 \operatorname{Csc}[a + b x]^3}{48 b} - \frac{7 \operatorname{Csc}[a + b x]^5}{80 b} + \frac{\operatorname{Csc}[a + b x]^5 \operatorname{Sec}[a + b x]^2}{16 b}$$

Result (type 3, 222 leaves):

$$-\frac{1}{3840 b} \left( 818 \operatorname{Cot}\left[\frac{1}{2}(a + b x)\right] + 1680 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + b x)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]\right] - 1680 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + b x)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]\right] - \frac{120}{\left(\operatorname{Cos}\left[\frac{1}{2}(a + b x)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]\right)^2} + 392 \operatorname{Csc}[a + b x]^3 \operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]^4 + 96 \operatorname{Csc}[a + b x]^5 \operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]^6 + \frac{120}{\left(\operatorname{Cos}\left[\frac{1}{2}(a + b x)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + b x)\right]\right)^2} + \frac{49}{2} \operatorname{Csc}\left[\frac{1}{2}(a + b x)\right]^4 \operatorname{Sin}[a + b x] + \frac{3}{2} \operatorname{Csc}\left[\frac{1}{2}(a + b x)\right]^6 \operatorname{Sin}[a + b x] + 818 \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right] \right)$$

### Problem 72: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Csc}[a + b x]^3 \operatorname{Csc}[2 a + 2 b x]^4 dx$$

Optimal (type 3, 112 leaves, 8 steps):

$$-\frac{105 \operatorname{ArcTanh}[\operatorname{Cos}[a + b x]]}{256 b} + \frac{105 \operatorname{Sec}[a + b x]}{256 b} + \frac{35 \operatorname{Sec}[a + b x]^3}{256 b} - \frac{21 \operatorname{Csc}[a + b x]^2 \operatorname{Sec}[a + b x]^3}{256 b} - \frac{3 \operatorname{Csc}[a + b x]^4 \operatorname{Sec}[a + b x]^3}{128 b} - \frac{\operatorname{Csc}[a + b x]^6 \operatorname{Sec}[a + b x]^3}{96 b}$$

Result (type 3, 278 leaves):



$$\frac{1}{3072 b \left( \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2 - \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right)^3}$$

$$\operatorname{Csc}[a+bx]^{12} \left( 1150 - 4752 \operatorname{Cos}[2(a+bx)] + 1600 \operatorname{Cos}[3(a+bx)] + 504 \operatorname{Cos}[4(a+bx)] + 1680 \operatorname{Cos}[6(a+bx)] - 600 \operatorname{Cos}[7(a+bx)] - \right.$$

$$630 \operatorname{Cos}[8(a+bx)] + 200 \operatorname{Cos}[9(a+bx)] + 2520 \operatorname{Cos}[3(a+bx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right] - 945 \operatorname{Cos}[7(a+bx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right] +$$

$$315 \operatorname{Cos}[9(a+bx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right] - 30 \operatorname{Cos}[a+bx] \left( 40 + 63 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right] - 63 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right] \right) -$$

$$\left. 2520 \operatorname{Cos}[3(a+bx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right] + 945 \operatorname{Cos}[7(a+bx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right] - 315 \operatorname{Cos}[9(a+bx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right] \right)$$

**Problem 123: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Sin}[a+bx]^3 \operatorname{Sin}[2a+2bx]^m dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{1}{b(4+m)} \left( \operatorname{Cos}[a+bx]^2 \right)^{\frac{1-m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \operatorname{Sin}[a+bx]^2\right] \operatorname{Sin}[a+bx]^3 \operatorname{Sin}[2a+2bx]^m \operatorname{Tan}[a+bx]$$

Result (type 6, 5212 leaves):

$$\left( 2^{4+m} (4+m) \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]^6 \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Sin}[a+bx]^3 \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] \left( -\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{3}{2}(a+bx)\right] \right) \right)^m \right.$$

$$\operatorname{Sin}[2(a+bx)]^m \left( \left( \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right) / \right.$$

$$\left( (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - \right.$$

$$2 \left( m \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right.$$

$$\left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) -$$

$$\operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] / \left( (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \right.$$

$$\left. \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right.$$

$$\left. \left. 2(2+m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \right) /$$

$$\left( b(2+m) \left( \frac{1}{2+m} 2^{4+m} (4+m) \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]^7 \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] \left( -\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{3}{2}(a+bx)\right] \right) \right)^m \right)$$



$$\begin{aligned}
& \text{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \tan\left[\frac{1}{2}(a+bx)\right]^2 - \\
& \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] / \left( (4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \right. \right. \\
& \left. \left. (2+m) \text{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \frac{1}{2+m} 2^{4+m} (4+m) \cos\left[\frac{1}{2}(a+bx)\right]^6 \sin\left[\frac{1}{2}(a+bx)\right]^2 \left( \cos\left[\frac{1}{2}(a+bx)\right] \left( -\sin\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{3}{2}(a+bx)\right] \right) \right)^m \\
& \left( \left( \text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) / \right. \\
& \left( (4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \left. 2 \left( m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + (3+2m) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \left( \sec\left[\frac{1}{2}(a+bx)\right]^2 \left( -\frac{1}{4+m} m (2+m) \text{AppellF1}\left[1 + \frac{2+m}{2}, 1-m, 3+2m, 1 + \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{4+m} (2+m) (3+2m) \text{AppellF1}\left[1 + \frac{2+m}{2}, -m, 4+2m, 1 + \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) / \left( (4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + (3+2m) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( -\frac{1}{4+m} m (2+m) \text{AppellF1}\left[1 + \frac{2+m}{2}, 1-m, 2(2+m), 1 + \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \right. \\
& \left. \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{4+m} 2(2+m)^2 \text{AppellF1}\left[1 + \frac{2+m}{2}, -m, 1+2(2+m), 1 + \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \\
& \left. \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) / \left( (4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \left. 2 \left( m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2(2+m) \right. \right. \\
& \left. \left. \text{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \left( \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left( -2 \left( m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 2(2+m), \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 + 2(2+m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \\
& \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + (4+m) \left(-\frac{1}{4+m}m(2+m) \operatorname{AppellF1}\left[1+\frac{2+m}{2}, 1-m, 2(2+m), \right. \right. \\
& \left. \left. 1+\frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{4+m}2(2+m)^2 \operatorname{AppellF1}\left[1+\frac{2+m}{2}, \right. \right. \\
& \left. \left. -m, 1+2(2+m), 1+\frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right) - 2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \\
& \left(m \left(-\frac{1}{6+m}2(2+m)(4+m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 1-m, 1+2(2+m), 1+\frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{6+m}(1-m)(4+m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 2-m, 2(2+m), 1+\frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right) + 2(2+m) \left(-\frac{1}{6+m}m(4+m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 1-m, 5+2m, \right. \right. \\
& \left. \left. 1+\frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{6+m}(4+m)(5+2m) \operatorname{AppellF1}\left[ \right. \right. \\
& \left. \left. 1+\frac{4+m}{2}, -m, 6+2m, 1+\frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right) \right) \Bigg) / \\
& \left( (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left(m \operatorname{AppellF1}\left[\frac{4+m}{2}, \right. \right. \right. \\
& \left. \left. 1-m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \left. 2(2+m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 - \\
& \left( \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right. \\
& \left. \left(-2 \left(m \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + (3+2m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + (4+m) \left(-\frac{1}{4+m}m(2+m) \operatorname{AppellF1}\left[1+\frac{2+m}{2}, \right. \right. \right. \\
& \left. \left. 1-m, 3+2m, 1+\frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{4+m}(2+m)(3+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[1+\frac{2+m}{2}, -m, 4+2m, 1+\frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right) - \right. \\
& \left. 2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \left(m \left(-\frac{1}{6+m}(4+m)(3+2m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 1-m, 4+2m, 1+\frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{6+m}(1-m)(4+m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 2-m, 3+2m, 1+\frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right) + (3+2m) \left(-\frac{1}{6+m}m(4+m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 1-m, 2(2+m), \right. \right. \right.
\end{aligned}$$

$$\begin{aligned} & \left( 1 + \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] - \frac{1}{6+m} 2(2+m)(4+m) \operatorname{AppellF1}\left[ \right. \\ & \left. 1 + \frac{4+m}{2}, -m, 1+2(2+m), 1 + \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \right) \Big/ \\ & \left( (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \operatorname{AppellF1}\left[\frac{4+m}{2}, \right. \right. \right. \\ & \left. \left. 1-m, 3+2m, \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\ & \left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \Big) \Big) \Big) \Big/ \end{aligned}$$

**Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sin[a+bx]^2 \sin[2a+2bx]^m dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{1}{b(3+m)} (\cos[a+bx]^2)^{\frac{1-m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \sin[a+bx]^2\right] \sin[a+bx]^2 \sin[2a+2bx]^m \operatorname{Tan}[a+bx]$$

Result (type 6, 5195 leaves):

$$\begin{aligned} & \left( 2^{3+m} (3+m) \cos\left[\frac{1}{2}(a+bx)\right]^5 \sin\left[\frac{1}{2}(a+bx)\right] \sin[a+bx]^2 \left( \cos\left[\frac{1}{2}(a+bx)\right] \left( -\sin\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{3}{2}(a+bx)\right] \right) \right)^m \sin[2(a+bx)]^m \right. \\ & \left( - \left( \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \Big/ \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \right. \right. \right. \\ & \left. \left. \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\ & \left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \right) + \\ & \left( \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right) \Big/ \\ & \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\ & \left. 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\ & \left. \left. 2(1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \Big) \Big/ \\ & \left( b(1+m) \left( \frac{1}{1+m} 2^{2+m} (3+m) \cos\left[\frac{1}{2}(a+bx)\right]^6 \left( \cos\left[\frac{1}{2}(a+bx)\right] \left( -\sin\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{3}{2}(a+bx)\right] \right) \right)^m \right. \right. \end{aligned}$$



$$\begin{aligned}
& \left( \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right) / \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \quad 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2(1+m) \right. \\
& \quad \quad \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \frac{1}{1+m} 2^{3+m} (3+m) \cos\left[\frac{1}{2}(a+bx)\right]^5 \sin\left[\frac{1}{2}(a+bx)\right] \left( \cos\left[\frac{1}{2}(a+bx)\right] \left( -\sin\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{3}{2}(a+bx)\right] \right) \right)^m \\
& \left( - \left( \left( -\frac{1}{3+m} m(1+m) \text{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 3+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} (1+m) (3+2m) \text{AppellF1}\left[1+\frac{1+m}{2}, -m, 4+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) / \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \quad 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \quad \quad \left. \left. (3+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \left( \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) / \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \quad 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2(1+m) \right. \\
& \quad \quad \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \left( \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \left( -\frac{1}{3+m} m(1+m) \text{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 2(1+m), 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} 2(1+m)^2 \text{AppellF1}\left[1+\frac{1+m}{2}, -m, 1+2(1+m), 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) / \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \right. \right. \\
& \quad \quad \left. \left. (1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2(1+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] + \right. \\
& \quad (3+m) \left( -\frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, 1-m, 2(1+m), 1 + \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right. \\
& \quad \left. \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] - \frac{1}{3+m} 2(1+m)^2 \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, -m, 1+2(1+m), 1 + \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) - 2 \tan \left[ \frac{1}{2} (a+bx) \right]^2 \\
& \quad \left( m \left( -\frac{1}{5+m} 2(1+m) (3+m) \operatorname{AppellF1} \left[ 1 + \frac{3+m}{2}, 1-m, 1+2(1+m), 1 + \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] + \frac{1}{5+m} (1-m) (3+m) \operatorname{AppellF1} \left[ 1 + \frac{3+m}{2}, 2-m, 2(1+m), 1 + \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) + 2(1+m) \left( -\frac{1}{5+m} m (3+m) \operatorname{AppellF1} \left[ 1 + \frac{3+m}{2}, 1-m, 3+2m, \right. \right. \\
& \quad \left. \left. 1 + \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] - \frac{1}{5+m} (3+m) (3+2m) \operatorname{AppellF1} \left[ \right. \right. \\
& \quad \left. \left. 1 + \frac{3+m}{2}, -m, 4+2m, 1 + \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \right) \Big/ \\
& \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. 1-m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \\
& \quad \left. \left. 2(1+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2 + \\
& \left( \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \left( -2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 3+2m, \right. \right. \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) + (3+2m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] + (3+m) \left( -\frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, 1-m, 3+2m, \right. \right. \\
& \quad \left. \left. 1 + \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] - \frac{1}{3+m} (1+m) (3+2m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, -m, 4+2m, 1 + \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \right) - \\
& \quad \left. 2 \tan \left[ \frac{1}{2} (a+bx) \right]^2 \left( m \left( -\frac{1}{5+m} (3+m) (3+2m) \operatorname{AppellF1} \left[ 1 + \frac{3+m}{2}, 1-m, 4+2m, 1 + \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right. \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] + \frac{1}{5+m} (1-m) (3+m) \operatorname{AppellF1} \left[ 1 + \frac{3+m}{2}, 2-m, 3+2m, 1 + \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \right) \Big/
\end{aligned}$$



$$\begin{aligned}
& -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] + (3+2m) \left( -\frac{1}{5+m} m(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2(2+m), \right. \right. \\
& \left. \left. 1+\frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m} 2(2+m)(3+m) \operatorname{AppellF1}\left[ \right. \right. \\
& \left. \left. 1+\frac{3+m}{2}, -m, 1+2(2+m), 1+\frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \right) \right) / \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. 1-m, 3+2m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \right)
\end{aligned}$$

**Problem 125: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin[a+bx] \sin[2a+2bx]^m dx$$

Optimal (type 5, 82 leaves, 2 steps):

$$\frac{1}{b(2+m)} \left( \cos[a+bx]^2 \right)^{\frac{1-m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \sin[a+bx]^2\right] \sin[a+bx] \sin[2a+2bx]^m \operatorname{Tan}[a+bx]$$

Result (type 5, 170 leaves):

$$\begin{aligned}
& \frac{1}{b(-1+4m^2)} 2^{-1-m} e^{-i(a+bx)} \left(1 - e^{4i(a+bx)}\right)^{-m} \left(-i e^{-2i(a+bx)} (-1 + e^{4i(a+bx)})\right)^m \\
& \left( (1-2m) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}(-1-2m), -m, \frac{1}{4}(3-2m), e^{4i(a+bx)}\right] + \right. \\
& \left. e^{2i(a+bx)} (1+2m) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}(1-2m), -m, \frac{1}{4}(5-2m), e^{4i(a+bx)}\right] \right)
\end{aligned}$$

**Problem 126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \csc[a+bx] \sin[2a+2bx]^m dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{\left( \cos[a+bx]^2 \right)^{\frac{1-m}{2}} \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{m}{2}, \frac{2+m}{2}, \sin[a+bx]^2\right] \operatorname{Sec}[a+bx] \sin[2a+2bx]^m}{bm}$$

Result (type 6, 1737 leaves):

$$\begin{aligned}
& \left( (2+m) \operatorname{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Csc}[a+bx] \operatorname{Sin}[2(a+bx)]^{2m}\right) / \\
& \left( b m \left( (2+m) \operatorname{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \operatorname{AppellF1}\left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \\
& \left( \left( 2(2+m) \operatorname{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Cos}[2(a+bx)] \operatorname{Sin}[2(a+bx)]^{-1+m} \right) / \right. \\
& \left( (2+m) \operatorname{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \quad 2m \left( \operatorname{AppellF1}\left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \left( (2+m) \operatorname{Sin}[2(a+bx)]^m \right. \\
& \quad \left. \left( -\frac{1}{2+m} m^2 \operatorname{AppellF1}\left[1+\frac{m}{2}, 1-m, 2m, 1+\frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \right. \right. \\
& \quad \left. \left. \frac{1}{2+m} 2m^2 \operatorname{AppellF1}\left[1+\frac{m}{2}, -m, 1+2m, 1+\frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) / \\
& \left( m \left( (2+m) \operatorname{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \operatorname{AppellF1}\left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( (2+m) \operatorname{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sin}[2(a+bx)]^m \right. \\
& \quad \left. \left( -2m \left( \operatorname{AppellF1}\left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \right. \\
& \quad \left. (2+m) \left( -\frac{1}{2+m} m^2 \operatorname{AppellF1}\left[1+\frac{m}{2}, 1-m, 2m, 1+\frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \right. \right. \\
& \quad \left. \left. \frac{1}{2+m} 2m^2 \operatorname{AppellF1}\left[1+\frac{m}{2}, -m, 1+2m, 1+\frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) - \\
& \quad 2m \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( -\frac{1}{4+m} 2m(2+m) \operatorname{AppellF1}\left[1+\frac{2+m}{2}, 1-m, 1+2m, 1+\frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \\
& \quad \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{4+m} (1-m)(2+m) \operatorname{AppellF1}\left[1+\frac{2+m}{2}, 2-m, 2m, 1+\frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + 2 \left( -\frac{1}{4+m} m(2+m) \operatorname{AppellF1}\left[1+\frac{2+m}{2}, 1-m, 1+2m, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. 1 + \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] - \frac{1}{4+m}(2+m)(1+2m) \\
& \operatorname{AppellF1}\left[1 + \frac{2+m}{2}, -m, 2+2m, 1 + \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \right) \right) \Bigg) \Bigg) / \\
& \left( m \left( (2+m) \operatorname{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \operatorname{AppellF1}\left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \right. \right. \\
& \left. \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \right) \right) \Bigg) \Bigg)
\end{aligned}$$

**Problem 127: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[a+bx]^2 \operatorname{Sin}[2a+2bx]^m dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$-\frac{1}{b(1-m)} \left( \operatorname{Cos}[a+bx]^2 \right)^{\frac{1+m}{2}} \operatorname{Csc}[a+bx] \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1}{2}(-1+m), \frac{1+m}{2}, \operatorname{Sin}[a+bx]^2\right] \operatorname{Sec}[a+bx] \operatorname{Sin}[2a+2bx]^m$$

Result (type 6, 4498 leaves):

$$\begin{aligned}
& \left( 2^{-1+m} \operatorname{Cot}\left[\frac{1}{2}(a+bx)\right] \operatorname{Csc}[a+bx]^2 \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] \left( -\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{3}{2}(a+bx)\right] \right) \right) \right)^m \\
& \operatorname{Sin}[2(a+bx)]^m \left( \left( (1+m)^2 \operatorname{AppellF1}\left[\frac{1}{2}(-1+m), -m, 2m, \frac{1+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \right) / \\
& \left( (-1+m) \left( (1+m) \operatorname{AppellF1}\left[\frac{1}{2}(-1+m), -m, 2m, \frac{1+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \right. \\
& \quad \left. \left. 2m \left( \operatorname{AppellF1}\left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \right. \right. \\
& \quad \left. \left. \left. 2 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \right) + \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) / \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 1+2m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) \Bigg) \Bigg) / \\
& \left( b(1+m) \left( -\frac{1}{1+m} 2^{-2+m} \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2 \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] \left( -\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{3}{2}(a+bx)\right] \right) \right) \right)^m \right. \\
& \left. \left( \left( (1+m)^2 \operatorname{AppellF1}\left[\frac{1}{2}(-1+m), -m, 2m, \frac{1+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \right) / \left( (-1+m) \left( (1+m) \operatorname{AppellF1}\left[\frac{1}{2}(-1+m), \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -m, 2m, \frac{1+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 - 2m \left( \text{AppellF1}\left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \Big) + \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) / \left( (3+m) \text{AppellF1}\left[ \right. \right. \\
& \left. \left. \frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \text{AppellF1}\left[\frac{3+m}{2}, -m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \Big) + \\
& \frac{1}{1+m} 2^{-1+m} m \cot\left[\frac{1}{2}(a+bx)\right] \left( \cos\left[\frac{1}{2}(a+bx)\right] \left( -\sin\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{3}{2}(a+bx)\right] \right) \right)^{-1+m} \\
& \left( \cos\left[\frac{1}{2}(a+bx)\right] \left( -\frac{1}{2} \cos\left[\frac{1}{2}(a+bx)\right] + \frac{3}{2} \cos\left[\frac{3}{2}(a+bx)\right] \right) - \frac{1}{2} \sin\left[\frac{1}{2}(a+bx)\right] \left( -\sin\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{3}{2}(a+bx)\right] \right) \right) \\
& \left( \left( (1+m)^2 \text{AppellF1}\left[\frac{1}{2}(-1+m), -m, 2m, \frac{1+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) / \left( (-1+m) \left( (1+m) \text{AppellF1}\left[\frac{1}{2}(-1+m), \right. \right. \right. \right. \\
& \left. \left. -m, 2m, \frac{1+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \text{AppellF1}\left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \Big) + \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) / \left( (3+m) \right. \\
& \left. \text{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \text{AppellF1}\left[\frac{3+m}{2}, -m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \Big) + \\
& \frac{1}{1+m} 2^{-1+m} \cot\left[\frac{1}{2}(a+bx)\right] \left( \cos\left[\frac{1}{2}(a+bx)\right] \left( -\sin\left[\frac{1}{2}(a+bx)\right] + \sin\left[\frac{3}{2}(a+bx)\right] \right) \right)^m \\
& \left( \left( (1+m)^2 \left( -\frac{1}{1+m} (-1+m) m \text{AppellF1}\left[1 + \frac{1}{2}(-1+m), 1-m, 2m, 1 + \frac{1+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{1+m} 2^{-1+m} (-1+m) m \text{AppellF1}\left[1 + \frac{1}{2}(-1+m), -m, 1+2m, 1 + \frac{1+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) / \left( (-1+m) \left( (1+m) \text{AppellF1}\left[\frac{1}{2}(-1+m), -m, 2m, \frac{1+m}{2}, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \text{AppellF1}\left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. 2 \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& 2m \left( \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right]\right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \left( (3+m) \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( -\frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} 2m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 1+2m, 1+\frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) / \left( (3+m) \right. \\
& \left. \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( (1+m)^2 \operatorname{AppellF1}\left[\frac{1}{2}(-1+m), -m, 2m, \frac{1+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left( -2m \left( \operatorname{AppellF1}\left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \right) \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + (1+m) \left( -\frac{1}{1+m} (-1+m) m \operatorname{AppellF1}\left[1+\frac{1}{2}(-1+m), 1-m, 2m, 1+\frac{1+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{1+m} 2(-1+m) m \operatorname{AppellF1}\left[ \right. \right. \\
& \left. \left. 1+\frac{1}{2}(-1+m), -m, 1+2m, 1+\frac{1+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) - \\
& 2m \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( -\frac{1}{3+m} 2m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 1+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{3+m} (1-m) (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 2-m, 2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + 2 \left( -\frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 1+2m, 1+\frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} (1+m) (1+2m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, \right. \right. \\
& \left. \left. -m, 2+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) \right) / \\
& \left( (-1+m) \left( (1+m) \operatorname{AppellF1}\left[\frac{1}{2}(-1+m), -m, 2m, \frac{1+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \right. \\
& \left. \left. 2m \left( \operatorname{AppellF1}\left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \tan\left[\frac{1}{2}(a+bx)\right]^2 \Big)^2 - \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \tan\left[\frac{1}{2}(a+bx)\right]^2 \right. \\
& \left. \left( -2m \left( \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 1+2m, \frac{5+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + (3+m) \left( -\frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. 1-m, 2m, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} 2m (1+m) \right. \right. \right. \\
& \left. \left. \left. \operatorname{AppellF1}\left[1 + \frac{1+m}{2}, -m, 1+2m, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) - \right. \\
& \left. 2m \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( -\frac{1}{5+m} 2m (3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, 1+2m, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{5+m} (1-m) (3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 2-m, 2m, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + 2 \left( -\frac{1}{5+m} m (3+m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, 1+2m, 1 + \frac{5+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m} (3+m) (1+2m) \operatorname{AppellF1}\left[1 + \frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. \left. -m, 2+2m, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) \right) \Big) \Big) \Big) \Big) \Big) / \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2m \left( \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, \right. \right. \right. \right. \\
& \left. \left. \left. 2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. \left. 2 \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \right) \Big) \Big) \Big) \Big) \Big)
\end{aligned}$$

**Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \csc[a+bx]^3 \sin[2a+2bx]^m dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$-\frac{1}{b(2-m)} (\cos[a+bx]^2)^{\frac{1-m}{2}} \csc[a+bx]^2 \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1}{2}(-2+m), \frac{m}{2}, \sin[a+bx]^2\right] \operatorname{Sec}[a+bx] \sin[2a+2bx]^m$$

Result (type 6, 5872 leaves):



$$\begin{aligned}
& \left( (4+m) \operatorname{AppellF1} \left[ \frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
& \left( (2+m) \left( (4+m) \operatorname{AppellF1} \left[ \frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \right. \\
& \quad 2m \left( \operatorname{AppellF1} \left[ \frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1} \left[ \frac{4+m}{2}, -m, 1+2m, \frac{6+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) \\
& \left( \frac{\frac{1}{2} \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 - \frac{3}{2} \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right]^2}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2} - \frac{2 \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right]^2 \left( \tan \left[ \frac{1}{2} (a+bx) \right] - \tan \left[ \frac{1}{2} (a+bx) \right]^3 \right)}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^3} \right) + \\
& 4^{-1+m} \left( \frac{\tan \left[ \frac{1}{2} (a+bx) \right] - \tan \left[ \frac{1}{2} (a+bx) \right]^3}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2} \right)^m \\
& \left( \left( \operatorname{AppellF1} \left[ \frac{1}{2} (-2+m), -m, 2m, \frac{m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \cot \left[ \frac{1}{2} (a+bx) \right] \operatorname{Csc} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \left( (-2+m) \right. \right. \\
& \quad \left( -\operatorname{AppellF1} \left[ \frac{1}{2} (-2+m), -m, 2m, \frac{m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + 2 \left( \operatorname{AppellF1} \left[ \frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) + 2 \operatorname{AppellF1} \left[ \frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) - \\
& \left( \cot \left[ \frac{1}{2} (a+bx) \right]^2 \left( -(-2+m) \operatorname{AppellF1} \left[ 1 + \frac{1}{2} (-2+m), 1-m, 2m, 1 + \frac{m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] - 2(-2+m) \operatorname{AppellF1} \left[ 1 + \frac{1}{2} (-2+m), -m, 1+2m, 1 + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \right) / \left( (-2+m) \right. \\
& \quad \left( -\operatorname{AppellF1} \left[ \frac{1}{2} (-2+m), -m, 2m, \frac{m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + 2 \left( \operatorname{AppellF1} \left[ \frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) + 2 \operatorname{AppellF1} \left[ \frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) + \\
& \left( 2(2+m) \left( -\frac{1}{2+m} m^2 \operatorname{AppellF1} \left[ 1 + \frac{m}{2}, 1-m, 2m, 1 + \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] - \right. \right. \\
& \quad \left. \left. \frac{1}{2+m} 2m^2 \operatorname{AppellF1} \left[ 1 + \frac{m}{2}, -m, 1+2m, 1 + \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \right) / \\
& \left( m \left( (2+m) \operatorname{AppellF1} \left[ \frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2m \left( \operatorname{AppellF1} \left[ \frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& 2 \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \tan\left[\frac{1}{2}(a+bx)\right]^2 \Bigg) + \\
& \left( (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \Bigg) / \\
& \left( (2+m) \left( (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \right. \\
& \quad 2m \left( \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 1+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \Bigg) + \\
& \left( (4+m) \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( -\frac{1}{4+m} m (2+m) \operatorname{AppellF1}\left[1+\frac{2+m}{2}, 1-m, 2m, 1+\frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{4+m} \right. \right. \\
& \quad \left. \left. 2m (2+m) \operatorname{AppellF1}\left[1+\frac{2+m}{2}, -m, 1+2m, 1+\frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) \Bigg) / \\
& \left( (2+m) \left( (4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \right. \\
& \quad 2m \left( \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \quad \left. \left. 2 \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 1+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \Bigg) + \\
& \left( \operatorname{AppellF1}\left[\frac{1}{2}(-2+m), -m, 2m, \frac{m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \cot\left[\frac{1}{2}(a+bx)\right]^2 \right. \\
& \quad \left( (-2+m) \operatorname{AppellF1}\left[1+\frac{1}{2}(-2+m), 1-m, 2m, 1+\frac{m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \right. \\
& \quad \left. 2(-2+m) \operatorname{AppellF1}\left[1+\frac{1}{2}(-2+m), -m, 1+2m, 1+\frac{m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \right. \\
& \quad \left. 2 \left( \operatorname{AppellF1}\left[\frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + 2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( -\frac{1}{2+m} 2m^2 \operatorname{AppellF1}\left[1+\frac{m}{2}, 1-m, 1+2m, \right. \right. \right. \\
& \quad \left. \left. 1+\frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{2+m} (1-m) m \operatorname{AppellF1}\left[1+\frac{m}{2}, 2-m, \right. \right. \\
& \quad \left. \left. 2m, 1+\frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + 2 \left( -\frac{1}{2+m} m^2 \operatorname{AppellF1}\left[1+\frac{m}{2}, \right. \right. \right. \\
& \quad \left. \left. 1-m, 1+2m, 1+\frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{2+m} m (1+2m) \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[1+\frac{m}{2}, -m, 2+2m, 1+\frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) \Bigg) \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left( (-2+m) \left( -\text{AppellF1} \left[ \frac{1}{2} (-2+m), -m, 2m, \frac{m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + 2 \left( \text{AppellF1} \left[ \frac{m}{2}, 1-m, 2m, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + 2 \text{AppellF1} \left[ \frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \right. \\
& \quad \left. \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2 - \left( 2(2+m) \text{AppellF1} \left[ \frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right. \\
& \quad \left. (-2m \left( \text{AppellF1} \left[ \frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + 2 \text{AppellF1} \left[ \frac{2+m}{2}, -m, 1+2m, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] + \right. \\
& \quad (2+m) \left( -\frac{1}{2+m} m^2 \text{AppellF1} \left[ 1 + \frac{m}{2}, 1-m, 2m, 1 + \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] - \right. \\
& \quad \left. \frac{1}{2+m} 2m^2 \text{AppellF1} \left[ 1 + \frac{m}{2}, -m, 1+2m, 1 + \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) - \\
& \quad 2m \tan \left[ \frac{1}{2} (a+bx) \right]^2 \left( -\frac{1}{4+m} 2m(2+m) \text{AppellF1} \left[ 1 + \frac{2+m}{2}, 1-m, 1+2m, 1 + \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right. \\
& \quad \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] + \frac{1}{4+m} (1-m)(2+m) \text{AppellF1} \left[ 1 + \frac{2+m}{2}, 2-m, 2m, 1 + \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, \right. \\
& \quad \left. -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] + 2 \left( -\frac{1}{4+m} m(2+m) \text{AppellF1} \left[ 1 + \frac{2+m}{2}, 1-m, 1+2m, 1 + \frac{4+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] - \frac{1}{4+m} (2+m)(1+2m) \text{AppellF1} \left[ 1 + \frac{2+m}{2}, \right. \right. \\
& \quad \left. \left. -m, 2+2m, 1 + \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \right) \right) \Big/ \\
& \left( m \left( (2+m) \text{AppellF1} \left[ \frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - 2m \left( \text{AppellF1} \left[ \frac{2+m}{2}, 1-m, \right. \right. \right. \right. \\
& \quad \left. \left. \left. 2m, \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. 2 \text{AppellF1} \left[ \frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2 - \\
& \left( (4+m) \text{AppellF1} \left[ \frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right. \\
& \quad \left. (-2m \left( \text{AppellF1} \left[ \frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + 2 \text{AppellF1} \left[ \frac{4+m}{2}, -m, 1+2m, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{6+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] + (4+m) \left( -\frac{1}{4+m} m(2+m) \right. \right. \\
& \quad \left. \left. \text{AppellF1} \left[ 1 + \frac{2+m}{2}, 1-m, 2m, 1 + \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] - \frac{1}{4+m} 2m \right. \right. \\
& \quad \left. \left. (2+m) \text{AppellF1} \left[ 1 + \frac{2+m}{2}, -m, 1+2m, 1 + \frac{4+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& 2 m \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2 \left(-\frac{1}{6+m} 2 m(4+m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 1-m, 1+2 m, 1+\frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2\right]\right. \\
& \quad \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right] + \frac{1}{6+m}(1-m)(4+m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 2-m, 2 m, 1+\frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2, \right. \\
& \quad \left. -\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right] + 2\left(-\frac{1}{6+m} m(4+m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, 1-m, 1+2 m, 1+\frac{6+m}{2}, \right. \right. \\
& \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right] - \frac{1}{6+m}(4+m)(1+2 m) \operatorname{AppellF1}\left[1+\frac{4+m}{2}, \right. \right. \\
& \quad \left. \left. -m, 2+2 m, 1+\frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right) / \\
& \left((2+m)\left((4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2 m, \frac{4+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2\right] - 2 m\right.\right. \\
& \quad \left.\left(\operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 2 m, \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2\right] + \right.\right. \\
& \quad \left.\left. 2 \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 1+2 m, \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]^2\right)\right)
\end{aligned}$$

**Problem 136: Result more than twice size of optimal antiderivative.**

$$\int \cos [a+b x] \operatorname{Csc}[2 a+2 b x] d x$$

Optimal (type 3, 14 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\cos [a+b x]]}{2 b}$$

Result (type 3, 42 leaves):

$$\frac{1}{2} \left( -\frac{\operatorname{Log}\left[\cos \left[\frac{a}{2}+\frac{b x}{2}\right]\right]}{b} + \frac{\operatorname{Log}\left[\sin \left[\frac{a}{2}+\frac{b x}{2}\right]\right]}{b} \right)$$

**Problem 137: Result more than twice size of optimal antiderivative.**

$$\int \cos [a+b x] \operatorname{Csc}[2 a+2 b x]^2 d x$$

Optimal (type 3, 28 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin [a+b x]]}{4 b} - \frac{\operatorname{Csc}[a+b x]}{4 b}$$

Result (type 3, 94 leaves):

$$\frac{1}{4} \left( -\frac{\operatorname{Cot}\left[\frac{1}{2}(a+bx)\right]}{2b} - \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{b} + \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{b} - \frac{\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{2b} \right)$$

Problem 138: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cos}[a+bx] \operatorname{Csc}[2a+2bx]^3 dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{3 \operatorname{ArcTanh}[\operatorname{Cos}[a+bx]]}{16b} + \frac{3 \operatorname{Sec}[a+bx]}{16b} - \frac{\operatorname{Csc}[a+bx]^2 \operatorname{Sec}[a+bx]}{16b}$$

Result (type 3, 143 leaves):

$$\left( \operatorname{Csc}[a+bx]^4 \left( 2 - 6 \operatorname{Cos}[2(a+bx)] + 2 \operatorname{Cos}[3(a+bx)] + 3 \operatorname{Cos}[3(a+bx)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right] - 3 \operatorname{Cos}[3(a+bx)] \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right] \right) + \operatorname{Cos}[a+bx] \left( -2 - 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right] + 3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right] \right) \right) / \left( 16b \left( \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2 - \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right) \right)$$

Problem 139: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cos}[a+bx] \operatorname{Csc}[2a+2bx]^4 dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$\frac{5 \operatorname{ArcTanh}[\operatorname{Sin}[a+bx]]}{32b} - \frac{5 \operatorname{Csc}[a+bx]}{32b} - \frac{5 \operatorname{Csc}[a+bx]^3}{96b} + \frac{\operatorname{Csc}[a+bx]^3 \operatorname{Sec}[a+bx]^2}{32b}$$

Result (type 3, 215 leaves):

$$\frac{13 \operatorname{Cot}\left[\frac{1}{2}(a+bx)\right]}{192b} - \frac{\operatorname{Cot}\left[\frac{1}{2}(a+bx)\right] \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{384b} - \frac{5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{32b} + \frac{5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{32b} + \frac{1}{64b \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)^2} - \frac{1}{64b \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)^2} - \frac{13 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{192b} - \frac{\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{384b}$$

### Problem 140: Result more than twice size of optimal antiderivative.

$$\int \cos [a + b x] \operatorname{Csc} [2 a + 2 b x]^5 dx$$

Optimal (type 3, 89 leaves, 7 steps):

$$-\frac{35 \operatorname{ArcTanh} [\cos [a + b x]]}{256 b} + \frac{35 \operatorname{Sec} [a + b x]}{256 b} + \frac{35 \operatorname{Sec} [a + b x]^3}{768 b} - \frac{7 \operatorname{Csc} [a + b x]^2 \operatorname{Sec} [a + b x]^3}{256 b} - \frac{\operatorname{Csc} [a + b x]^4 \operatorname{Sec} [a + b x]^3}{128 b}$$

Result (type 3, 268 leaves):

$$\begin{aligned} & -\frac{1}{768 b \left( \operatorname{Csc} \left[ \frac{1}{2} (a + b x) \right]^2 - \operatorname{Sec} \left[ \frac{1}{2} (a + b x) \right]^2 \right)^3} \\ & \operatorname{Csc} [a + b x]^{10} \left( -204 + 658 \cos [2 (a + b x)] - 228 \cos [3 (a + b x)] + 140 \cos [4 (a + b x)] - 76 \cos [5 (a + b x)] - 210 \cos [6 (a + b x)] + \right. \\ & \quad 76 \cos [7 (a + b x)] - 315 \cos [3 (a + b x)] \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] \right] - 105 \cos [5 (a + b x)] \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] \right] + \\ & \quad 105 \cos [7 (a + b x)] \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] \right] + 3 \cos [a + b x] \left( 76 + 105 \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] \right] - 105 \operatorname{Log} \left[ \sin \left[ \frac{1}{2} (a + b x) \right] \right] \right) + \\ & \quad \left. 315 \cos [3 (a + b x)] \operatorname{Log} \left[ \sin \left[ \frac{1}{2} (a + b x) \right] \right] + 105 \cos [5 (a + b x)] \operatorname{Log} \left[ \sin \left[ \frac{1}{2} (a + b x) \right] \right] - 105 \cos [7 (a + b x)] \operatorname{Log} \left[ \sin \left[ \frac{1}{2} (a + b x) \right] \right] \right) \end{aligned}$$

### Problem 158: Result more than twice size of optimal antiderivative.

$$\int \cos [a + b x]^3 \operatorname{Csc} [2 a + 2 b x]^3 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh} [\cos [a + b x]]}{16 b} - \frac{\operatorname{Cot} [a + b x] \operatorname{Csc} [a + b x]}{16 b}$$

Result (type 3, 79 leaves):

$$\frac{1}{8} \left( -\frac{\operatorname{Csc} \left[ \frac{1}{2} (a + b x) \right]^2}{8 b} - \frac{\operatorname{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] \right]}{2 b} + \frac{\operatorname{Log} \left[ \sin \left[ \frac{1}{2} (a + b x) \right] \right]}{2 b} + \frac{\operatorname{Sec} \left[ \frac{1}{2} (a + b x) \right]^2}{8 b} \right)$$

### Problem 159: Result more than twice size of optimal antiderivative.

$$\int \cos [a + b x]^3 \operatorname{Csc} [2 a + 2 b x]^4 dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{16 b} - \frac{\text{Csc}[a + b x]}{16 b} - \frac{\text{Csc}[a + b x]^3}{48 b}$$

Result (type 3, 152 leaves):

$$\frac{1}{16} \left( -\frac{7 \text{Cot}\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\text{Cot}\left[\frac{1}{2}(a + b x)\right] \text{Csc}\left[\frac{1}{2}(a + b x)\right]^2}{24 b} - \frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] - \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right]}{b} + \frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right] + \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right]}{b} - \frac{7 \text{Tan}\left[\frac{1}{2}(a + b x)\right]}{12 b} - \frac{\text{Sec}\left[\frac{1}{2}(a + b x)\right]^2 \text{Tan}\left[\frac{1}{2}(a + b x)\right]}{24 b} \right)$$

**Problem 160: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[a + b x]^3 \text{Csc}[2 a + 2 b x]^5 dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$-\frac{15 \text{ArcTanh}[\text{Cos}[a + b x]]}{256 b} + \frac{15 \text{Sec}[a + b x]}{256 b} - \frac{5 \text{Csc}[a + b x]^2 \text{Sec}[a + b x]}{256 b} - \frac{\text{Csc}[a + b x]^4 \text{Sec}[a + b x]}{128 b}$$

Result (type 3, 195 leaves):

$$-\frac{7 \text{Csc}\left[\frac{1}{2}(a + b x)\right]^2}{1024 b} - \frac{\text{Csc}\left[\frac{1}{2}(a + b x)\right]^4}{2048 b} - \frac{15 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(a + b x)\right]\right]}{256 b} + \frac{15 \text{Log}\left[\text{Sin}\left[\frac{1}{2}(a + b x)\right]\right]}{256 b} + \frac{7 \text{Sec}\left[\frac{1}{2}(a + b x)\right]^2}{1024 b} + \frac{\text{Sec}\left[\frac{1}{2}(a + b x)\right]^4}{2048 b} + \frac{\text{Sin}\left[\frac{1}{2}(a + b x)\right]}{32 b \left(\text{Cos}\left[\frac{1}{2}(a + b x)\right] - \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right)} - \frac{\text{Sin}\left[\frac{1}{2}(a + b x)\right]}{32 b \left(\text{Cos}\left[\frac{1}{2}(a + b x)\right] + \text{Sin}\left[\frac{1}{2}(a + b x)\right]\right)}$$

**Problem 187: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[a + b x]^3 \text{Sin}[2 a + 2 b x]^m dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$-\frac{1}{b(4+m)} \text{Cos}[a + b x]^3 \text{Cot}[a + b x] \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \text{Cos}[a + b x]^2\right] (\text{Sin}[a + b x]^2)^{\frac{1-m}{2}} \text{Sin}[2 a + 2 b x]^m$$

Result (type 6, 10498 leaves):

$$\begin{aligned}
& - \left( \left( 2^{1+2m} (3+m) \cos[a+bx]^3 \sin[2(a+bx)]^m \tan\left[\frac{1}{2}(a+bx)\right] \left( \frac{\tan\left[\frac{1}{2}(a+bx)\right] - \tan\left[\frac{1}{2}(a+bx)\right]^3}{(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2)^2} \right)^m \right. \right. \\
& \left( \left( \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^3 \right) / \right. \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \left. 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. (1+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \left( 12 \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right) \right) / \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \left. 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. (3+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( 6 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^2 \right) / \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \left. 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( 8 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) / \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \left. 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. 2(2+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 5+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \Big) / \\
& \left( b(1+m) \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^4 \left( \frac{1}{(1+m) \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^5} 2^{3+2m} (3+m) \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{\tan\left[\frac{1}{2}(a+bx)\right] - \tan\left[\frac{1}{2}(a+bx)\right]^3}{\left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^2} \right)^m \left( \left( \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^3 \right) / \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \right. \\
& \left. \left. 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. (1+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \right. \\
& \left. \left( 12 \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right) \right) / \right. \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \left. \left. 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. (3+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \right. \\
& \left. \left( 6 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right) \right) / \right. \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& \left. \left. 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \right. \\
& \left. \left( 8 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) / \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. \left. 2(2+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 5+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \right) - \\
& \frac{1}{(1+m) \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^4} 2^{2m} (3+m) \sec\left[\frac{1}{2}(a+bx)\right]^2 \left( \frac{\tan\left[\frac{1}{2}(a+bx)\right] - \tan\left[\frac{1}{2}(a+bx)\right]^3}{\left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^2} \right)^m \\
& \left( \left( \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^3 \right) / \right. \\
& \left. \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \right.
\end{aligned}$$



$$\begin{aligned}
& 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \\
& \quad \left. (1+2m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 + \\
& \left( 12 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) / \\
& \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \\
& \quad 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \\
& \quad \left. (3+2m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 - \\
& \left( 6 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) / \\
& \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \\
& \quad 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \\
& \quad \left. 2(1+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 - \\
& \left( 8 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) / \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2(2+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 5+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) - \\
& \frac{1}{(1+m) \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^4} 2^{1+2m} m (3+m) \tan \left[ \frac{1}{2} (a+bx) \right] \left( \frac{\tan \left[ \frac{1}{2} (a+bx) \right] - \tan \left[ \frac{1}{2} (a+bx) \right]^3}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2} \right)^{-1+m} \\
& \left( \left( \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^3 \right) \right) / \\
& \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \\
& \quad 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \\
& \quad \left. (1+2m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 + \\
& \left( 12 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( 6 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left( 1 + \tan\left[\frac{1}{2}(a+bx)\right]^2 \right)^2 \right) / \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \left. 2(1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( 8 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) / \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2(2+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 5+2m, \frac{5+m}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \left( \frac{\frac{1}{2} \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 - \frac{3}{2} \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]^2}{\left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^2} - \right. \\
& \left. \frac{2 \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \left(\tan\left[\frac{1}{2}(a+bx)\right] - \tan\left[\frac{1}{2}(a+bx)\right]^3\right)}{\left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^3} \right) - \frac{1}{(1+m) \left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^4} \\
& 2^{1+2m} (3+m) \tan\left[\frac{1}{2}(a+bx)\right] \left( \frac{\tan\left[\frac{1}{2}(a+bx)\right] - \tan\left[\frac{1}{2}(a+bx)\right]^3}{\left(1 + \tan\left[\frac{1}{2}(a+bx)\right]^2\right)^2} \right)^m \\
& \left( \left( 3 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right. \right. \\
& \left. \left. \left( 1 + \tan\left[\frac{1}{2}(a+bx)\right]^2 \right)^2 \right) \right) / \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \\
& 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \left. \left. (1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) +
\end{aligned}$$



$$\begin{aligned}
& \left. \left( \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] \right) \left( 1 + \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right)^2 \right) / \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \right. \right. \\
& \left. \left. \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) - \\
& \left( 8 \left( -\frac{1}{3+m} m(1+m) \text{AppellF1}\left[1 + \frac{1+m}{2}, 1-m, 2(2+m), 1 + \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} 2(1+m)(2+m) \text{AppellF1}\left[1 + \frac{1+m}{2}, -m, 1+2(2+m), 1 + \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] \right) \right) / \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \\
& \left. \left. -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \left. \left. 2(2+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 5+2m, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right) + \\
& \left( 6 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \left( 1 + \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \right)^2 \right. \\
& \left. \left( -2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \right. \\
& \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] + \right. \\
& \left. (3+m) \left( -\frac{1}{3+m} m(1+m) \text{AppellF1}\left[1 + \frac{1+m}{2}, 1-m, 2(1+m), 1 + \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right. \right. \\
& \left. \left. \text{Tan}\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} 2(1+m)^2 \text{AppellF1}\left[1 + \frac{1+m}{2}, -m, 1+2(1+m), 1 + \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \right. \\
& \left. \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] \right) - 2 \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2 \left( m \left( -\frac{1}{5+m} 2(1+m)(3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, 1+2(1+m), \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] + \frac{1}{5+m} (1-m)(3+m) \text{AppellF1}\left[ \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{3+m}{2}, 2-m, 2(1+m), 1 + \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] \right) \right) + \\
& \left. 2(1+m) \left( -\frac{1}{5+m} m(3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, 3+2m, 1 + \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \left. \left. \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m} (3+m)(3+2m) \text{AppellF1}\left[1 + \frac{3+m}{2}, -m, 4+2m, \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{5+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \text{Tan}\left[\frac{1}{2}(a+bx)\right] \right) \right) \right) / \\
& \left( (3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \text{Tan}\left[\frac{1}{2}(a+bx)\right]^2, -\text{Tan}\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \text{AppellF1}\left[\frac{3+m}{2}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 1 - m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \Big] + \\
& 2(1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \tan\left[\frac{1}{2}(a+bx)\right]^2 \Big)^2 + \\
& \left(8 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(-2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, \right.\right.\right.\right. \\
& \left.\left.\left.2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2(2+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 5+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right.\right.\right. \\
& \left.\left.\left.-\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + (3+m) \left(-\frac{1}{3+m} m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 2(2+m), \right.\right.\right. \right. \\
& \left.\left.\left.1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} 2(1+m)(2+m) \operatorname{AppellF1}\left[\right.\right. \right. \\
& \left.\left.\left.1+\frac{1+m}{2}, -m, 1+2(2+m), 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right] \right) - \\
& 2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \left(m \left(-\frac{1}{5+m} 2(2+m)(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 1+2(2+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right.\right.\right. \right. \\
& \left.\left.\left.-\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{5+m} (1-m)(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 2-m, \right.\right. \right. \\
& \left.\left.\left.2(2+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right] \right) + \\
& 2(2+m) \left(-\frac{1}{5+m} m(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 5+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m} (3+m)(5+2m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, -m, 6+2m, \right.\right. \\
& \left.\left.\left.1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right] \right) \Big) \Big) \Big) \Big) / \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, \right.\right.\right. \right. \\
& \left.\left.\left.1-m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right.\right. \\
& \left.\left.2(2+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 5+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \Big)^2 - \\
& \left(\operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(1+\tan\left[\frac{1}{2}(a+bx)\right]^2\right)^3 \right. \\
& \left. \left(-2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right.\right.\right. \right. \\
& \left.\left.\left.(1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \right.\right. \\
& \left.\left.\left.(3+m) \left(-\frac{1}{3+m} m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 1+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \right.\right.\right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m}(1+m)(1+2m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 2+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \\
& \left. \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) - 2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( m \left( -\frac{1}{5+m}(3+m)(1+2m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2+2m, \right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{5+m}(1-m)(3+m) \operatorname{AppellF1}\left[ \right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{3+m}{2}, 2-m, 1+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) + \\
& (1+2m) \left( -\frac{1}{5+m}m(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2(1+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \\
& \quad \left. \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m}2(1+m)(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, -m, 1+2(1+m), \right. \right. \\
& \quad \left. \left. 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) \Bigg/ \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \\
& \quad \left. \left. (1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right)^2 - \\
& \left( 12 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left( 1 + \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \right. \\
& \quad \left. \left( -2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \right. \right. \\
& \quad \left. \left. (3+m) \left( -\frac{1}{3+m}m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 3+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m}(1+m)(3+2m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 4+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) - 2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( m \left( -\frac{1}{5+m}(3+m)(3+2m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 4+2m, \right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{5+m}(1-m)(3+m) \operatorname{AppellF1}\left[ \right. \right. \right. \\
& \quad \left. \left. \left. 1+\frac{3+m}{2}, 2-m, 3+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) + \\
& (3+2m) \left( -\frac{1}{5+m}m(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2(2+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \\
& \quad \left. \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m}2(2+m)(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, -m, 1+2(2+m), \right. \right.
\end{aligned}$$

$$\left( \left( \left( \left( \left( \left( \left( \left( 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) \right) \right) \right) \right) \right) \right) \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \right) \right) \right)$$

**Problem 188: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[a+bx]^2 \sin[2a+2bx]^m dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$-\frac{1}{b(3+m)} \cos[a+bx]^2 \cot[a+bx] \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos[a+bx]^2\right] (\sin[a+bx]^2)^{\frac{1-m}{2}} \sin[2a+2bx]^m$$

Result (type 6, 7926 leaves):

$$\left( 2^{1+2m} (3+m) \cos[a+bx]^2 \sin[2(a+bx)]^m \tan\left[\frac{1}{2}(a+bx)\right] \left( \frac{\tan\left[\frac{1}{2}(a+bx)\right] - \tan\left[\frac{1}{2}(a+bx)\right]^3}{(1+\tan\left[\frac{1}{2}(a+bx)\right]^2)^2} \right)^m \right) \left( \left( \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \left(1+\tan\left[\frac{1}{2}(a+bx)\right]^2\right)^2 \right) \right) \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + (1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) + \left( 4 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) -$$

$$\begin{aligned}
& \left( 4 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) / \\
& \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \\
& \quad \left. 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2(1+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \Big) / \\
& \left( b(1+m) \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^3 \left( -\frac{1}{(1+m) \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^4} 3 \times 2^{1+2m} (3+m) \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right. \right. \\
& \quad \left. \left. \left( \frac{\tan \left[ \frac{1}{2} (a+bx) \right] - \tan \left[ \frac{1}{2} (a+bx) \right]^3}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2} \right)^m \left( \left( \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right. \right. \right. \right. \\
& \quad \left. \left. \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2 \right) \right) / \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. (1+2m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) + \\
& \quad \left( 4 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) / \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (3+2m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) - \\
& \quad \left( 4 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) / \\
& \quad \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \\
& \quad \left. 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2(1+m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \Big) + \\
& \frac{1}{(1+m) \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^3} 2^{2m} (3+m) \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \left( \frac{\tan \left[ \frac{1}{2} (a+bx) \right] - \tan \left[ \frac{1}{2} (a+bx) \right]^3}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2} \right)^m
\end{aligned}$$





$$\begin{aligned}
& \left( 4 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) / \\
& \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \\
& \quad 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + 2(1+m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \\
& \left. \left( \frac{\frac{1}{2} \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 - \frac{3}{2} \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right]^2}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2} - \frac{2 \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \left( \tan \left[ \frac{1}{2} (a+bx) \right] - \tan \left[ \frac{1}{2} (a+bx) \right]^3 \right)}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^3} \right) \right) + \\
& \frac{1}{(1+m) \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^3} 2^{1+2m} (3+m) \tan \left[ \frac{1}{2} (a+bx) \right] \left( \frac{\tan \left[ \frac{1}{2} (a+bx) \right] - \tan \left[ \frac{1}{2} (a+bx) \right]^3}{\left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2} \right)^m \\
& \left( \left( 2 \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right. \right. \\
& \quad \left. \left. \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) \right) \right) / \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right. \\
& \quad 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + (1+2m) \right. \\
& \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) + \\
& \left( \left( -\frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, 1-m, 1+2m, 1 + \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (a+bx) \right] - \frac{1}{3+m} (1+m) (1+2m) \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, -m, 2+2m, 1 + \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \right. \\
& \quad \left. \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \left( 1 + \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right)^2 / \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - 2 \left( m \operatorname{AppellF1} \left[ \frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1+2m) \operatorname{AppellF1} \left[ \frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \tan \left[ \frac{1}{2} (a+bx) \right]^2 \right) + \\
& \left( 4 \left( -\frac{1}{3+m} m (1+m) \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, 1-m, 3+2m, 1 + \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right. \right. \\
& \quad \left. \left. \tan \left[ \frac{1}{2} (a+bx) \right] - \frac{1}{3+m} (1+m) (3+2m) \operatorname{AppellF1} \left[ 1 + \frac{1+m}{2}, -m, 4+2m, 1 + \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] \right) \right. \\
& \quad \left. \left. \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \tan \left[ \frac{1}{2} (a+bx) \right] \right) \right) / \left( (3+m) \operatorname{AppellF1} \left[ \frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[ \frac{1}{2} (a+bx) \right]^2, -\tan \left[ \frac{1}{2} (a+bx) \right]^2 \right] - \right.
\end{aligned}$$



$$\begin{aligned}
& -\tan\left[\frac{1}{2}(a+bx)\right]^2 \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + 2(1+m) \left( -\frac{1}{5+m} m(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 3+2m, \right. \right. \\
& \left. \left. 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m} (3+m)(3+2m) \operatorname{AppellF1}\left[ \right. \right. \\
& \left. \left. 1+\frac{3+m}{2}, -m, 4+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) \Big/ \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] - 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] + \right. \right. \\
& \left. \left. 2(1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right)^2 - \\
& \left( \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \left( 1+\tan\left[\frac{1}{2}(a+bx)\right]^2 \right)^2 \right. \\
& \left. \left( -2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] + (1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right) \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + (3+m) \left( -\frac{1}{3+m} m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, \right. \right. \right. \\
& \left. \left. 1-m, 1+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m} (1+m)(1+2m) \right. \right. \\
& \left. \left. \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 2+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) - \right. \right. \\
& \left. \left. 2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \left( m \left( -\frac{1}{5+m} (3+m)(1+2m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \right. \right. \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{5+m} (1-m)(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 2-m, 1+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) + (1+2m) \left( -\frac{1}{5+m} m(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2(1+m), \right. \right. \right. \\
& \left. \left. 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m} 2(1+m)(3+m) \operatorname{AppellF1}\left[ \right. \right. \\
& \left. \left. 1+\frac{3+m}{2}, -m, 1+2(1+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] \right) \right) \right) \Big/ \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] - 2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] + \right. \right. \\
& \left. \left. (1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right)^2 - \\
& \left( 4 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 \right] \left( -2 \left( m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2 + (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \\
& \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + (3+m) \left(-\frac{1}{3+m}m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 3+2m, \right. \right. \\
& \left. \left. 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{3+m}(1+m)(3+2m) \right. \\
& \left. \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 4+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right) - \\
& 2 \tan\left[\frac{1}{2}(a+bx)\right]^2 \left(m \left(-\frac{1}{5+m}(3+m)(3+2m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 4+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] + \frac{1}{5+m}(1-m)(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 2-m, 3+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, \right. \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right) + (3+2m) \left(-\frac{1}{5+m}m(3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2(2+m), \right. \right. \\
& \left. \left. 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right] - \frac{1}{5+m}2(2+m)(3+m) \operatorname{AppellF1}\left[ \right. \right. \\
& \left. \left. 1+\frac{3+m}{2}, -m, 1+2(2+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \sec\left[\frac{1}{2}(a+bx)\right]^2 \tan\left[\frac{1}{2}(a+bx)\right]\right) \right) \Bigg) / \\
& \left( (3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, \right. \right. \right. \\
& \left. \left. 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + \right. \\
& \left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \tan\left[\frac{1}{2}(a+bx)\right]^2\right) \right) \right)
\end{aligned}$$

**Problem 189: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[a+bx] \sin[2a+2bx]^m dx$$

Optimal (type 5, 83 leaves, 2 steps):

$$-\frac{1}{b(2+m)} \cos[a+bx] \cot[a+bx] \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[a+bx]^2\right] (\sin[a+bx]^2)^{\frac{1-m}{2}} \sin[2a+2bx]^m$$

Result (type 5, 173 leaves):

$$\frac{1}{b(-1+4m^2)} i 2^{-1-m} e^{-i(a+bx)} \left(1 - e^{4i(a+bx)}\right)^{-m} \left(-i e^{-2i(a+bx)} (-1 + e^{4i(a+bx)})\right)^m$$

$$\left( (-1+2m) \text{Hypergeometric2F1}\left[\frac{1}{4}(-1-2m), -m, \frac{1}{4}(3-2m), e^{4i(a+bx)}\right] + \right.$$

$$\left. e^{2i(a+bx)} (1+2m) \text{Hypergeometric2F1}\left[\frac{1}{4}(1-2m), -m, \frac{1}{4}(5-2m), e^{4i(a+bx)}\right] \right)$$

**Problem 196: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Csc}[c+bx]^2 \text{Sin}[a+bx] dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\text{Cos}[c+bx]] \text{Cos}[a-c]}{b} - \frac{\text{Csc}[c+bx] \text{Sin}[a-c]}{b}$$

Result (type 3, 90 leaves):

$$\frac{2i \text{ArcTan}\left[\frac{\text{Cos}[c]-i \text{Sin}[c]}{i \text{Cos}[c] \text{Cos}\left[\frac{bx}{2}\right] + \text{Cos}\left[\frac{bx}{2}\right] \text{Sin}[c]}\right] \text{Cos}[a-c]}{b} - \frac{\text{Csc}[c+bx] \text{Sin}[a-c]}{b}$$

**Problem 201: Unable to integrate problem.**

$$\int \text{Sin}[a+bx]^2 \text{Sin}[c+dx]^n dx$$

Optimal (type 5, 410 leaves, 15 steps):

$$-\frac{1}{2b+dn} i 2^{-2-n} e^{-i(2a+cn)-i(2b+dn)x+in(c+dx)} \left(1 - e^{2ic+2idx}\right)^{-n}$$

$$\left(i e^{-i(c+dx)} - i e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left[\frac{1}{2}\left(-\frac{2b}{d}-n\right), -n, \frac{1}{2}\left(2-\frac{2b}{d}-n\right), e^{2i(c+dx)}\right] + \frac{1}{2b-dn}$$

$$i 2^{-2-n} e^{i(2a-cn)+i(2b-dn)x+in(c+dx)} \left(1 - e^{2ic+2idx}\right)^{-n} \left(i e^{-i(c+dx)} - i e^{i(c+dx)}\right)^n \text{Hypergeometric2F1}\left[\frac{1}{2}\left(\frac{2b}{d}-n\right), -n, \frac{1}{2}\left(2+\frac{2b}{d}-n\right), e^{2i(c+dx)}\right] +$$

$$\frac{i 2^{-1-n} \left(i e^{-i(c+dx)} - i e^{i(c+dx)}\right)^n \left(1 - e^{2ic+2idx}\right)^{-n} \text{Hypergeometric2F1}\left[-n, -\frac{n}{2}, 1-\frac{n}{2}, e^{2i(c+dx)}\right]}{dn}$$

Result (type 8, 19 leaves):

$$\int \text{Sin}[a+bx]^2 \text{Sin}[c+dx]^n dx$$

### Problem 205: Unable to integrate problem.

$$\int \sin[a + bx]^3 \sin[c + dx]^n dx$$

Optimal (type 5, 600 leaves, 18 steps):

$$\frac{1}{3b-dn} 2^{-3-n} e^{i(3a-cn)+i(3b-dn)x+in(c+dx)} (1 - e^{2ic+2idx})^{-n} \\ \left( i e^{-i(c+dx)} - i e^{i(c+dx)} \right)^n \text{Hypergeometric2F1} \left[ \frac{1}{2} \left( \frac{3b}{d} - n \right), -n, \frac{1}{2} \left( 2 + \frac{3b}{d} - n \right), e^{2i(c+dx)} \right] - \frac{1}{b-dn} \\ 3 \times 2^{-3-n} e^{i(a-cn)+i(b-dn)x+in(c+dx)} (1 - e^{2ic+2idx})^{-n} \left( i e^{-i(c+dx)} - i e^{i(c+dx)} \right)^n \text{Hypergeometric2F1} \left[ -n, \frac{b-dn}{2d}, \frac{1}{2} \left( 2 + \frac{b}{d} - n \right), e^{2i(c+dx)} \right] - \frac{1}{b+dn} \\ 3 \times 2^{-3-n} e^{-i(a-cn)-i(b+dn)x+in(c+dx)} (1 - e^{2ic+2idx})^{-n} \left( i e^{-i(c+dx)} - i e^{i(c+dx)} \right)^n \text{Hypergeometric2F1} \left[ -n, -\frac{b+dn}{2d}, 1 - \frac{b+dn}{2d}, e^{2i(c+dx)} \right] + \frac{1}{3b+dn} \\ 2^{-3-n} e^{-i(3a+cn)-i(3b+dn)x+in(c+dx)} (1 - e^{2ic+2idx})^{-n} \left( i e^{-i(c+dx)} - i e^{i(c+dx)} \right)^n \text{Hypergeometric2F1} \left[ -n, -\frac{3b+dn}{2d}, \frac{1}{2} \left( 2 - \frac{3b}{d} - n \right), e^{2i(c+dx)} \right]$$

Result (type 8, 19 leaves):

$$\int \sin[a + bx]^3 \sin[c + dx]^n dx$$

### Problem 214: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sec[c + bx]^2 \sin[a + bx] dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$\frac{\cos[a - c] \sec[c + bx]}{b} + \frac{\text{ArcTanh}[\sin[c + bx]] \sin[a - c]}{b}$$

Result (type 3, 88 leaves):

$$\frac{\cos[a - c] \sec[c + bx]}{b} - \frac{2i \text{ArcTan} \left[ \frac{(i \cos[c] + \sin[c]) \left( \cos\left[\frac{bx}{2}\right] \sin[c] + \cos[c] \sin\left[\frac{bx}{2}\right] \right)}{\cos[c] \cos\left[\frac{bx}{2}\right] - i \cos\left[\frac{bx}{2}\right] \sin[c]} \right] \sin[a - c]}{b}$$

### Problem 227: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos[a + bx] \csc[c + bx] dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$\frac{\cos[a - c] \log[\sin[c + bx]]}{b} - x \sin[a - c]$$

Result (type 3, 58 leaves):

$$\frac{-2i \operatorname{ArcTan}[\tan[c + bx]] \cos[a - c] + \cos[a - c] (2i bx + \log[\sin[c + bx]^2]) - 2bx \sin[a - c]}{2b}$$

**Problem 228: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[a + bx] \csc[c + bx]^2 dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{\cos[a - c] \csc[c + bx]}{b} + \frac{\operatorname{ArcTanh}[\cos[c + bx]] \sin[a - c]}{b}$$

Result (type 3, 90 leaves):

$$-\frac{\cos[a - c] \csc[c + bx]}{b} + \frac{2i \operatorname{ArcTan}\left[\frac{(\cos[c] - i \sin[c]) (\cos[c] \cos\left[\frac{bx}{2}\right] - \sin[c] \sin\left[\frac{bx}{2}\right])}{i \cos[c] \cos\left[\frac{bx}{2}\right] + \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \sin[a - c]}{b}$$

**Problem 231: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin[a + bx] \tan[c + bx]^2 dx$$

Optimal (type 3, 44 leaves, 6 steps):

$$\frac{\cos[a + bx]}{b} + \frac{\cos[a - c] \sec[c + bx]}{b} + \frac{\operatorname{ArcTanh}[\sin[c + bx]] \sin[a - c]}{b}$$

Result (type 3, 109 leaves):

$$\frac{\cos[a] \cos[bx]}{b} + \frac{\cos[a - c] \sec[c + bx]}{b} - \frac{2i \operatorname{ArcTan}\left[\frac{(i \cos[c] + \sin[c]) (\cos\left[\frac{bx}{2}\right] \sin[c] + \cos[c] \sin\left[\frac{bx}{2}\right])}{\cos[c] \cos\left[\frac{bx}{2}\right] - i \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \sin[a - c]}{b} - \frac{\sin[a] \sin[bx]}{b}$$

**Problem 232: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin[a + bx] \tan[c + bx] dx$$

Optimal (type 3, 29 leaves, 3 steps):



$$\frac{\text{ArcTanh}[\text{Sin}[c + b x]] \text{Cos}[a - c]}{b} - \frac{\text{Sin}[a + b x]}{b}$$

Result (type 3, 94 leaves):

$$- \frac{2 i \text{ArcTan}\left[\frac{(i \text{Cos}[c] + \text{Sin}[c]) \left(\text{Cos}\left[\frac{b x}{2}\right] \text{Sin}[c] + \text{Cos}[c] \text{Sin}\left[\frac{b x}{2}\right]\right)}{\text{Cos}[c] \text{Cos}\left[\frac{b x}{2}\right] - i \text{Cos}\left[\frac{b x}{2}\right] \text{Sin}[c]}\right] \text{Cos}[a - c]}{b} - \frac{\text{Cos}[b x] \text{Sin}[a]}{b} - \frac{\text{Cos}[a] \text{Sin}[b x]}{b}$$

**Problem 233: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cot}[c + b x] \text{Sin}[a + b x] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$- \frac{\text{ArcTanh}[\text{Cos}[c + b x]] \text{Sin}[a - c]}{b} + \frac{\text{Sin}[a + b x]}{b}$$

Result (type 3, 93 leaves):

$$\frac{\text{Cos}[b x] \text{Sin}[a]}{b} - \frac{2 i \text{ArcTan}\left[\frac{(\text{Cos}[c] - i \text{Sin}[c]) \left(\text{Cos}[c] \text{Cos}\left[\frac{b x}{2}\right] - \text{Sin}[c] \text{Sin}\left[\frac{b x}{2}\right]\right)}{i \text{Cos}[c] \text{Cos}\left[\frac{b x}{2}\right] + \text{Cos}\left[\frac{b x}{2}\right] \text{Sin}[c]}\right] \text{Sin}[a - c]}{b} + \frac{\text{Cos}[a] \text{Sin}[b x]}{b}$$

**Problem 234: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cot}[c + b x]^2 \text{Sin}[a + b x] dx$$

Optimal (type 3, 46 leaves, 6 steps):

$$- \frac{\text{ArcTanh}[\text{Cos}[c + b x]] \text{Cos}[a - c]}{b} + \frac{\text{Cos}[a + b x]}{b} - \frac{\text{Csc}[c + b x] \text{Sin}[a - c]}{b}$$

Result (type 3, 111 leaves):

$$- \frac{2 i \text{ArcTan}\left[\frac{(\text{Cos}[c] - i \text{Sin}[c]) \left(\text{Cos}[c] \text{Cos}\left[\frac{b x}{2}\right] - \text{Sin}[c] \text{Sin}\left[\frac{b x}{2}\right]\right)}{i \text{Cos}[c] \text{Cos}\left[\frac{b x}{2}\right] + \text{Cos}\left[\frac{b x}{2}\right] \text{Sin}[c]}\right] \text{Cos}[a - c]}{b} + \frac{\text{Cos}[a] \text{Cos}[b x]}{b} - \frac{\text{Csc}[c + b x] \text{Sin}[a - c]}{b} - \frac{\text{Sin}[a] \text{Sin}[b x]}{b}$$

**Problem 242: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[a + b x] \text{Sec}[c + b x]^2 dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + b x]] \text{Cos}[a - c]}{b} - \frac{\text{Sec}[c + b x] \text{Sin}[a - c]}{b}$$

Result (type 3, 89 leaves):

$$-\frac{2 \text{ i ArcTan}\left[\frac{(\text{i Cos}[c] + \text{Sin}[c]) \left(\text{Cos}\left[\frac{b x}{2}\right] \text{Sin}[c] + \text{Cos}[c] \text{Sin}\left[\frac{b x}{2}\right]\right)}{\text{Cos}[c] \text{Cos}\left[\frac{b x}{2}\right] - \text{i Cos}\left[\frac{b x}{2}\right] \text{Sin}[c]}\right] \text{Cos}[a - c]}{b} - \frac{\text{Sec}[c + b x] \text{Sin}[a - c]}{b}$$

**Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[a + b x] \text{Tan}[c + b x]^2 dx$$

Optimal (type 3, 46 leaves, 6 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + b x]] \text{Cos}[a - c]}{b} - \frac{\text{Sec}[c + b x] \text{Sin}[a - c]}{b} - \frac{\text{Sin}[a + b x]}{b}$$

Result (type 3, 111 leaves):

$$-\frac{2 \text{ i ArcTan}\left[\frac{(\text{i Cos}[c] + \text{Sin}[c]) \left(\text{Cos}\left[\frac{b x}{2}\right] \text{Sin}[c] + \text{Cos}[c] \text{Sin}\left[\frac{b x}{2}\right]\right)}{\text{Cos}[c] \text{Cos}\left[\frac{b x}{2}\right] - \text{i Cos}\left[\frac{b x}{2}\right] \text{Sin}[c]}\right] \text{Cos}[a - c]}{b} - \frac{\text{Cos}[b x] \text{Sin}[a]}{b} - \frac{\text{Sec}[c + b x] \text{Sin}[a - c]}{b} - \frac{\text{Cos}[a] \text{Sin}[b x]}{b}$$

**Problem 249: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[a + b x] \text{Tan}[c + b x] dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$-\frac{\text{Cos}[a + b x]}{b} - \frac{\text{ArcTanh}[\text{Sin}[c + b x]] \text{Sin}[a - c]}{b}$$

Result (type 3, 93 leaves):

$$-\frac{\text{Cos}[a] \text{Cos}[b x]}{b} + \frac{2 \text{ i ArcTan}\left[\frac{(\text{i Cos}[c] + \text{Sin}[c]) \left(\text{Cos}\left[\frac{b x}{2}\right] \text{Sin}[c] + \text{Cos}[c] \text{Sin}\left[\frac{b x}{2}\right]\right)}{\text{Cos}[c] \text{Cos}\left[\frac{b x}{2}\right] - \text{i Cos}\left[\frac{b x}{2}\right] \text{Sin}[c]}\right] \text{Sin}[a - c]}{b} + \frac{\text{Sin}[a] \text{Sin}[b x]}{b}$$

**Problem 250: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [a + b x] \cot [c + b x] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}[\cos [c + b x]] \cos [a - c]}{b} + \frac{\cos [a + b x]}{b}$$

Result (type 3, 94 leaves):

$$-\frac{2 i \operatorname{ArcTan}\left[\frac{(\cos [c] - i \sin [c])\left(\cos [c] \cos \left[\frac{b x}{2}\right] - \sin [c] \sin \left[\frac{b x}{2}\right]\right)}{i \cos [c] \cos \left[\frac{b x}{2}\right] + \cos \left[\frac{b x}{2}\right] \sin [c]}\right] \cos [a - c]}{b} + \frac{\cos [a] \cos [b x]}{b} - \frac{\sin [a] \sin [b x]}{b}$$

**Problem 251: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [a + b x] \cot [c + b x]^2 dx$$

Optimal (type 3, 46 leaves, 6 steps):

$$-\frac{\cos [a - c] \operatorname{Csc}[c + b x]}{b} + \frac{\operatorname{ArcTanh}[\cos [c + b x]] \sin [a - c]}{b} - \frac{\sin [a + b x]}{b}$$

Result (type 3, 112 leaves):

$$-\frac{\cos [a - c] \operatorname{Csc}[c + b x]}{b} - \frac{\cos [b x] \sin [a]}{b} + \frac{2 i \operatorname{ArcTan}\left[\frac{(\cos [c] - i \sin [c])\left(\cos [c] \cos \left[\frac{b x}{2}\right] - \sin [c] \sin \left[\frac{b x}{2}\right]\right)}{i \cos [c] \cos \left[\frac{b x}{2}\right] + \cos \left[\frac{b x}{2}\right] \sin [c]}\right] \sin [a - c]}{b} - \frac{\cos [a] \sin [b x]}{b}$$

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**Test results for the 294 problems in "4.7.2 trig^m (a trig+b trig)^n.m"**

**Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[x]^2 (a \cos [x] + b \sin [x]) dx$$

Optimal (type 3, 12 leaves, 5 steps):

$$-b \operatorname{ArcTanh}[\cos [x]] - a \operatorname{Csc}[x]$$

Result (type 3, 25 leaves):

$$-a \operatorname{Csc}[x] - b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] + b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right]$$

**Problem 8: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[x]^3}{a \operatorname{Cos}[x] + b \operatorname{Sin}[x]} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{a^2 b x}{(a^2 + b^2)^2} + \frac{b x}{2 (a^2 + b^2)} - \frac{a^3 \operatorname{Log}[a \operatorname{Cos}[x] + b \operatorname{Sin}[x]]}{(a^2 + b^2)^2} - \frac{b \operatorname{Cos}[x] \operatorname{Sin}[x]}{2 (a^2 + b^2)} - \frac{a \operatorname{Sin}[x]^2}{2 (a^2 + b^2)}$$

Result (type 3, 94 leaves):

$$\frac{1}{4 (a^2 + b^2)^2} \left( -4 i a^3 x + 6 a^2 b x + 2 b^3 x + 4 i a^3 \operatorname{ArcTan}[\operatorname{Tan}[x]] + a (a^2 + b^2) \operatorname{Cos}[2x] - 2 a^3 \operatorname{Log}[(a \operatorname{Cos}[x] + b \operatorname{Sin}[x])^2] - a^2 b \operatorname{Sin}[2x] - b^3 \operatorname{Sin}[2x] \right)$$

**Problem 10: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[x]}{a \operatorname{Cos}[x] + b \operatorname{Sin}[x]} dx$$

Optimal (type 3, 35 leaves, 2 steps):

$$\frac{b x}{a^2 + b^2} - \frac{a \operatorname{Log}[a \operatorname{Cos}[x] + b \operatorname{Sin}[x]]}{a^2 + b^2}$$

Result (type 3, 47 leaves):

$$\frac{2 (-i a + b) x + 2 i a \operatorname{ArcTan}[\operatorname{Tan}[x]] - a \operatorname{Log}[(a \operatorname{Cos}[x] + b \operatorname{Sin}[x])^2]}{2 (a^2 + b^2)}$$

**Problem 16: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Sin}[x]^2}{(a \operatorname{Cos}[x] + b \operatorname{Sin}[x])^2} dx$$

Optimal (type 3, 64 leaves, 4 steps):

$$-\frac{(a^2 - b^2) x}{(a^2 + b^2)^2} + \frac{a}{(a^2 + b^2) (b + a \operatorname{Cot}[x])} - \frac{2 a b \operatorname{Log}[a \operatorname{Cos}[x] + b \operatorname{Sin}[x]]}{(a^2 + b^2)^2}$$

Result (type 3, 121 leaves):

$$\left( -a \cos [x] \left( (a + i b)^2 x + a b \operatorname{Log} \left[ (a \cos [x] + b \sin [x])^2 \right] \right) + \left( a^3 + a b^2 (1 - 2 i x) - a^2 b x + b^3 x - a b^2 \operatorname{Log} \left[ (a \cos [x] + b \sin [x])^2 \right] \right) \sin [x] + 2 i a b \operatorname{ArcTan} [\tan [x]] (a \cos [x] + b \sin [x]) \right) / \left( (a^2 + b^2)^2 (a \cos [x] + b \sin [x]) \right)$$

**Problem 21: Result more than twice size of optimal antiderivative.**

$$\int \frac{\csc [x]^3}{(a \cos [x] + b \sin [x])^2} dx$$

Optimal (type 3, 118 leaves, 11 steps):

$$\begin{aligned} & -\frac{\operatorname{ArcTanh} [\cos [x]]}{2 a^2} - \frac{2 b^2 \operatorname{ArcTanh} [\cos [x]]}{a^4} - \frac{(a^2 + b^2) \operatorname{ArcTanh} [\cos [x]]}{a^4} + \\ & \frac{3 b \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[ \frac{b \cos [x] - a \sin [x]}{\sqrt{a^2 + b^2}} \right]}{a^4} + \frac{2 b \csc [x]}{a^3} - \frac{\cot [x] \csc [x]}{2 a^2} + \frac{a^2 + b^2}{a^3 (a \cos [x] + b \sin [x])} \end{aligned}$$

Result (type 3, 270 leaves):

$$\begin{aligned} & \frac{1}{8 a^4 (b + a \cot [x])} \left( -48 b \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[ \frac{-b + a \tan \left[ \frac{x}{2} \right]}{\sqrt{a^2 + b^2}} \right] (b + a \cot [x]) + 8 a^3 \csc [x] + \right. \\ & 8 a b^2 \csc [x] - 12 a^2 b \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] \right] - 24 b^3 \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] \right] - 12 a^3 \cot [x] \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] \right] - 24 a b^2 \cot [x] \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] \right] + \\ & 12 a^2 b \operatorname{Log} \left[ \sin \left[ \frac{x}{2} \right] \right] + 24 b^3 \operatorname{Log} \left[ \sin \left[ \frac{x}{2} \right] \right] + 12 a^3 \cot [x] \operatorname{Log} \left[ \sin \left[ \frac{x}{2} \right] \right] + 24 a b^2 \cot [x] \operatorname{Log} \left[ \sin \left[ \frac{x}{2} \right] \right] + a^2 b \sec \left[ \frac{x}{2} \right]^2 + \\ & \left. a^3 \cot [x] \sec \left[ \frac{x}{2} \right]^2 - a \csc \left[ \frac{x}{2} \right]^2 (-4 a b \cos [x] + a^2 \cot [x] + b (a - 4 b \sin [x])) + 8 a b^2 \tan \left[ \frac{x}{2} \right] + 8 a^2 b \cot [x] \tan \left[ \frac{x}{2} \right] \right) \end{aligned}$$

**Problem 22: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sin [x]^3}{(a \cos [x] + b \sin [x])^3} dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$-\frac{b (3 a^2 - b^2) x}{(a^2 + b^2)^3} + \frac{a}{2 (a^2 + b^2) (b + a \cot [x])^2} + \frac{2 a b}{(a^2 + b^2)^2 (b + a \cot [x])} + \frac{a (a^2 - 3 b^2) \operatorname{Log} [a \cos [x] + b \sin [x]]}{(a^2 + b^2)^3}$$

Result (type 3, 114 leaves):

$$\frac{b(-3a^2 + b^2)x}{(a^2 + b^2)^3} + \frac{a(a^2 - 3b^2)\text{Log}[a\text{Cos}[x] + b\text{Sin}[x]]}{(a^2 + b^2)^3} + \frac{a^3}{2(a - ib)^2(a + ib)^2(a\text{Cos}[x] + b\text{Sin}[x])^2} + \frac{3ab\text{Sin}[x]}{(a^2 + b^2)^2(a\text{Cos}[x] + b\text{Sin}[x])}$$

**Problem 24: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[x]}{(a\text{Cos}[x] + b\text{Sin}[x])^3} dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{1}{2a(b + a\text{Cot}[x])^2}$$

Result (type 3, 47 leaves):

$$\frac{2b^2\text{Sin}[x]^2 + a(a + b\text{Sin}[2x])}{2a(a^2 + b^2)(a\text{Cos}[x] + b\text{Sin}[x])^2}$$

**Problem 25: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a\text{Cos}[x] + b\text{Sin}[x])^3} dx$$

Optimal (type 3, 73 leaves, 3 steps):

$$-\frac{\text{ArcTanh}\left[\frac{b\text{Cos}[x] - a\text{Sin}[x]}{\sqrt{a^2 + b^2}}\right]}{2(a^2 + b^2)^{3/2}} - \frac{b\text{Cos}[x] - a\text{Sin}[x]}{2(a^2 + b^2)(a\text{Cos}[x] + b\text{Sin}[x])^2}$$

Result (type 3, 101 leaves):

$$\frac{(a^2 + b^2)(-b\text{Cos}[x] + a\text{Sin}[x]) + 2\sqrt{a^2 + b^2}\text{ArcTanh}\left[\frac{-b + a\text{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2 + b^2}}\right](a\text{Cos}[x] + b\text{Sin}[x])^2}{2(a - ib)^2(a + ib)^2(a\text{Cos}[x] + b\text{Sin}[x])^2}$$

**Problem 29: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \text{Sin}[c + dx]^{-n} (a\text{Cos}[c + dx] + ib\text{Sin}[c + dx])^n dx$$

Optimal (type 5, 66 leaves, 1 step):

$$-\frac{1}{2dn} i \operatorname{Hypergeometric2F1}\left[1, n, 1+n, -\frac{1}{2} i (\operatorname{Cot}[c+dx])\right] \operatorname{Sin}[c+dx]^{-n} (a \operatorname{Cos}[c+dx] + i a \operatorname{Sin}[c+dx])^n$$

Result (type 6, 2971 leaves):

$$\begin{aligned} & \left( e^{-i n (c+dx) + n \operatorname{Log}[\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]]} (\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx])^{\frac{i n (c+dx)}{\operatorname{Log}[\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]]}} (a (\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx]))^n \right. \\ & \operatorname{Sin}[c+dx]^{-2n} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \left( -\operatorname{Hypergeometric2F1}\left[1-2n, 1-n, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^{-2n} + \right. \\ & \left. \left( (-2+n) \operatorname{AppellF1}\left[1-n, -2n, 1, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) / \right. \\ & \left. \left( \left( i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left( i (-2+n) \operatorname{AppellF1}\left[1-n, -2n, 1, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \right. \\ & \left. \left. \left( 2n \operatorname{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \right. \\ & \left. \left. \left. \operatorname{AppellF1}\left[2-n, -2n, 2, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) / \right. \\ & \left. \left( d (-1+n) \left( \frac{1}{2(-1+n)} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 (\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx])^n \operatorname{Sin}[c+dx]^{-n} \right. \right. \right. \\ & \left. \left. \left( -\operatorname{Hypergeometric2F1}\left[1-2n, 1-n, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^{-2n} + \right. \right. \right. \\ & \left. \left. \left( (-2+n) \operatorname{AppellF1}\left[1-n, -2n, 1, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) / \left( \left( i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left( i (-2+n) \operatorname{AppellF1}\left[ \right. \right. \right. \right. \\ & \left. \left. \left. 1-n, -2n, 1, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right] + \left( 2n \operatorname{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \right. \right. \right. \right. \\ & \left. \left. \left. i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right] + \operatorname{AppellF1}\left[2-n, -2n, 2, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) - \\ & \frac{1}{-1+n} n \operatorname{Cos}[c+dx] (\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx])^n \operatorname{Sin}[c+dx]^{-1-n} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \\ & \left( -\operatorname{Hypergeometric2F1}\left[1-2n, 1-n, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right)^{-2n} + \right. \\ & \left. \left( (-2+n) \operatorname{AppellF1}\left[1-n, -2n, 1, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) / \left( \left( i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left( i (-2+n) \operatorname{AppellF1}\left[ \right. \right. \right. \right. \\ & \left. \left. \left. 1-n, -2n, 1, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right] + \left( 2n \operatorname{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], \right. \right. \right. \right. \\ & \left. \left. \left. i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right] + \operatorname{AppellF1}\left[2-n, -2n, 2, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) + \\ & \frac{1}{-1+n} n (i \operatorname{Cos}[c+dx] - \operatorname{Sin}[c+dx]) (\operatorname{Cos}[c+dx] + i \operatorname{Sin}[c+dx])^{-1+n} \operatorname{Sin}[c+dx]^{-n} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \end{aligned}$$

$$\begin{aligned}
& \left( -\text{Hypergeometric2F1}\left[1-2n, 1-n, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right]\right] \left(1+i \tan\left[\frac{1}{2}(c+dx)\right]\right)^{-2n} + \right. \\
& \left. \left( (-2+n) \text{AppellF1}\left[1-n, -2n, 1, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \right) / \right. \\
& \left. \left( \left( i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left( i(-2+n) \text{AppellF1}\left[1-n, -2n, 1, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \right. \\
& \left. \left. \left( 2n \text{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[2-n, -2n, 2, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \right) \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) + \\
& \frac{1}{-1+n} (\cos[c+dx] + i \sin[c+dx])^n \sin[c+dx]^{-n} \tan\left[\frac{1}{2}(c+dx)\right] \left( i n \text{Hypergeometric2F1}\left[1-2n, 1-n, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right]\right] \right. \\
& \left. \sec\left[\frac{1}{2}(c+dx)\right]^2 \left(1+i \tan\left[\frac{1}{2}(c+dx)\right]\right)^{-1-2n} - \frac{1}{2}(1-n) \csc\left[\frac{1}{2}(c+dx)\right] \sec\left[\frac{1}{2}(c+dx)\right] \right. \\
& \left. \left( -\text{Hypergeometric2F1}\left[1-2n, 1-n, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right]\right] + \left(1+i \tan\left[\frac{1}{2}(c+dx)\right]\right)^{-1+2n} \right) \left(1+i \tan\left[\frac{1}{2}(c+dx)\right]\right)^{-2n} - \right. \\
& \left. \left( (-2+n) \text{AppellF1}\left[1-n, -2n, 1, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) / \right. \\
& \left. \left( 2 \left( i + \tan\left[\frac{1}{2}(c+dx)\right] \right)^2 \left( i(-2+n) \text{AppellF1}\left[1-n, -2n, 1, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \right. \\
& \left. \left. \left( 2n \text{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \right) + \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[2-n, -2n, 2, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \right) \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) \right) + \\
& \left( (-2+n) \left( \frac{1}{2-n} i(1-n)n \text{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 + \right. \right. \\
& \left. \left. \frac{1}{2(2-n)} i(1-n) \text{AppellF1}\left[2-n, -2n, 2, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \sec\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) / \right. \\
& \left. \left( \left( i + \tan\left[\frac{1}{2}(c+dx)\right] \right) \left( i(-2+n) \text{AppellF1}\left[1-n, -2n, 1, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \right. \\
& \left. \left. \left( 2n \text{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] + \text{AppellF1}\left[2-n, -2n, 2, 3-n, \right. \right. \right. \\
& \left. \left. \left. -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \right) \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) - \left( (-2+n) \text{AppellF1}\left[1-n, -2n, 1, 2-n, -i \tan\left[\frac{1}{2}(c+dx)\right], \right. \right. \\
& \left. \left. i \tan\left[\frac{1}{2}(c+dx)\right] \right) \left( \frac{1}{2} \left( 2n \text{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \right. \\
& \left. \left. \left. \text{AppellF1}\left[2-n, -2n, 2, 3-n, -i \tan\left[\frac{1}{2}(c+dx)\right], i \tan\left[\frac{1}{2}(c+dx)\right]\right] \right) \sec\left[\frac{1}{2}(c+dx)\right]^2 + \right. \right.
\end{aligned}$$



$$\begin{aligned}
& i(-2+n) \left( \frac{1}{2-n} i(1-n) n \operatorname{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \right. \\
& \quad \left. \frac{1}{2(2-n)} i(1-n) \operatorname{AppellF1}\left[2-n, -2n, 2, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right) + \\
& \left( \frac{1}{3-n} i(2-n) n \operatorname{AppellF1}\left[3-n, 1-2n, 2, 4-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + \frac{1}{3-n} \right. \\
& \quad i(2-n) \operatorname{AppellF1}\left[3-n, -2n, 3, 4-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 + 2n \left( \frac{1}{2(3-n)} i(2-n) \right. \\
& \quad \left. \operatorname{AppellF1}\left[3-n, 1-2n, 2, 4-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 - \frac{1}{2(3-n)} i(1-2n)(2-n) \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[3-n, 2-2n, 1, 4-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^2 \right) \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \Big) / \\
& \left( \left( i + \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right] \right) \left( i(-2+n) \operatorname{AppellF1}\left[1-n, -2n, 1, 2-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \\
& \quad \left. \left( 2n \operatorname{AppellF1}\left[2-n, 1-2n, 1, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] + \right. \right. \\
& \quad \left. \left. \operatorname{AppellF1}\left[2-n, -2n, 2, 3-n, -i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right], i \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]^2 \right) \Big) \Big)
\end{aligned}$$

### Problem 37: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sec}[c+dx]^2 (a \operatorname{Cos}[c+dx] + b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 24 leaves, 5 steps):

$$\frac{a \operatorname{ArcTanh}[\operatorname{Sin}[c+dx]]}{d} + \frac{b \operatorname{Sec}[c+dx]}{d}$$

Result (type 3, 81 leaves):

$$-\frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \operatorname{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{b \operatorname{Sec}[c+dx]}{d}$$

### Problem 41: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sec}[c+dx]^6 (a \operatorname{Cos}[c+dx] + b \operatorname{Sin}[c+dx]) dx$$

Optimal (type 3, 74 leaves, 7 steps):

$$\frac{3 a \operatorname{ArcTanh}[\sin [c+d x]]}{8 d} + \frac{b \operatorname{Sec}[c+d x]^5}{5 d} + \frac{3 a \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{8 d} + \frac{a \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{4 d}$$

Result (type 3, 207 leaves):

$$\begin{aligned} & -\frac{3 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \frac{3 a \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]}{8 d} + \\ & \frac{b \operatorname{Sec}[c+d x]^5}{5 d} + \frac{a}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} + \frac{3 a}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} - \\ & \frac{a}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4} - \frac{3 a}{16 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} \end{aligned}$$

**Problem 51: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^3 (a \cos [c+d x]+b \sin [c+d x])^2 dx$$

Optimal (type 3, 67 leaves, 7 steps):

$$\frac{a^2 \operatorname{ArcTanh}[\sin [c+d x]]}{d} - \frac{b^2 \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} + \frac{2 a b \operatorname{Sec}[c+d x]}{d} + \frac{b^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{2 d}$$

Result (type 3, 181 leaves):

$$\begin{aligned} & \frac{1}{4 d} \left( 8 a b + (-4 a^2 + 2 b^2) \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right] + \right. \\ & \left. 4 a^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right] - 2 b^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right] + \right. \\ & \left. \frac{b^2}{\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} + 16 a b \operatorname{Sec}[c+d x] \sin \left[\frac{1}{2}(c+d x)\right]^2 - \frac{b^2}{\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2} \right) \end{aligned}$$

**Problem 53: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^5 (a \cos [c+d x]+b \sin [c+d x])^2 dx$$

Optimal (type 3, 120 leaves, 9 steps):

$$\frac{a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 d} - \frac{b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} + \frac{2 a b \operatorname{Sec}[c + d x]^3}{3 d} +$$

$$\frac{a^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 d} - \frac{b^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{b^2 \operatorname{Sec}[c + d x]^3 \operatorname{Tan}[c + d x]}{4 d}$$

Result (type 3, 851 leaves):

$$\frac{a b \operatorname{Cos}[c + d x]^2 (a + b \operatorname{Tan}[c + d x])^2}{3 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} + \frac{(-4 a^2 + b^2) \operatorname{Cos}[c + d x]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Tan}[c + d x])^2}{8 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} +$$

$$\frac{(4 a^2 - b^2) \operatorname{Cos}[c + d x]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Tan}[c + d x])^2}{8 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} +$$

$$\frac{b^2 \operatorname{Cos}[c + d x]^2 (a + b \operatorname{Tan}[c + d x])^2}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} +$$

$$\frac{(12 a^2 + 8 a b - 3 b^2) \operatorname{Cos}[c + d x]^2 (a + b \operatorname{Tan}[c + d x])^2}{48 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} +$$

$$\frac{a b \operatorname{Cos}[c + d x]^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a + b \operatorname{Tan}[c + d x])^2}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} +$$

$$\frac{a b \operatorname{Cos}[c + d x]^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a + b \operatorname{Tan}[c + d x])^2}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} -$$

$$\frac{b^2 \operatorname{Cos}[c + d x]^2 (a + b \operatorname{Tan}[c + d x])^2}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} +$$

$$\frac{a b \operatorname{Cos}[c + d x]^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a + b \operatorname{Tan}[c + d x])^2}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} +$$

$$\frac{(-12 a^2 + 8 a b + 3 b^2) \operatorname{Cos}[c + d x]^2 (a + b \operatorname{Tan}[c + d x])^2}{48 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} -$$

$$\frac{a b \operatorname{Cos}[c + d x]^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a + b \operatorname{Tan}[c + d x])^2}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2}$$

### Problem 55: Result more than twice size of optimal antiderivative.

$$\int \sec [c+d x]^7 (a \cos [c+d x]+b \sin [c+d x])^2 d x$$

Optimal (type 3, 168 leaves, 11 steps):

$$\frac{3 a^2 \operatorname{ArcTanh}[\sin [c+d x]]}{8 d}-\frac{b^2 \operatorname{ArcTanh}[\sin [c+d x]]}{16 d}+\frac{2 a b \sec [c+d x]^5}{5 d}+\frac{3 a^2 \sec [c+d x] \tan [c+d x]}{8 d}-\frac{b^2 \sec [c+d x] \tan [c+d x]}{16 d}+\frac{a^2 \sec [c+d x]^3 \tan [c+d x]}{4 d}-\frac{b^2 \sec [c+d x]^3 \tan [c+d x]}{24 d}+\frac{b^2 \sec [c+d x]^5 \tan [c+d x]}{6 d}$$

Result (type 3, 1175 leaves):

$$\begin{aligned} & \frac{3 a b \cos [c+d x]^2 (a+b \tan [c+d x])^2}{20 d (a \cos [c+d x]+b \sin [c+d x])^2}+\frac{(-6 a^2+b^2) \cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right](a+b \tan [c+d x])^2}{16 d (a \cos [c+d x]+b \sin [c+d x])^2} \\ & +\frac{(6 a^2-b^2) \cos [c+d x]^2 \operatorname{Log}\left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right](a+b \tan [c+d x])^2}{16 d (a \cos [c+d x]+b \sin [c+d x])^2}+ \\ & \frac{b^2 \cos [c+d x]^2 (a+b \tan [c+d x])^2}{48 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^6(a \cos [c+d x]+b \sin [c+d x])^2} \\ & +\frac{(5 a^2+4 a b) \cos [c+d x]^2 (a+b \tan [c+d x])^2}{80 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^4(a \cos [c+d x]+b \sin [c+d x])^2} \\ & +\frac{(30 a^2+12 a b-5 b^2) \cos [c+d x]^2 (a+b \tan [c+d x])^2}{160 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^2(a \cos [c+d x]+b \sin [c+d x])^2} \\ & +\frac{a b \cos [c+d x]^2 \sin \left[\frac{1}{2}(c+d x)\right](a+b \tan [c+d x])^2}{10 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^5(a \cos [c+d x]+b \sin [c+d x])^2} \\ & +\frac{3 a b \cos [c+d x]^2 \sin \left[\frac{1}{2}(c+d x)\right](a+b \tan [c+d x])^2}{20 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^3(a \cos [c+d x]+b \sin [c+d x])^2} \\ & -\frac{3 a b \cos [c+d x]^2 \sin \left[\frac{1}{2}(c+d x)\right](a+b \tan [c+d x])^2}{20 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)(a \cos [c+d x]+b \sin [c+d x])^2} \\ & -\frac{b^2 \cos [c+d x]^2 (a+b \tan [c+d x])^2}{48 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^6(a \cos [c+d x]+b \sin [c+d x])^2} \end{aligned}$$

$$\begin{aligned}
& \frac{a b \cos [c+d x]^2 \sin \left[\frac{1}{2}(c+d x)\right](a+b \tan [c+d x])^2}{10 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^5(a \cos [c+d x]+b \sin [c+d x])^2} + \\
& \frac{(-5 a^2+4 a b) \cos [c+d x]^2(a+b \tan [c+d x])^2}{80 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4(a \cos [c+d x]+b \sin [c+d x])^2} - \\
& \frac{3 a b \cos [c+d x]^2 \sin \left[\frac{1}{2}(c+d x)\right](a+b \tan [c+d x])^2}{20 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^3(a \cos [c+d x]+b \sin [c+d x])^2} + \\
& \frac{(-30 a^2+12 a b+5 b^2) \cos [c+d x]^2(a+b \tan [c+d x])^2}{160 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2(a \cos [c+d x]+b \sin [c+d x])^2} - \\
& \frac{3 a b \cos [c+d x]^2 \sin \left[\frac{1}{2}(c+d x)\right](a+b \tan [c+d x])^2}{20 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)(a \cos [c+d x]+b \sin [c+d x])^2}
\end{aligned}$$

**Problem 66: Result more than twice size of optimal antiderivative.**

$$\int \sec [c+d x]^4(a \cos [c+d x]+b \sin [c+d x])^3 d x$$

Optimal (type 3, 103 leaves, 9 steps):

$$\frac{a^3 \operatorname{ArcTanh}[\sin [c+d x]]}{d} - \frac{3 a b^2 \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} + \frac{3 a^2 b \sec [c+d x]}{d} - \frac{b^3 \sec [c+d x]}{d} + \frac{b^3 \sec [c+d x]^3}{3 d} + \frac{3 a b^2 \sec [c+d x] \tan [c+d x]}{2 d}$$

Result (type 3, 293 leaves):

$$\begin{aligned}
& \frac{1}{12 d} \left( 36 a^2 b - 10 b^3 - 6 a (2 a^2 - 3 b^2) \operatorname{Log} \left[ \cos \left[ \frac{1}{2}(c+d x) \right] - \sin \left[ \frac{1}{2}(c+d x) \right] \right] + 12 a^3 \operatorname{Log} \left[ \cos \left[ \frac{1}{2}(c+d x) \right] + \sin \left[ \frac{1}{2}(c+d x) \right] \right] - \right. \\
& 18 a b^2 \operatorname{Log} \left[ \cos \left[ \frac{1}{2}(c+d x) \right] + \sin \left[ \frac{1}{2}(c+d x) \right] \right] + \frac{9 a b^2}{\left( \cos \left[ \frac{1}{2}(c+d x) \right] - \sin \left[ \frac{1}{2}(c+d x) \right] \right)^2} + \frac{b^3}{\left( \cos \left[ \frac{1}{2}(c+d x) \right] - \sin \left[ \frac{1}{2}(c+d x) \right] \right)^2} + \\
& 2 b (18 a^2 - b^2 + 2 b^2 \cos [c+d x] + (18 a^2 - 5 b^2) \cos [2(c+d x)]) \sec [c+d x]^3 \sin \left[ \frac{1}{2}(c+d x) \right]^2 - \\
& \left. \frac{9 a b^2}{\left( \cos \left[ \frac{1}{2}(c+d x) \right] + \sin \left[ \frac{1}{2}(c+d x) \right] \right)^2} + \frac{b^3}{\left( \cos \left[ \frac{1}{2}(c+d x) \right] + \sin \left[ \frac{1}{2}(c+d x) \right] \right)^2} \right)
\end{aligned}$$

### Problem 67: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[c + d x]^5 (a \text{Cos}[c + d x] + b \text{Sin}[c + d x])^3 dx$$

Optimal (type 3, 30 leaves, 2 steps):

$$\frac{(b + a \text{Cot}[c + d x])^4 \text{Tan}[c + d x]^4}{4 b d}$$

Result (type 3, 79 leaves):

$$\frac{1}{8 d} \text{Sec}[c + d x]^4 \left( (6 a^2 b - 2 b^3) \text{Cos}[2(c + d x)] + a (6 a b + 2(a^2 + b^2) \text{Sin}[2(c + d x)] + (a^2 - b^2) \text{Sin}[4(c + d x)]) \right)$$

### Problem 68: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[c + d x]^6 (a \text{Cos}[c + d x] + b \text{Sin}[c + d x])^3 dx$$

Optimal (type 3, 158 leaves, 12 steps):

$$\frac{a^3 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 d} - \frac{3 a b^2 \text{ArcTanh}[\text{Sin}[c + d x]]}{8 d} + \frac{a^2 b \text{Sec}[c + d x]^3}{d} - \frac{b^3 \text{Sec}[c + d x]^3}{3 d} +$$

$$\frac{b^3 \text{Sec}[c + d x]^5}{5 d} + \frac{a^3 \text{Sec}[c + d x] \text{Tan}[c + d x]}{2 d} - \frac{3 a b^2 \text{Sec}[c + d x] \text{Tan}[c + d x]}{8 d} + \frac{3 a b^2 \text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{4 d}$$

Result (type 3, 464 leaves):

$$\frac{1}{1920 d} \text{Sec}[c + d x]^5 \left( 960 a^2 b + 64 b^3 + 320 (3 a^2 b - b^3) \text{Cos}[2(c + d x)] - \right.$$

$$300 a^3 \text{Cos}[3(c + d x)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 225 a b^2 \text{Cos}[3(c + d x)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] -$$

$$60 a^3 \text{Cos}[5(c + d x)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 45 a b^2 \text{Cos}[5(c + d x)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] -$$

$$150 a (4 a^2 - 3 b^2) \text{Cos}[c + d x] \left( \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) +$$

$$300 a^3 \text{Cos}[3(c + d x)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 225 a b^2 \text{Cos}[3(c + d x)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] +$$

$$60 a^3 \text{Cos}[5(c + d x)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - 45 a b^2 \text{Cos}[5(c + d x)] \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] +$$

$$240 a^3 \text{Sin}[2(c + d x)] + 540 a b^2 \text{Sin}[2(c + d x)] + 120 a^3 \text{Sin}[4(c + d x)] - 90 a b^2 \text{Sin}[4(c + d x)] \left. \right)$$

### Problem 70: Result more than twice size of optimal antiderivative.

$$\int \sec [c+d x]^8 (a \cos [c+d x]+b \sin [c+d x])^3 d x$$

Optimal (type 3, 210 leaves, 14 steps):

$$\frac{3 a^3 \operatorname{ArcTanh}[\sin [c+d x]]}{8 d}-\frac{3 a b^2 \operatorname{ArcTanh}[\sin [c+d x]]}{16 d}+\frac{3 a^2 b \sec [c+d x]^5}{5 d}-\frac{b^3 \sec [c+d x]^5}{5 d}+\frac{b^3 \sec [c+d x]^7}{7 d}+\frac{3 a^3 \sec [c+d x] \tan [c+d x]}{8 d}-\frac{3 a b^2 \sec [c+d x] \tan [c+d x]}{16 d}+\frac{a^3 \sec [c+d x]^3 \tan [c+d x]}{4 d}-\frac{a b^2 \sec [c+d x]^3 \tan [c+d x]}{8 d}+\frac{a b^2 \sec [c+d x]^5 \tan [c+d x]}{2 d}$$

Result (type 3, 637 leaves):

$$\frac{1}{35840 d} \sec [c+d x]^7\left(10752 a^2 b+1536 b^3+3584\left(3 a^2 b-b^3\right) \cos [2(c+d x)]-4410 a^3 \cos [3(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]+2205 a b^2 \cos [3(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]-1470 a^3 \cos [5(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]+735 a b^2 \cos [5(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]-210 a^3 \cos [7(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]+105 a b^2 \cos [7(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]-3675 a\left(2 a^2-b^2\right) \cos [c+d x]\left(\log \left[\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right]-\log \left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]\right)+4410 a^3 \cos [3(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]-2205 a b^2 \cos [3(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]+1470 a^3 \cos [5(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]-735 a b^2 \cos [5(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]+210 a^3 \cos [7(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]-105 a b^2 \cos [7(c+d x)] \log \left[\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right]+4340 a^3 \sin [2(c+d x)]+6790 a b^2 \sin [2(c+d x)]+2800 a^3 \sin [4(c+d x)]-1400 a b^2 \sin [4(c+d x)]+420 a^3 \sin [6(c+d x)]-210 a b^2 \sin [6(c+d x)]\right)$$

### Problem 72: Result more than twice size of optimal antiderivative.

$$\int \sec [c+d x]^{10}(a \cos [c+d x]+b \sin [c+d x])^3 d x$$

Optimal (type 3, 259 leaves, 16 steps):

$$\frac{5 a^3 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{16 d} - \frac{15 a b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{128 d} + \frac{3 a^2 b \operatorname{Sec}[c+d x]^7}{7 d} - \frac{b^3 \operatorname{Sec}[c+d x]^7}{7 d} +$$

$$\frac{b^3 \operatorname{Sec}[c+d x]^9}{9 d} + \frac{5 a^3 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{16 d} - \frac{15 a b^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{128 d} + \frac{5 a^3 \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{24 d} -$$

$$\frac{5 a b^2 \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{64 d} + \frac{a^3 \operatorname{Sec}[c+d x]^5 \operatorname{Tan}[c+d x]}{6 d} - \frac{a b^2 \operatorname{Sec}[c+d x]^5 \operatorname{Tan}[c+d x]}{16 d} + \frac{3 a b^2 \operatorname{Sec}[c+d x]^7 \operatorname{Tan}[c+d x]}{8 d}$$

Result (type 3, 1924 leaves):

$$\frac{5 b (-216 a^2 + 23 b^2) \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Tan}[c+d x])^3}{8064 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} -$$

$$\frac{5 (8 a^3 - 3 a b^2) \operatorname{Cos}[c+d x]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Tan}[c+d x])^3}{128 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} +$$

$$\frac{5 (8 a^3 - 3 a b^2) \operatorname{Cos}[c+d x]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Tan}[c+d x])^3}{128 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} +$$

$$\frac{(27 a b^2 + 4 b^3) \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Tan}[c+d x])^3}{1152 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^8 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} +$$

$$\frac{(84 a^3 + 108 a^2 b + 63 a b^2 - b^3) \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Tan}[c+d x])^3}{4032 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} +$$

$$\frac{(336 a^3 + 288 a^2 b - 63 a b^2 - 26 b^3) \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Tan}[c+d x])^3}{5376 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} +$$

$$\frac{5 (504 a^3 + 216 a^2 b - 189 a b^2 - 23 b^3) \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Tan}[c+d x])^3}{16128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} -$$

$$\frac{b^3 \operatorname{Cos}[c+d x]^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^3}{144 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^9 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} -$$

$$\frac{b^3 \operatorname{Cos}[c+d x]^3 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^3}{144 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^9 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} +$$

$$\frac{(-27 a b^2 + 4 b^3) \operatorname{Cos}[c+d x]^3 (a+b \operatorname{Tan}[c+d x])^3}{1152 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^8 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^3} +$$



$$\begin{aligned}
& \frac{(-84 a^3 + 108 a^2 b - 63 a b^2 - b^3) \cos [c + d x]^3 (a + b \tan [c + d x])^3}{4032 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^6 (a \cos [c + d x] + b \sin [c + d x])^3} + \\
& \frac{(-336 a^3 + 288 a^2 b + 63 a b^2 - 26 b^3) \cos [c + d x]^3 (a + b \tan [c + d x])^3}{5376 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4 (a \cos [c + d x] + b \sin [c + d x])^3} - \\
& \frac{5 (504 a^3 - 216 a^2 b - 189 a b^2 + 23 b^3) \cos [c + d x]^3 (a + b \tan [c + d x])^3}{16128 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 (a \cos [c + d x] + b \sin [c + d x])^3} + \\
& \frac{\cos [c + d x]^3 \left( 144 a^2 b \sin \left[ \frac{1}{2} (c + d x) \right] - 13 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^3}{1344 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5 (a \cos [c + d x] + b \sin [c + d x])^3} + \\
& \frac{\cos [c + d x]^3 \left( 108 a^2 b \sin \left[ \frac{1}{2} (c + d x) \right] - b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^3}{2016 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^7 (a \cos [c + d x] + b \sin [c + d x])^3} + \\
& \frac{\cos [c + d x]^3 \left( -108 a^2 b \sin \left[ \frac{1}{2} (c + d x) \right] + b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^3}{2016 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^7 (a \cos [c + d x] + b \sin [c + d x])^3} + \\
& \frac{\cos [c + d x]^3 \left( -144 a^2 b \sin \left[ \frac{1}{2} (c + d x) \right] + 13 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^3}{1344 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5 (a \cos [c + d x] + b \sin [c + d x])^3} - \\
& \frac{5 \cos [c + d x]^3 \left( -216 a^2 b \sin \left[ \frac{1}{2} (c + d x) \right] + 23 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^3}{8064 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3 (a \cos [c + d x] + b \sin [c + d x])^3} - \\
& \frac{5 \cos [c + d x]^3 \left( -216 a^2 b \sin \left[ \frac{1}{2} (c + d x) \right] + 23 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^3}{8064 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a \cos [c + d x] + b \sin [c + d x])^3} + \\
& \frac{5 \cos [c + d x]^3 \left( -216 a^2 b \sin \left[ \frac{1}{2} (c + d x) \right] + 23 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^3}{8064 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3 (a \cos [c + d x] + b \sin [c + d x])^3} + \\
& \frac{5 \cos [c + d x]^3 \left( -216 a^2 b \sin \left[ \frac{1}{2} (c + d x) \right] + 23 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^3}{8064 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a \cos [c + d x] + b \sin [c + d x])^3}
\end{aligned}$$

**Problem 77:** Result unnecessarily involves imaginary or complex numbers.

$$\int \cos [c + d x]^2 (a \cos [c + d x] + b \sin [c + d x])^4 dx$$

Optimal (type 3, 301 leaves, 19 steps):

$$\begin{aligned} & \frac{5 a^4 x}{16} + \frac{3}{8} a^2 b^2 x + \frac{b^4 x}{16} - \frac{2 a^3 b \cos [c + d x]^6}{3 d} + \frac{5 a^4 \cos [c + d x] \sin [c + d x]}{16 d} + \frac{3 a^2 b^2 \cos [c + d x] \sin [c + d x]}{8 d} + \\ & \frac{b^4 \cos [c + d x] \sin [c + d x]}{16 d} + \frac{5 a^4 \cos [c + d x]^3 \sin [c + d x]}{24 d} + \frac{a^2 b^2 \cos [c + d x]^3 \sin [c + d x]}{4 d} - \frac{b^4 \cos [c + d x]^3 \sin [c + d x]}{8 d} + \\ & \frac{a^4 \cos [c + d x]^5 \sin [c + d x]}{6 d} - \frac{a^2 b^2 \cos [c + d x]^5 \sin [c + d x]}{d} - \frac{b^4 \cos [c + d x]^3 \sin [c + d x]^3}{6 d} + \frac{a b^3 \sin [c + d x]^4}{d} - \frac{2 a b^3 \sin [c + d x]^6}{3 d} \end{aligned}$$

Result (type 3, 178 leaves):

$$\begin{aligned} & \frac{1}{192 d} (12 (a - i b) (a + i b) (5 a^2 + b^2) (c + d x) - 12 a b (5 a^2 + 3 b^2) \cos [2 (c + d x)] - 24 a^3 b \cos [4 (c + d x)] - 4 a b (a^2 - b^2) \cos [6 (c + d x)] + \\ & 3 (15 a^4 + 6 a^2 b^2 - b^4) \sin [2 (c + d x)] + 3 (3 a^4 - 6 a^2 b^2 - b^4) \sin [4 (c + d x)] + (a^4 - 6 a^2 b^2 + b^4) \sin [6 (c + d x)]) \end{aligned}$$

**Problem 84: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x]^5 (a \cos [c + d x] + b \sin [c + d x])^4 dx$$

Optimal (type 3, 168 leaves, 12 steps):

$$\begin{aligned} & \frac{a^4 \operatorname{ArcTanh}[\sin [c + d x]]}{d} - \frac{3 a^2 b^2 \operatorname{ArcTanh}[\sin [c + d x]]}{d} + \frac{3 b^4 \operatorname{ArcTanh}[\sin [c + d x]]}{8 d} + \frac{4 a^3 b \sec [c + d x]}{d} - \frac{4 a b^3 \sec [c + d x]}{d} + \\ & \frac{4 a b^3 \sec [c + d x]^3}{3 d} + \frac{3 a^2 b^2 \sec [c + d x] \tan [c + d x]}{d} - \frac{3 b^4 \sec [c + d x] \tan [c + d x]}{8 d} + \frac{b^4 \sec [c + d x] \tan [c + d x]^3}{4 d} \end{aligned}$$

Result (type 3, 936 leaves):

$$\begin{aligned}
& \frac{2 a b (6 a^2 - 5 b^2) \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Tan}[c + d x])^4}{3 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} + \\
& \frac{(-8 a^4 + 24 a^2 b^2 - 3 b^4) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Tan}[c + d x])^4}{8 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} + \\
& \frac{(8 a^4 - 24 a^2 b^2 + 3 b^4) \operatorname{Cos}[c + d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Tan}[c + d x])^4}{8 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} + \\
& \frac{b^4 \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Tan}[c + d x])^4}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} + \\
& \frac{(72 a^2 b^2 + 16 a b^3 - 15 b^4) \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Tan}[c + d x])^4}{48 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} + \\
& \frac{2 a b^3 \operatorname{Cos}[c + d x]^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a + b \operatorname{Tan}[c + d x])^4}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} - \\
& \frac{b^4 \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Tan}[c + d x])^4}{16 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} - \\
& \frac{2 a b^3 \operatorname{Cos}[c + d x]^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a + b \operatorname{Tan}[c + d x])^4}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} + \\
& \frac{(-72 a^2 b^2 + 16 a b^3 + 15 b^4) \operatorname{Cos}[c + d x]^4 (a + b \operatorname{Tan}[c + d x])^4}{48 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} + \\
& \frac{2 \operatorname{Cos}[c + d x]^4 \left(6 a^3 b \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - 5 a b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a + b \operatorname{Tan}[c + d x])^4}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4} - \\
& \frac{2 \operatorname{Cos}[c + d x]^4 \left(6 a^3 b \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - 5 a b^3 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a + b \operatorname{Tan}[c + d x])^4}{3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4}
\end{aligned}$$

**Problem 85: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + d x]^6 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^4 dx$$

Optimal (type 3, 30 leaves, 2 steps):

$$\frac{(b + a \operatorname{Cot}[c + dx])^5 \operatorname{Tan}[c + dx]^5}{5bd}$$

Result (type 3, 131 leaves):

$$\frac{\left( (a + b \operatorname{Tan}[c + dx])^4 \left( 10ab(a^2 - b^2) \operatorname{Cos}[c + dx]^2 + (5a^4 - 10a^2b^2 + b^4) \operatorname{Cos}[c + dx]^3 \operatorname{Sin}[c + dx] + b^2 \left( (5a^2 - b^2) \operatorname{Sin}[2(c + dx)] + b(5a + b \operatorname{Tan}[c + dx]) \right) \right) \right)}{(5d(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^4)}$$

**Problem 86: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^7 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^4 dx$$

Optimal (type 3, 258 leaves, 16 steps):

$$\frac{a^4 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2d} - \frac{3a^2b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{4d} + \frac{b^4 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{16d} + \frac{4a^3b \operatorname{Sec}[c + dx]^3}{3d} - \frac{4ab^3 \operatorname{Sec}[c + dx]^3}{3d} + \frac{4ab^3 \operatorname{Sec}[c + dx]^5}{5d} + \frac{a^4 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2d} - \frac{3a^2b^2 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{4d} + \frac{b^4 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{16d} + \frac{3a^2b^2 \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{2d} - \frac{b^4 \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{8d} + \frac{b^4 \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]^3}{6d}$$

Result (type 3, 1342 leaves):

$$\frac{ab(20a^2 - 11b^2) \operatorname{Cos}[c + dx]^4 (a + b \operatorname{Tan}[c + dx])^4}{30d(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^4} + \frac{(-8a^4 + 12a^2b^2 - b^4) \operatorname{Cos}[c + dx]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] (a + b \operatorname{Tan}[c + dx])^4}{16d(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^4} + \frac{(8a^4 - 12a^2b^2 + b^4) \operatorname{Cos}[c + dx]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] (a + b \operatorname{Tan}[c + dx])^4}{16d(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^4} + \frac{b^4 \operatorname{Cos}[c + dx]^4 (a + b \operatorname{Tan}[c + dx])^4}{48d \left( \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^6 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^4} + \frac{(30a^2b^2 + 8ab^3 - 5b^4) \operatorname{Cos}[c + dx]^4 (a + b \operatorname{Tan}[c + dx])^4}{80d \left( \operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] \right)^4 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^4}$$

$$\begin{aligned}
& \frac{(120 a^4 + 160 a^3 b - 180 a^2 b^2 - 88 a b^3 + 15 b^4) \cos [c + d x]^4 (a + b \tan [c + d x])^4}{480 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 (a \cos [c + d x] + b \sin [c + d x])^4} + \\
& \frac{a b^3 \cos [c + d x]^4 \sin \left[ \frac{1}{2} (c + d x) \right] (a + b \tan [c + d x])^4}{5 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5 (a \cos [c + d x] + b \sin [c + d x])^4} - \\
& \frac{b^4 \cos [c + d x]^4 (a + b \tan [c + d x])^4}{48 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^6 (a \cos [c + d x] + b \sin [c + d x])^4} - \\
& \frac{a b^3 \cos [c + d x]^4 \sin \left[ \frac{1}{2} (c + d x) \right] (a + b \tan [c + d x])^4}{5 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5 (a \cos [c + d x] + b \sin [c + d x])^4} + \\
& \frac{(-30 a^2 b^2 + 8 a b^3 + 5 b^4) \cos [c + d x]^4 (a + b \tan [c + d x])^4}{80 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4 (a \cos [c + d x] + b \sin [c + d x])^4} + \\
& \frac{(-120 a^4 + 160 a^3 b + 180 a^2 b^2 - 88 a b^3 - 15 b^4) \cos [c + d x]^4 (a + b \tan [c + d x])^4}{480 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 (a \cos [c + d x] + b \sin [c + d x])^4} + \\
& \frac{\cos [c + d x]^4 \left( 20 a^3 b \sin \left[ \frac{1}{2} (c + d x) \right] - 11 a b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^4}{30 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3 (a \cos [c + d x] + b \sin [c + d x])^4} + \\
& \frac{\cos [c + d x]^4 \left( 20 a^3 b \sin \left[ \frac{1}{2} (c + d x) \right] - 11 a b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^4}{30 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a \cos [c + d x] + b \sin [c + d x])^4} + \\
& \frac{\cos [c + d x]^4 \left( -20 a^3 b \sin \left[ \frac{1}{2} (c + d x) \right] + 11 a b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^4}{30 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3 (a \cos [c + d x] + b \sin [c + d x])^4} + \\
& \frac{\cos [c + d x]^4 \left( -20 a^3 b \sin \left[ \frac{1}{2} (c + d x) \right] + 11 a b^3 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^4}{30 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a \cos [c + d x] + b \sin [c + d x])^4}
\end{aligned}$$

**Problem 88: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x]^9 (a \cos [c + d x] + b \sin [c + d x])^4 dx$$

Optimal (type 3, 330 leaves, 19 steps):

$$\frac{3 a^4 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{8 d} - \frac{3 a^2 b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{8 d} + \frac{3 b^4 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{128 d} + \frac{4 a^3 b \operatorname{Sec}[c+d x]^5}{5 d} - \frac{4 a b^3 \operatorname{Sec}[c+d x]^5}{5 d} + \frac{4 a b^3 \operatorname{Sec}[c+d x]^7}{7 d} + \frac{3 a^4 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{8 d} - \frac{3 a^2 b^2 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{8 d} + \frac{3 b^4 \operatorname{Sec}[c+d x] \operatorname{Tan}[c+d x]}{128 d} + \frac{a^4 \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{4 d} - \frac{a^2 b^2 \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{4 d} + \frac{b^4 \operatorname{Sec}[c+d x]^3 \operatorname{Tan}[c+d x]}{64 d} + \frac{a^2 b^2 \operatorname{Sec}[c+d x]^5 \operatorname{Tan}[c+d x]}{d} - \frac{b^4 \operatorname{Sec}[c+d x]^5 \operatorname{Tan}[c+d x]}{16 d} + \frac{b^4 \operatorname{Sec}[c+d x]^5 \operatorname{Tan}[c+d x]^3}{8 d}$$

Result (type 3, 1732 leaves):

$$\frac{a b (42 a^2 - 17 b^2) \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Tan}[c+d x])^4}{140 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4} - \frac{3 (16 a^4 - 16 a^2 b^2 + b^4) \operatorname{Cos}[c+d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Tan}[c+d x])^4}{128 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4} + \frac{3 (16 a^4 - 16 a^2 b^2 + b^4) \operatorname{Cos}[c+d x]^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Tan}[c+d x])^4}{128 d (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4} + \frac{b^4 \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Tan}[c+d x])^4}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^8 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4} + \frac{(56 a^2 b^2 + 16 a b^3 - 7 b^4) \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Tan}[c+d x])^4}{448 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^6 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4} + \frac{(560 a^4 + 896 a^3 b - 256 a b^3 - 35 b^4) \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Tan}[c+d x])^4}{8960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^4 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4} + \frac{(1680 a^4 + 1344 a^3 b - 1680 a^2 b^2 - 544 a b^3 + 105 b^4) \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Tan}[c+d x])^4}{8960 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^2 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4} + \frac{a b^3 \operatorname{Cos}[c+d x]^4 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^4}{14 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^7 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4} - \frac{b^4 \operatorname{Cos}[c+d x]^4 (a+b \operatorname{Tan}[c+d x])^4}{128 d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right)^8 (a \operatorname{Cos}[c+d x] + b \operatorname{Sin}[c+d x])^4}$$

$$\begin{aligned}
& \frac{a b^3 \cos [c+d x]^4 \sin \left[\frac{1}{2}(c+d x)\right] (a+b \tan [c+d x])^4}{14 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^7 (a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\left(-56 a^2 b^2+16 a b^3+7 b^4\right) \cos [c+d x]^4 (a+b \tan [c+d x])^4}{448 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^6 (a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\left(-560 a^4+896 a^3 b-256 a b^3+35 b^4\right) \cos [c+d x]^4 (a+b \tan [c+d x])^4}{8960 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^4 (a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\left(-1680 a^4+1344 a^3 b+1680 a^2 b^2-544 a b^3-105 b^4\right) \cos [c+d x]^4 (a+b \tan [c+d x])^4}{8960 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^2 (a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\cos [c+d x]^4\left(42 a^3 b \sin \left[\frac{1}{2}(c+d x)\right]-17 a b^3 \sin \left[\frac{1}{2}(c+d x)\right]\right)(a+b \tan [c+d x])^4}{140 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^3 (a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\cos [c+d x]^4\left(42 a^3 b \sin \left[\frac{1}{2}(c+d x)\right]-17 a b^3 \sin \left[\frac{1}{2}(c+d x)\right]\right)(a+b \tan [c+d x])^4}{140 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)(a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\cos [c+d x]^4\left(7 a^3 b \sin \left[\frac{1}{2}(c+d x)\right]-2 a b^3 \sin \left[\frac{1}{2}(c+d x)\right]\right)(a+b \tan [c+d x])^4}{35 d\left(\cos \left[\frac{1}{2}(c+d x)\right]-\sin \left[\frac{1}{2}(c+d x)\right]\right)^5 (a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\cos [c+d x]^4\left(-7 a^3 b \sin \left[\frac{1}{2}(c+d x)\right]+2 a b^3 \sin \left[\frac{1}{2}(c+d x)\right]\right)(a+b \tan [c+d x])^4}{35 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^5 (a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\cos [c+d x]^4\left(-42 a^3 b \sin \left[\frac{1}{2}(c+d x)\right]+17 a b^3 \sin \left[\frac{1}{2}(c+d x)\right]\right)(a+b \tan [c+d x])^4}{140 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)^3 (a \cos [c+d x]+b \sin [c+d x])^4} + \\
& \frac{\cos [c+d x]^4\left(-42 a^3 b \sin \left[\frac{1}{2}(c+d x)\right]+17 a b^3 \sin \left[\frac{1}{2}(c+d x)\right]\right)(a+b \tan [c+d x])^4}{140 d\left(\cos \left[\frac{1}{2}(c+d x)\right]+\sin \left[\frac{1}{2}(c+d x)\right]\right)(a \cos [c+d x]+b \sin [c+d x])^4}
\end{aligned}$$

**Problem 94: Result unnecessarily involves imaginary or complex numbers.**

$$\int \cos [c+d x]^3 (a \cos [c+d x]+b \sin [c+d x])^5 dx$$

Optimal (type 3, 426 leaves, 25 steps):

$$\begin{aligned} & \frac{35 a^5 x}{128} + \frac{25}{64} a^3 b^2 x + \frac{15}{128} a b^4 x - \frac{5 a^2 b^3 \cos [c+d x]^6}{3 d} - \frac{5 a^4 b \cos [c+d x]^8}{8 d} + \frac{5 a^2 b^3 \cos [c+d x]^8}{4 d} + \frac{35 a^5 \cos [c+d x] \sin [c+d x]}{128 d} + \\ & \frac{25 a^3 b^2 \cos [c+d x] \sin [c+d x]}{64 d} + \frac{15 a b^4 \cos [c+d x] \sin [c+d x]}{128 d} + \frac{35 a^5 \cos [c+d x]^3 \sin [c+d x]}{192 d} + \frac{25 a^3 b^2 \cos [c+d x]^3 \sin [c+d x]}{96 d} + \\ & \frac{5 a b^4 \cos [c+d x]^3 \sin [c+d x]}{64 d} + \frac{7 a^5 \cos [c+d x]^5 \sin [c+d x]}{48 d} + \frac{5 a^3 b^2 \cos [c+d x]^5 \sin [c+d x]}{24 d} - \frac{5 a b^4 \cos [c+d x]^5 \sin [c+d x]}{16 d} + \\ & \frac{a^5 \cos [c+d x]^7 \sin [c+d x]}{8 d} - \frac{5 a^3 b^2 \cos [c+d x]^7 \sin [c+d x]}{4 d} - \frac{5 a b^4 \cos [c+d x]^5 \sin [c+d x]^3}{8 d} + \frac{b^5 \sin [c+d x]^6}{6 d} - \frac{b^5 \sin [c+d x]^8}{8 d} \end{aligned}$$

Result (type 3, 259 leaves):

$$\begin{aligned} & \frac{1}{3072 d} (120 a (a - i b) (a + i b) (7 a^2 + 3 b^2) (c + d x) - 24 b (35 a^4 + 30 a^2 b^2 + 3 b^4) \cos [2 (c + d x)] + 12 b (-35 a^4 - 10 a^2 b^2 + b^4) \cos [4 (c + d x)] + \\ & 8 b (-15 a^4 + 10 a^2 b^2 + b^4) \cos [6 (c + d x)] - 3 b (5 a^4 - 10 a^2 b^2 + b^4) \cos [8 (c + d x)] + 96 a^3 (7 a^2 + 5 b^2) \sin [2 (c + d x)] + \\ & 24 a (7 a^4 - 10 a^2 b^2 - 5 b^4) \sin [4 (c + d x)] + 32 a^3 (a^2 - 5 b^2) \sin [6 (c + d x)] + 3 a (a^4 - 10 a^2 b^2 + 5 b^4) \sin [8 (c + d x)]) \end{aligned}$$

Problem 98: Result more than twice size of optimal antiderivative.

$$\int \sec [c+d x] (a \cos [c+d x] + b \sin [c+d x])^5 dx$$

Optimal (type 3, 170 leaves, 8 steps):

$$\begin{aligned} & \frac{1}{8} a (3 a^4 + 10 a^2 b^2 + 15 b^4) x - \frac{b^5 \operatorname{Log}[\sin [c+d x]]}{d} + \frac{b^5 \operatorname{Log}[\tan [c+d x]]}{d} + \\ & \frac{(4 b (5 a^4 - b^4) + 5 a (a^2 - 3 b^2) (a^2 + b^2) \cot [c+d x]) \sin [c+d x]^2}{8 d} - \frac{(b (5 a^4 - 10 a^2 b^2 + b^4) + a (a^4 - 10 a^2 b^2 + 5 b^4) \cot [c+d x]) \sin [c+d x]^4}{4 d} \end{aligned}$$

Result (type 3, 408 leaves):

$$\begin{aligned} & \frac{a (3 a^4 + 10 a^2 b^2 + 15 b^4) (c + d x) \cos [c+d x]^5 (a + b \tan [c+d x])^5}{8 d (a \cos [c+d x] + b \sin [c+d x])^5} - \frac{b (5 a^4 + 10 a^2 b^2 - 3 b^4) \cos [c+d x]^5 \cos [2 (c + d x)] (a + b \tan [c+d x])^5}{8 d (a \cos [c+d x] + b \sin [c+d x])^5} - \\ & \frac{b (5 a^4 - 10 a^2 b^2 + b^4) \cos [c+d x]^5 \cos [4 (c + d x)] (a + b \tan [c+d x])^5}{32 d (a \cos [c+d x] + b \sin [c+d x])^5} - \frac{b^5 \cos [c+d x]^5 \operatorname{Log}[\cos [c+d x]] (a + b \tan [c+d x])^5}{d (a \cos [c+d x] + b \sin [c+d x])^5} + \\ & \frac{a (a^4 - 5 b^4) \cos [c+d x]^5 \sin [2 (c + d x)] (a + b \tan [c+d x])^5}{4 d (a \cos [c+d x] + b \sin [c+d x])^5} + \frac{a (a^4 - 10 a^2 b^2 + 5 b^4) \cos [c+d x]^5 \sin [4 (c + d x)] (a + b \tan [c+d x])^5}{32 d (a \cos [c+d x] + b \sin [c+d x])^5} \end{aligned}$$

Problem 99: Result more than twice size of optimal antiderivative.

$$\int \sec [c+d x]^2 (a \cos [c+d x] + b \sin [c+d x])^5 dx$$



Optimal (type 3, 205 leaves, 17 steps):

$$\frac{5 a b^4 \operatorname{ArcTanh}[\operatorname{Sin}[c+d x]]}{d} - \frac{10 a^2 b^3 \operatorname{Cos}[c+d x]}{d} + \frac{2 b^5 \operatorname{Cos}[c+d x]}{d} - \frac{5 a^4 b \operatorname{Cos}[c+d x]^3}{3 d} + \frac{10 a^2 b^3 \operatorname{Cos}[c+d x]^3}{3 d} - \frac{b^5 \operatorname{Cos}[c+d x]^3}{3 d} + \frac{b^5 \operatorname{Sec}[c+d x]}{d} + \frac{a^5 \operatorname{Sin}[c+d x]}{d} - \frac{5 a b^4 \operatorname{Sin}[c+d x]}{d} - \frac{a^5 \operatorname{Sin}[c+d x]^3}{3 d} + \frac{10 a^3 b^2 \operatorname{Sin}[c+d x]^3}{3 d} - \frac{5 a b^4 \operatorname{Sin}[c+d x]^3}{3 d}$$

Result (type 3, 632 leaves):

$$\frac{b^5 \operatorname{Cos}[c+d x]^5 (a+b \operatorname{Tan}[c+d x])^5}{d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5} - \frac{b (5 a^4+30 a^2 b^2-7 b^4) \operatorname{Cos}[c+d x]^6 (a+b \operatorname{Tan}[c+d x])^5}{4 d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5} - \frac{b (5 a^4-10 a^2 b^2+b^4) \operatorname{Cos}[c+d x]^5 \operatorname{Cos}[3(c+d x)] (a+b \operatorname{Tan}[c+d x])^5}{12 d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5} - \frac{5 a b^4 \operatorname{Cos}[c+d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Tan}[c+d x])^5}{d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5} + \frac{5 a b^4 \operatorname{Cos}[c+d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] (a+b \operatorname{Tan}[c+d x])^5}{d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5} + \frac{b^5 \operatorname{Cos}[c+d x]^5 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^5}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]-\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5} - \frac{b^5 \operatorname{Cos}[c+d x]^5 \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right] (a+b \operatorname{Tan}[c+d x])^5}{d \left(\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right]+\operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right) (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5} + \frac{a (3 a^4+10 a^2 b^2-25 b^4) \operatorname{Cos}[c+d x]^5 \operatorname{Sin}[c+d x] (a+b \operatorname{Tan}[c+d x])^5}{4 d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5} + \frac{a (a^4-10 a^2 b^2+5 b^4) \operatorname{Cos}[c+d x]^5 \operatorname{Sin}[3(c+d x)] (a+b \operatorname{Tan}[c+d x])^5}{12 d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5}$$

**Problem 100: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c+d x]^3 (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^5 dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$\frac{1}{2} a (a^4+10 a^2 b^2-15 b^4) x - \frac{2 b^3 (5 a^2-b^2) \operatorname{Log}[\operatorname{Sin}[c+d x]]}{d} + \frac{2 b^3 (5 a^2-b^2) \operatorname{Log}[\operatorname{Tan}[c+d x]]}{d} + \frac{(b (5 a^4-10 a^2 b^2+b^4)+a (a^4-10 a^2 b^2+5 b^4) \operatorname{Cot}[c+d x]) \operatorname{Sin}[c+d x]^2}{2 d} + \frac{5 a b^4 \operatorname{Tan}[c+d x]}{d} + \frac{b^5 \operatorname{Tan}[c+d x]^2}{2 d}$$

Result (type 3, 382 leaves):

$$\frac{b^5 \cos [c+d x]^3 (a+b \tan [c+d x])^5}{2 d (a \cos [c+d x]+b \sin [c+d x])^5} + \frac{a\left(a^4+10 a^2 b^2-15 b^4\right)(c+d x) \cos [c+d x]^5 (a+b \tan [c+d x])^5}{2 d (a \cos [c+d x]+b \sin [c+d x])^5} -$$

$$\frac{b\left(5 a^4-10 a^2 b^2+b^4\right) \cos [c+d x]^5 \cos [2(c+d x)](a+b \tan [c+d x])^5}{4 d (a \cos [c+d x]+b \sin [c+d x])^5} - \frac{2\left(5 a^2 b^3-b^5\right) \cos [c+d x]^5 \log [\cos [c+d x]](a+b \tan [c+d x])^5}{d (a \cos [c+d x]+b \sin [c+d x])^5} +$$

$$\frac{5 a b^4 \cos [c+d x]^4 \sin [c+d x](a+b \tan [c+d x])^5}{d (a \cos [c+d x]+b \sin [c+d x])^5} + \frac{a\left(a^4-10 a^2 b^2+5 b^4\right) \cos [c+d x]^5 \sin [2(c+d x)](a+b \tan [c+d x])^5}{4 d (a \cos [c+d x]+b \sin [c+d x])^5}$$

Problem 101: Result more than twice size of optimal antiderivative.

$$\int \sec [c+d x]^4 (a \cos [c+d x]+b \sin [c+d x])^5 dx$$

Optimal (type 3, 204 leaves, 17 steps):

$$\frac{10 a^3 b^2 \operatorname{ArcTanh}[\sin [c+d x]]}{d} - \frac{15 a b^4 \operatorname{ArcTanh}[\sin [c+d x]]}{2 d} - \frac{5 a^4 b \cos [c+d x]}{d} + \frac{10 a^2 b^3 \cos [c+d x]}{d} - \frac{b^5 \cos [c+d x]}{d} + \frac{10 a^2 b^3 \sec [c+d x]}{d} -$$

$$\frac{2 b^5 \sec [c+d x]}{d} + \frac{b^5 \sec [c+d x]^3}{3 d} + \frac{a^5 \sin [c+d x]}{d} - \frac{10 a^3 b^2 \sin [c+d x]}{d} + \frac{15 a b^4 \sin [c+d x]}{2 d} + \frac{5 a b^4 \sin [c+d x] \tan [c+d x]^2}{2 d}$$

Result (type 3, 892 leaves):

$$\begin{aligned}
& - \frac{b^3 (-60a^2 + 11b^2) \cos [c + dx]^5 (a + b \tan [c + dx])^5}{6d (a \cos [c + dx] + b \sin [c + dx])^5} - \frac{b (5a^4 - 10a^2b^2 + b^4) \cos [c + dx]^6 (a + b \tan [c + dx])^5}{d (a \cos [c + dx] + b \sin [c + dx])^5} \\
& \frac{5 (4a^3b^2 - 3ab^4) \cos [c + dx]^5 \log \left[ \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right] (a + b \tan [c + dx])^5}{2d (a \cos [c + dx] + b \sin [c + dx])^5} + \\
& \frac{5 (4a^3b^2 - 3ab^4) \cos [c + dx]^5 \log \left[ \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right] (a + b \tan [c + dx])^5}{2d (a \cos [c + dx] + b \sin [c + dx])^5} + \\
& \frac{(15ab^4 + b^5) \cos [c + dx]^5 (a + b \tan [c + dx])^5}{12d \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2 (a \cos [c + dx] + b \sin [c + dx])^5} + \\
& \frac{b^5 \cos [c + dx]^5 \sin \left[ \frac{1}{2} (c + dx) \right] (a + b \tan [c + dx])^5}{6d \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right)^3 (a \cos [c + dx] + b \sin [c + dx])^5} - \\
& \frac{b^5 \cos [c + dx]^5 \sin \left[ \frac{1}{2} (c + dx) \right] (a + b \tan [c + dx])^5}{6d \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^3 (a \cos [c + dx] + b \sin [c + dx])^5} + \\
& \frac{(-15ab^4 + b^5) \cos [c + dx]^5 (a + b \tan [c + dx])^5}{12d \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right)^2 (a \cos [c + dx] + b \sin [c + dx])^5} + \\
& \frac{\cos [c + dx]^5 \left( 60a^2b^3 \sin \left[ \frac{1}{2} (c + dx) \right] - 11b^5 \sin \left[ \frac{1}{2} (c + dx) \right] \right) (a + b \tan [c + dx])^5}{6d \left( \cos \left[ \frac{1}{2} (c + dx) \right] - \sin \left[ \frac{1}{2} (c + dx) \right] \right) (a \cos [c + dx] + b \sin [c + dx])^5} + \\
& \frac{\cos [c + dx]^5 \left( -60a^2b^3 \sin \left[ \frac{1}{2} (c + dx) \right] + 11b^5 \sin \left[ \frac{1}{2} (c + dx) \right] \right) (a + b \tan [c + dx])^5}{6d \left( \cos \left[ \frac{1}{2} (c + dx) \right] + \sin \left[ \frac{1}{2} (c + dx) \right] \right) (a \cos [c + dx] + b \sin [c + dx])^5} + \\
& \frac{a (a^4 - 10a^2b^2 + 5b^4) \cos [c + dx]^5 \sin [c + dx] (a + b \tan [c + dx])^5}{d (a \cos [c + dx] + b \sin [c + dx])^5}
\end{aligned}$$

**Problem 102: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + dx]^5 (a \cos [c + dx] + b \sin [c + dx])^5 dx$$

Optimal (type 3, 147 leaves, 6 steps):

$$a (a^4 - 10 a^2 b^2 + 5 b^4) x - \frac{b (5 a^4 - 10 a^2 b^2 + b^4) \operatorname{Log}[\operatorname{Cos}[c + d x]]}{d} + \frac{4 a b^2 (a^2 - b^2) \operatorname{Tan}[c + d x]}{d} +$$

$$\frac{b (3 a^2 - b^2) (a + b \operatorname{Tan}[c + d x])^2}{2 d} + \frac{2 a b (a + b \operatorname{Tan}[c + d x])^3}{3 d} + \frac{b (a + b \operatorname{Tan}[c + d x])^4}{4 d}$$

Result (type 3, 369 leaves):

$$\frac{b^5 \operatorname{Cos}[c + d x] (a + b \operatorname{Tan}[c + d x])^5}{4 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5} - \frac{b^3 (-5 a^2 + b^2) \operatorname{Cos}[c + d x]^3 (a + b \operatorname{Tan}[c + d x])^5}{d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5} +$$

$$\frac{a (a^4 - 10 a^2 b^2 + 5 b^4) (c + d x) \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Tan}[c + d x])^5}{d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5} + \frac{(-5 a^4 b + 10 a^2 b^3 - b^5) \operatorname{Cos}[c + d x]^5 \operatorname{Log}[\operatorname{Cos}[c + d x]] (a + b \operatorname{Tan}[c + d x])^5}{d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5} +$$

$$\frac{5 a b^4 \operatorname{Cos}[c + d x]^2 \operatorname{Sin}[c + d x] (a + b \operatorname{Tan}[c + d x])^5}{3 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5} + \frac{10 \operatorname{Cos}[c + d x]^4 (3 a^3 b^2 \operatorname{Sin}[c + d x] - 2 a b^4 \operatorname{Sin}[c + d x]) (a + b \operatorname{Tan}[c + d x])^5}{3 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5}$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Sec}[c + d x]^6 (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5 dx$$

Optimal (type 3, 224 leaves, 15 steps):

$$\frac{a^5 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} - \frac{5 a^3 b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{d} + \frac{15 a b^4 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{8 d} +$$

$$\frac{5 a^4 b \operatorname{Sec}[c + d x]}{d} - \frac{10 a^2 b^3 \operatorname{Sec}[c + d x]}{d} + \frac{b^5 \operatorname{Sec}[c + d x]}{d} + \frac{10 a^2 b^3 \operatorname{Sec}[c + d x]^3}{3 d} - \frac{2 b^5 \operatorname{Sec}[c + d x]^3}{3 d} +$$

$$\frac{b^5 \operatorname{Sec}[c + d x]^5}{5 d} + \frac{5 a^3 b^2 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{d} - \frac{15 a b^4 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{8 d} + \frac{5 a b^4 \operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]^3}{4 d}$$

Result (type 3, 1219 leaves):

$$\frac{b (600 a^4 - 1000 a^2 b^2 + 89 b^4) \operatorname{Cos}[c + d x]^5 (a + b \operatorname{Tan}[c + d x])^5}{120 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5} +$$

$$\frac{(-8 a^5 + 40 a^3 b^2 - 15 a b^4) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Tan}[c + d x])^5}{8 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5} +$$

$$\frac{(8 a^5 - 40 a^3 b^2 + 15 a b^4) \operatorname{Cos}[c + d x]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] (a + b \operatorname{Tan}[c + d x])^5}{8 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^5} +$$

$$\begin{aligned}
& \frac{(25 a b^4 + 2 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{80 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{(600 a^3 b^2 + 200 a^2 b^3 - 375 a b^4 - 31 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{240 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{b^5 \cos [c + d x]^5 \sin \left[ \frac{1}{2} (c + d x) \right] (a + b \tan [c + d x])^5}{20 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5 (a \cos [c + d x] + b \sin [c + d x])^5} - \\
& \frac{b^5 \cos [c + d x]^5 \sin \left[ \frac{1}{2} (c + d x) \right] (a + b \tan [c + d x])^5}{20 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{(-25 a b^4 + 2 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{80 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{(-600 a^3 b^2 + 200 a^2 b^3 + 375 a b^4 - 31 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{240 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \left( \cos [c + d x]^5 \left( -600 a^4 b \sin \left[ \frac{1}{2} (c + d x) \right] + 1000 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] - 89 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5 \right) / \\
& \left( 120 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a \cos [c + d x] + b \sin [c + d x])^5 \right) + \\
& \frac{\cos [c + d x]^5 \left( 200 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] - 31 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5}{120 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{\cos [c + d x]^5 \left( -200 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] + 31 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5}{120 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \left( \cos [c + d x]^5 \left( 600 a^4 b \sin \left[ \frac{1}{2} (c + d x) \right] - 1000 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] + 89 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5 \right) / \\
& \left( 120 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a \cos [c + d x] + b \sin [c + d x])^5 \right)
\end{aligned}$$

**Problem 104: Result more than twice size of optimal antiderivative.**

$$\int \sec [c + d x]^7 (a \cos [c + d x] + b \sin [c + d x])^5 dx$$

Optimal (type 3, 30 leaves, 2 steps):

$$\frac{(b + a \operatorname{Cot}[c + dx])^6 \operatorname{Tan}[c + dx]^6}{6bd}$$

Result (type 3, 370 leaves):

$$\begin{aligned} & - \frac{b^3 (-5a^2 + b^2) \operatorname{Cos}[c + dx] (a + b \operatorname{Tan}[c + dx])^5}{2d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} + \frac{b (5a^4 - 10a^2 b^2 + b^4) \operatorname{Cos}[c + dx]^3 (a + b \operatorname{Tan}[c + dx])^5}{2d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} + \frac{b^5 \operatorname{Sec}[c + dx] (a + b \operatorname{Tan}[c + dx])^5}{6d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} + \\ & \frac{a b^4 \operatorname{Sin}[c + dx] (a + b \operatorname{Tan}[c + dx])^5}{d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} + \frac{2 \operatorname{Cos}[c + dx]^2 (5a^3 b^2 \operatorname{Sin}[c + dx] - 3a b^4 \operatorname{Sin}[c + dx]) (a + b \operatorname{Tan}[c + dx])^5}{3d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} + \\ & \frac{\operatorname{Cos}[c + dx]^4 (3a^5 \operatorname{Sin}[c + dx] - 10a^3 b^2 \operatorname{Sin}[c + dx] + 3a b^4 \operatorname{Sin}[c + dx]) (a + b \operatorname{Tan}[c + dx])^5}{3d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} \end{aligned}$$

**Problem 105: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[c + dx]^8 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5 dx$$

Optimal (type 3, 318 leaves, 19 steps):

$$\begin{aligned} & \frac{a^5 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2d} - \frac{5a^3 b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{4d} + \frac{5a b^4 \operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{16d} + \frac{5a^4 b \operatorname{Sec}[c + dx]^3}{3d} - \frac{10a^2 b^3 \operatorname{Sec}[c + dx]^3}{3d} + \\ & \frac{b^5 \operatorname{Sec}[c + dx]^3}{3d} + \frac{2a^2 b^3 \operatorname{Sec}[c + dx]^5}{d} - \frac{2b^5 \operatorname{Sec}[c + dx]^5}{5d} + \frac{b^5 \operatorname{Sec}[c + dx]^7}{7d} + \frac{a^5 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2d} - \frac{5a^3 b^2 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{4d} + \\ & \frac{5a b^4 \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{16d} + \frac{5a^3 b^2 \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{2d} - \frac{5a b^4 \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]}{8d} + \frac{5a b^4 \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]^3}{6d} \end{aligned}$$

Result (type 3, 1677 leaves):

$$\begin{aligned} & \frac{b (1400a^4 - 1540a^2 b^2 + 103b^4) \operatorname{Cos}[c + dx]^5 (a + b \operatorname{Tan}[c + dx])^5}{1680d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} + \\ & \frac{(-8a^5 + 20a^3 b^2 - 5a b^4) \operatorname{Cos}[c + dx]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] (a + b \operatorname{Tan}[c + dx])^5}{16d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} + \\ & \frac{(8a^5 - 20a^3 b^2 + 5a b^4) \operatorname{Cos}[c + dx]^5 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] (a + b \operatorname{Tan}[c + dx])^5}{16d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} + \\ & \frac{(35a b^4 + 3b^5) \operatorname{Cos}[c + dx]^5 (a + b \operatorname{Tan}[c + dx])^5}{336d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^6 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^5} \end{aligned}$$

$$\begin{aligned}
& \frac{(350 a^3 b^2 + 140 a^2 b^3 - 175 a b^4 - 18 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{560 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{(840 a^5 + 1400 a^4 b - 2100 a^3 b^2 - 1540 a^2 b^3 + 525 a b^4 + 103 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{3360 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{b^5 \cos [c + d x]^5 \sin \left[ \frac{1}{2} (c + d x) \right] (a + b \tan [c + d x])^5}{56 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^7 (a \cos [c + d x] + b \sin [c + d x])^5} - \\
& \frac{b^5 \cos [c + d x]^5 \sin \left[ \frac{1}{2} (c + d x) \right] (a + b \tan [c + d x])^5}{56 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^7 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{(-35 a b^4 + 3 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{336 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^6 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{(-350 a^3 b^2 + 140 a^2 b^3 + 175 a b^4 - 18 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{560 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^4 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{(-840 a^5 + 1400 a^4 b + 2100 a^3 b^2 - 1540 a^2 b^3 - 525 a b^4 + 103 b^5) \cos [c + d x]^5 (a + b \tan [c + d x])^5}{3360 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \left( \cos [c + d x]^5 \left( -1400 a^4 b \sin \left[ \frac{1}{2} (c + d x) \right] + 1540 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] - 103 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5 \right) / \\
& \left( 1680 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3 (a \cos [c + d x] + b \sin [c + d x])^5 \right) + \\
& \left( \cos [c + d x]^5 \left( -1400 a^4 b \sin \left[ \frac{1}{2} (c + d x) \right] + 1540 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] - 103 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5 \right) / \\
& \left( 1680 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a \cos [c + d x] + b \sin [c + d x])^5 \right) + \\
& \frac{\cos [c + d x]^5 \left( 70 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] - 9 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5}{140 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \frac{\cos [c + d x]^5 \left( -70 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] + 9 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5}{140 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^5 (a \cos [c + d x] + b \sin [c + d x])^5} + \\
& \left( \cos [c + d x]^5 \left( 1400 a^4 b \sin \left[ \frac{1}{2} (c + d x) \right] - 1540 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] + 103 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5 \right) /
\end{aligned}$$

$$\begin{aligned} & \left( 1680 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) \right)^3 (a \cos [c + d x] + b \sin [c + d x])^5 + \\ & \left( \cos [c + d x]^5 \left( 1400 a^4 b \sin \left[ \frac{1}{2} (c + d x) \right] - 1540 a^2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] + 103 b^5 \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a + b \tan [c + d x])^5 \right) / \\ & \left( 1680 d \left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right) (a \cos [c + d x] + b \sin [c + d x])^5 \right) \end{aligned}$$

**Problem 112: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos [c + d x]^3}{a \cos [c + d x] + b \sin [c + d x]} dx$$

Optimal (type 3, 119 leaves, 5 steps):

$$\frac{a b^2 x}{(a^2 + b^2)^2} + \frac{a x}{2 (a^2 + b^2)} + \frac{b \cos [c + d x]^2}{2 (a^2 + b^2) d} + \frac{b^3 \log [a \cos [c + d x] + b \sin [c + d x]]}{(a^2 + b^2)^2 d} + \frac{a \cos [c + d x] \sin [c + d x]}{2 (a^2 + b^2) d}$$

Result (type 3, 143 leaves):

$$\begin{aligned} & \frac{1}{4 (a^2 + b^2)^2 d} \left( 2 a^3 c + 6 a b^2 c + 4 i b^3 c + 2 a^3 d x + 6 a b^2 d x + 4 i b^3 d x - 4 i b^3 \operatorname{ArcTan}[\tan [c + d x]] + \right. \\ & \left. b (a^2 + b^2) \cos [2 (c + d x)] + 2 b^3 \log [(a \cos [c + d x] + b \sin [c + d x])^2] + a^3 \sin [2 (c + d x)] + a b^2 \sin [2 (c + d x)] \right) \end{aligned}$$

**Problem 119: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^4}{a \cos [c + d x] + b \sin [c + d x]} dx$$

Optimal (type 3, 153 leaves, 7 steps):

$$\begin{aligned} & - \frac{a \operatorname{ArcTanh}[\sin [c + d x]]}{2 b^2 d} - \frac{a (a^2 + b^2) \operatorname{ArcTanh}[\sin [c + d x]]}{b^4 d} - \\ & \frac{(a^2 + b^2)^{3/2} \operatorname{ArcTanh} \left[ \frac{b \cos [c + d x] - a \sin [c + d x]}{\sqrt{a^2 + b^2}} \right]}{b^4 d} + \frac{(a^2 + b^2) \sec [c + d x]}{b^3 d} + \frac{\sec [c + d x]^3}{3 b d} - \frac{a \sec [c + d x] \tan [c + d x]}{2 b^2 d} \end{aligned}$$

Result (type 3, 321 leaves):



$$\frac{1}{24 b^4 d} \left( 48 (a^2 + b^2)^{3/2} \operatorname{ArcTanh} \left[ \frac{-b + a \operatorname{Tan} \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{a^2 + b^2}} \right] + \right. \\
\left. \operatorname{Sec} [c + d x]^3 \left( 12 a^2 b + 20 b^3 + 12 b (a^2 + b^2) \operatorname{Cos} [2 (c + d x)] + 6 a^3 \operatorname{Cos} [3 (c + d x)] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + \right. \right. \\
\left. 9 a b^2 \operatorname{Cos} [3 (c + d x)] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] + 9 a (2 a^2 + 3 b^2) \operatorname{Cos} [c + d x] \right. \\
\left. \left( \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] - \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] - \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] \right) - 6 a^3 \operatorname{Cos} [3 (c + d x)] \right. \\
\left. \left. \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] - 9 a b^2 \operatorname{Cos} [3 (c + d x)] \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] + \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] - 6 a b^2 \operatorname{Sin} [2 (c + d x)] \right) \right) \right)$$

**Problem 121:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [c + d x]^6}{a \operatorname{Cos} [c + d x] + b \operatorname{Sin} [c + d x]} dx$$

Optimal (type 3, 262 leaves, 11 steps):

$$\frac{3 a \operatorname{ArcTanh} [\operatorname{Sin} [c + d x]]}{8 b^2 d} - \frac{a (a^2 + b^2) \operatorname{ArcTanh} [\operatorname{Sin} [c + d x]]}{2 b^4 d} - \frac{a (a^2 + b^2)^2 \operatorname{ArcTanh} [\operatorname{Sin} [c + d x]]}{b^6 d} - \\
\frac{(a^2 + b^2)^{5/2} \operatorname{ArcTanh} \left[ \frac{b \operatorname{Cos} [c + d x] - a \operatorname{Sin} [c + d x]}{\sqrt{a^2 + b^2}} \right]}{b^6 d} + \frac{(a^2 + b^2)^2 \operatorname{Sec} [c + d x]}{b^5 d} + \frac{(a^2 + b^2) \operatorname{Sec} [c + d x]^3}{3 b^3 d} + \frac{\operatorname{Sec} [c + d x]^5}{5 b d} - \\
\frac{3 a \operatorname{Sec} [c + d x] \operatorname{Tan} [c + d x]}{8 b^2 d} - \frac{a (a^2 + b^2) \operatorname{Sec} [c + d x] \operatorname{Tan} [c + d x]}{2 b^4 d} - \frac{a \operatorname{Sec} [c + d x]^3 \operatorname{Tan} [c + d x]}{4 b^2 d}$$

Result (type 3, 1313 leaves):

$$\begin{aligned}
& \frac{(120 a^4 + 260 a^2 b^2 + 149 b^4) \operatorname{Sec}[c + d x] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{120 b^5 d (a + b \operatorname{Tan}[c + d x])} + \frac{1}{b^6 d (a + b \operatorname{Tan}[c + d x])} \\
& 2 (a - i b)^2 (a + i b)^2 \sqrt{a^2 + b^2} \operatorname{ArcTanh}\left[\frac{\sqrt{a^2 + b^2} (-b \operatorname{Cos}[\frac{1}{2}(c + d x)] + a \operatorname{Sin}[\frac{1}{2}(c + d x)])}{a^2 \operatorname{Cos}[\frac{1}{2}(c + d x)] + b^2 \operatorname{Cos}[\frac{1}{2}(c + d x)]}\right] \operatorname{Sec}[c + d x] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x]) + \\
& \frac{1}{8 b^6 d (a + b \operatorname{Tan}[c + d x])} (8 a^5 + 20 a^3 b^2 + 15 a b^4) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}[c + d x] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x]) + \\
& \frac{1}{8 b^6 d (a + b \operatorname{Tan}[c + d x])} (-8 a^5 - 20 a^3 b^2 - 15 a b^4) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \operatorname{Sec}[c + d x] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x]) + \\
& \frac{(-5 a + 2 b) \operatorname{Sec}[c + d x] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{80 b^2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4 (a + b \operatorname{Tan}[c + d x])} + \frac{(-60 a^3 + 20 a^2 b - 105 a b^2 + 29 b^3) \operatorname{Sec}[c + d x] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{240 b^4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 (a + b \operatorname{Tan}[c + d x])} + \\
& \frac{\operatorname{Sec}[c + d x] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{20 b d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^5 (a + b \operatorname{Tan}[c + d x])} - \frac{\operatorname{Sec}[c + d x] \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{20 b d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^5 (a + b \operatorname{Tan}[c + d x])} + \\
& \frac{(5 a + 2 b) \operatorname{Sec}[c + d x] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{80 b^2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^4 (a + b \operatorname{Tan}[c + d x])} + \frac{(60 a^3 + 20 a^2 b + 105 a b^2 + 29 b^3) \operatorname{Sec}[c + d x] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{240 b^4 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^2 (a + b \operatorname{Tan}[c + d x])} + \\
& \frac{\operatorname{Sec}[c + d x] \left(-20 a^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - 29 b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{120 b^3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3 (a + b \operatorname{Tan}[c + d x])} + \\
& \frac{\operatorname{Sec}[c + d x] \left(20 a^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 29 b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])}{120 b^3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right)^3 (a + b \operatorname{Tan}[c + d x])} + \\
& \left(\operatorname{Sec}[c + d x] \left(-120 a^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - 260 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] - 149 b^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])\right) / \\
& \left(120 b^5 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a + b \operatorname{Tan}[c + d x])\right) + \\
& \left(\operatorname{Sec}[c + d x] \left(120 a^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 260 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + 149 b^4 \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])\right) / \\
& \left(120 b^5 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + d x)\right]\right) (a + b \operatorname{Tan}[c + d x])\right)
\end{aligned}$$

**Problem 124:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cos}[c + d x]^2}{(a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} dx$$

Optimal (type 3, 82 leaves, 4 steps):

$$\frac{(a^2 - b^2) x}{(a^2 + b^2)^2} + \frac{2 a b \operatorname{Log}[a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x]]}{(a^2 + b^2)^2 d} - \frac{b}{(a^2 + b^2) d (a + b \operatorname{Tan}[c + d x])}$$

Result (type 3, 192 leaves):

$$\begin{aligned} & (a^2 \operatorname{Cos}[c + d x] \left( (a + i b)^2 (c + d x) + a b \operatorname{Log}[(a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2] \right) + \\ & b \left( (a + i b) (-i b^2 + a b (1 + i c + i d x) + a^2 (c + d x)) + a^2 b \operatorname{Log}[(a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2] \right) \operatorname{Sin}[c + d x] - \\ & 2 i a^2 b \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x]) \Big/ (a (a^2 + b^2)^2 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])) \end{aligned}$$

**Problem 129: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + d x]^3}{(a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])^2} dx$$

Optimal (type 3, 179 leaves, 11 steps):

$$\begin{aligned} & \frac{2 a^2 \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{b^4 d} + \frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{2 b^2 d} + \frac{(a^2 + b^2) \operatorname{ArcTanh}[\operatorname{Sin}[c + d x]]}{b^4 d} + \\ & \frac{3 a \sqrt{a^2 + b^2} \operatorname{ArcTanh}\left[\frac{b \operatorname{Cos}[c + d x] - a \operatorname{Sin}[c + d x]}{\sqrt{a^2 + b^2}}\right]}{b^4 d} - \frac{2 a \operatorname{Sec}[c + d x]}{b^3 d} - \frac{a^2 + b^2}{b^3 d (a \operatorname{Cos}[c + d x] + b \operatorname{Sin}[c + d x])} + \frac{\operatorname{Sec}[c + d x] \operatorname{Tan}[c + d x]}{2 b^2 d} \end{aligned}$$

Result (type 3, 709 leaves):

$$\begin{aligned}
& - \frac{(a - ib)(a + ib) \operatorname{Sec}[c + dx]^2 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])}{b^3 d (a + b \operatorname{Tan}[c + dx])^2} - \frac{2 a \operatorname{Sec}[c + dx]^2 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}{b^3 d (a + b \operatorname{Tan}[c + dx])^2} - \\
& \frac{6 a \sqrt{a^2 + b^2} \operatorname{ArcTanh}\left[\frac{\sqrt{a^2 + b^2} (-b \operatorname{Cos}[\frac{1}{2}(c + dx)] + a \operatorname{Sin}[\frac{1}{2}(c + dx)])}{a^2 \operatorname{Cos}[\frac{1}{2}(c + dx)] + b^2 \operatorname{Cos}[\frac{1}{2}(c + dx)]}\right] \operatorname{Sec}[c + dx]^2 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}{b^4 d (a + b \operatorname{Tan}[c + dx])^2} - \\
& \frac{3 (2 a^2 + b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^2 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}{2 b^4 d (a + b \operatorname{Tan}[c + dx])^2} + \\
& \frac{3 (2 a^2 + b^2) \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \operatorname{Sec}[c + dx]^2 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}{2 b^4 d (a + b \operatorname{Tan}[c + dx])^2} + \\
& \frac{\operatorname{Sec}[c + dx]^2 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}{4 b^2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2 (a + b \operatorname{Tan}[c + dx])^2} - \frac{2 a \operatorname{Sec}[c + dx]^2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}{b^3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right) (a + b \operatorname{Tan}[c + dx])^2} - \\
& \frac{\operatorname{Sec}[c + dx]^2 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}{4 b^2 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right)^2 (a + b \operatorname{Tan}[c + dx])^2} + \frac{2 a \operatorname{Sec}[c + dx]^2 \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right] (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}{b^3 d \left(\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right) (a + b \operatorname{Tan}[c + dx])^2}
\end{aligned}$$

**Problem 131: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c + dx]^4}{(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3} dx$$

Optimal (type 3, 216 leaves, ? steps):

$$\begin{aligned}
& - \frac{3 b^2 (4 a^2 - b^2) \operatorname{ArcTanh}\left[\frac{b - a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 + b^2}}\right]}{(a^2 + b^2)^{7/2} d} + \frac{b (3 a^2 - b^2) \operatorname{Cos}[c + dx]}{(a^2 + b^2)^3 d} + \frac{a (a^2 - 3 b^2) \operatorname{Sin}[c + dx]}{(a^2 + b^2)^3 d} + \\
& \frac{b^4 \operatorname{Sin}[c + dx]}{2 a (a^2 + b^2)^2 d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2} - \frac{b^3 (8 a^2 + b^2)}{2 a (a^2 + b^2)^3 d (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])}
\end{aligned}$$

Result (type 3, 211 leaves):

$$\frac{1}{2d} \left( \frac{6b^2(-4a^2 + b^2) \operatorname{ArcTanh}\left[\frac{-b + a \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 + b^2}}\right]}{(a^2 + b^2)^{7/2}} - \frac{2b(-3a^2 + b^2) \operatorname{Cos}[c + dx]}{(a^2 + b^2)^3} + \frac{2a(a^2 - 3b^2) \operatorname{Sin}[c + dx]}{(a^2 + b^2)^3} + \frac{b^4 \operatorname{Sin}[c + dx]}{a(a - ib)^2(a + ib)^2(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2} - \frac{b^3(8a^2 + b^2)}{a(a^2 + b^2)^3(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])} \right)$$

**Problem 132: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[c + dx]^3}{(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3} dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$\frac{a(a^2 - 3b^2)x}{(a^2 + b^2)^3} + \frac{b(3a^2 - b^2) \operatorname{Log}[a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx]]}{(a^2 + b^2)^3 d} - \frac{b}{2(a^2 + b^2)d(a + b \operatorname{Tan}[c + dx])^2} - \frac{2ab}{(a^2 + b^2)^2 d(a + b \operatorname{Tan}[c + dx])}$$

Result (type 3, 154 leaves):

$$\frac{1}{2d} \left( \frac{2a(a^2 - 3b^2)(c + dx)}{(a^2 + b^2)^3} - \frac{2b(-3a^2 + b^2) \operatorname{Log}[a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx]]}{(a^2 + b^2)^3} - \frac{b^3}{(a - ib)^2(a + ib)^2(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2} + \frac{6b^2 \operatorname{Sin}[c + dx]}{(a^2 + b^2)^2(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])} \right)$$

**Problem 134: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c + dx]}{(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3} dx$$

Optimal (type 3, 22 leaves, 2 steps):

$$-\frac{1}{2bd(a + b \operatorname{Tan}[c + dx])^2}$$

Result (type 3, 57 leaves):

$$\frac{-b \operatorname{Cos}[2(c + dx)] + a \operatorname{Sin}[2(c + dx)]}{2(a^2 + b^2)d(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2}$$

### Problem 135: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a \cos [c+d x]+b \sin [c+d x])^3} d x$$

Optimal (type 3, 103 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{b \cos [c+d x]-a \sin [c+d x]}{\sqrt{a^2+b^2}}\right]}{2\left(a^2+b^2\right)^{3/2} d}-\frac{b \cos [c+d x]-a \sin [c+d x]}{2\left(a^2+b^2\right) d\left(a \cos [c+d x]+b \sin [c+d x]\right)^2}$$

Result (type 3, 132 leaves):

$$\left(\left(a^2+b^2\right)\left(-b \cos [c+d x]+a \sin [c+d x]\right)+2 \sqrt{a^2+b^2} \operatorname{ArcTanh}\left[\frac{-b+a \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]\left(a \cos [c+d x]+b \sin [c+d x]\right)^2\right) / \left(2(a-i b)^2(a+i b)^2 d\left(a \cos [c+d x]+b \sin [c+d x]\right)^2\right)$$

### Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sec [c+d x]^4}{(a \cos [c+d x]+b \sin [c+d x])^3} d x$$

Optimal (type 3, 383 leaves, 31 steps):

$$\begin{aligned} & -\frac{4 a^3 \operatorname{ArcTanh}[\sin [c+d x]]}{b^6 d}-\frac{3 a \operatorname{ArcTanh}[\sin [c+d x]]}{2 b^4 d}-\frac{6 a\left(a^2+b^2\right) \operatorname{ArcTanh}[\sin [c+d x]]}{b^6 d}-\frac{8 a^2 \sqrt{a^2+b^2} \operatorname{ArcTanh}\left[\frac{b \cos [c+d x]-a \sin [c+d x]}{\sqrt{a^2+b^2}}\right]}{b^6 d} \\ & -\frac{\sqrt{a^2+b^2} \operatorname{ArcTanh}\left[\frac{b \cos [c+d x]-a \sin [c+d x]}{\sqrt{a^2+b^2}}\right]}{2 b^4 d}-\frac{2\left(a^2+b^2\right)^{3/2} \operatorname{ArcTanh}\left[\frac{b \cos [c+d x]-a \sin [c+d x]}{\sqrt{a^2+b^2}}\right]}{b^6 d}+\frac{4 a^2 \sec [c+d x]}{b^5 d}+\frac{2\left(a^2+b^2\right) \sec [c+d x]}{b^5 d}+ \\ & \frac{\sec [c+d x]^3}{3 b^3 d}-\frac{\left(a^2+b^2\right)\left(b \cos [c+d x]-a \sin [c+d x]\right)}{2 b^4 d\left(a \cos [c+d x]+b \sin [c+d x]\right)^2}+\frac{4 a\left(a^2+b^2\right)}{b^5 d\left(a \cos [c+d x]+b \sin [c+d x]\right)}-\frac{3 a \sec [c+d x] \tan [c+d x]}{2 b^4 d} \end{aligned}$$

Result (type 3, 688 leaves):

$$\frac{1}{12 b^6 d (a + b \tan [c + d x])^3} \sec [c + d x]^3 (a \cos [c + d x] + b \sin [c + d x]) \left( \frac{6 b^2 (a^2 + b^2)^2 \sin [c + d x]}{a} + \frac{6 (a - i b) (a + i b) b (8 a^2 - b^2) (a \cos [c + d x] + b \sin [c + d x])}{a} \right) +$$

$$2 b (36 a^2 + 13 b^2) (a \cos [c + d x] + b \sin [c + d x])^2 + 6 \theta \sqrt{a^2 + b^2} (4 a^2 + b^2) \operatorname{ArcTanh} \left[ \frac{-b + a \tan \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{a^2 + b^2}} \right] (a \cos [c + d x] + b \sin [c + d x])^2 +$$

$$30 a (4 a^2 + 3 b^2) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right] (a \cos [c + d x] + b \sin [c + d x])^2 -$$

$$30 a (4 a^2 + 3 b^2) \operatorname{Log} \left[ \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right] (a \cos [c + d x] + b \sin [c + d x])^2 +$$

$$\frac{b^2 (-9 a + b) (a \cos [c + d x] + b \sin [c + d x])^2}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} + \frac{2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] (a \cos [c + d x] + b \sin [c + d x])^2}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3} +$$

$$\frac{2 b (36 a^2 + 13 b^2) \sin \left[ \frac{1}{2} (c + d x) \right] (a \cos [c + d x] + b \sin [c + d x])^2}{\cos \left[ \frac{1}{2} (c + d x) \right] - \sin \left[ \frac{1}{2} (c + d x) \right]} - \frac{2 b^3 \sin \left[ \frac{1}{2} (c + d x) \right] (a \cos [c + d x] + b \sin [c + d x])^2}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^3} +$$

$$\left. \frac{b^2 (9 a + b) (a \cos [c + d x] + b \sin [c + d x])^2}{\left( \cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right] \right)^2} - \frac{2 b (36 a^2 + 13 b^2) \sin \left[ \frac{1}{2} (c + d x) \right] (a \cos [c + d x] + b \sin [c + d x])^2}{\cos \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{1}{2} (c + d x) \right]} \right)$$

**Problem 140: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec [c + d x]^5}{(a \cos [c + d x] + b \sin [c + d x])^3} dx$$

Optimal (type 3, 232 leaves, 3 steps):

$$-\frac{(a^2 + b^2)^3}{2 a^2 b^5 d (b + a \cot [c + d x])^2} - \frac{(5 a^2 - b^2) (a^2 + b^2)^2}{a^2 b^6 d (b + a \cot [c + d x])} + \frac{3 (a^2 + b^2) (5 a^2 + b^2) \operatorname{Log} [b + a \cot [c + d x]]}{b^7 d} +$$

$$\frac{3 (a^2 + b^2) (5 a^2 + b^2) \operatorname{Log} [\tan [c + d x]]}{b^7 d} - \frac{a (10 a^2 + 9 b^2) \tan [c + d x]}{b^6 d} + \frac{3 (2 a^2 + b^2) \tan [c + d x]^2}{2 b^5 d} - \frac{a \tan [c + d x]^3}{b^4 d} + \frac{\tan [c + d x]^4}{4 b^3 d}$$

Result (type 3, 530 leaves):

$$\begin{aligned}
& - \frac{(a - ib)^2 (a + ib)^2 \operatorname{Sec}[c + dx]^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])}{2 b^5 d (a + b \operatorname{Tan}[c + dx])^3} - \\
& \frac{3 (5 a^4 + 6 a^2 b^2 + b^4) \operatorname{Log}[\operatorname{Cos}[c + dx]] \operatorname{Sec}[c + dx]^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3}{b^7 d (a + b \operatorname{Tan}[c + dx])^3} + \\
& \frac{3 (5 a^4 + 6 a^2 b^2 + b^4) \operatorname{Log}[a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx]] \operatorname{Sec}[c + dx]^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3}{b^7 d (a + b \operatorname{Tan}[c + dx])^3} + \\
& \frac{(3 a^2 + b^2) \operatorname{Sec}[c + dx]^5 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3}{b^5 d (a + b \operatorname{Tan}[c + dx])^3} + \frac{\operatorname{Sec}[c + dx]^7 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3}{4 b^3 d (a + b \operatorname{Tan}[c + dx])^3} - \\
& \frac{2 \operatorname{Sec}[c + dx]^4 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3 (5 a^3 \operatorname{Sin}[c + dx] + 4 a b^2 \operatorname{Sin}[c + dx])}{b^6 d (a + b \operatorname{Tan}[c + dx])^3} - \\
& \frac{5 \operatorname{Sec}[c + dx]^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^2 (a^4 \operatorname{Sin}[c + dx] + 2 a^2 b^2 \operatorname{Sin}[c + dx] + b^4 \operatorname{Sin}[c + dx])}{b^6 d (a + b \operatorname{Tan}[c + dx])^3} - \\
& \frac{a \operatorname{Sec}[c + dx]^5 (a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^3 \operatorname{Tan}[c + dx]}{b^4 d (a + b \operatorname{Tan}[c + dx])^3}
\end{aligned}$$

**Problem 141:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cos}[c + dx]^4}{(a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx])^4} dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\begin{aligned}
& \frac{(a^4 - 6 a^2 b^2 + b^4) x}{(a^2 + b^2)^4} + \frac{4 a b (a^2 - b^2) \operatorname{Log}[a \operatorname{Cos}[c + dx] + b \operatorname{Sin}[c + dx]]}{(a^2 + b^2)^4 d} - \\
& \frac{b}{3 (a^2 + b^2) d (a + b \operatorname{Tan}[c + dx])^3} - \frac{a b}{(a^2 + b^2)^2 d (a + b \operatorname{Tan}[c + dx])^2} - \frac{b (3 a^2 - b^2)}{(a^2 + b^2)^3 d (a + b \operatorname{Tan}[c + dx])}
\end{aligned}$$

Result (type 3, 419 leaves):



$$\frac{(a^2 - 2ab - b^2)(a^2 + 2ab - b^2)(c + dx)}{(a - ib)^4(a + ib)^4d} + \frac{4(i a^{10}b + a^9b^2 + 2i a^8b^3 + 2a^7b^4 - 2i a^4b^7 - 2a^3b^8 - i a^2b^9 - a b^{10})(c + dx)}{(a - ib)^8(a + ib)^7d} - \frac{4i(a^3b - ab^3)\text{ArcTan}[\text{Tan}[c + dx]]}{(a^2 + b^2)^4d} + \frac{2(a^3b - ab^3)\text{Log}[(a \cos[c + dx] + b \sin[c + dx])^2]}{(a^2 + b^2)^4d} + \frac{b^4 \sin[c + dx]}{3a(a - ib)^2(a + ib)^2d(a \cos[c + dx] + b \sin[c + dx])^3} - \frac{b^3(6a^2 + b^2)}{3a(a - ib)^3(a + ib)^3d(a \cos[c + dx] + b \sin[c + dx])^2} + \frac{2(9a^2b^2 \sin[c + dx] - 2b^4 \sin[c + dx])}{3a(a - ib)^3(a + ib)^3d(a \cos[c + dx] + b \sin[c + dx])}$$

**Problem 142: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[c + dx]^3}{(a \cos[c + dx] + b \sin[c + dx])^4} dx$$

Optimal (type 3, 157 leaves, ? steps):

$$\frac{a(2a^2 - 3b^2)\text{ArcTanh}\left[\frac{-b + a \tan\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 + b^2}}\right]}{(a^2 + b^2)^{7/2}d} + \frac{-3(3a^4b - a^2b^3 + b^5)\cos[2(c + dx)] + \frac{1}{2}b(-9a^2 + b^2)(2(a^2 + b^2) + 3ab \sin[2(c + dx)])}{6(a^2 + b^2)^3d(a \cos[c + dx] + b \sin[c + dx])^3}$$

Result (type 3, 165 leaves):

$$\frac{6a(2a^2 - 3b^2)\text{ArcTanh}\left[\frac{-b + a \tan\left[\frac{1}{2}(c + dx)\right]}{\sqrt{a^2 + b^2}}\right]}{(a^2 + b^2)^{7/2}} + \frac{-3(3a^4b - a^2b^3 + b^5)\cos[2(c + dx)] + \frac{1}{2}b(-9a^2 + b^2)(2(a^2 + b^2) + 3ab \sin[2(c + dx)])}{(a - ib)^3(a + ib)^3(a \cos[c + dx] + b \sin[c + dx])^3}$$

6 d

**Problem 143: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[c + dx]^2}{(a \cos[c + dx] + b \sin[c + dx])^4} dx$$

Optimal (type 3, 30 leaves, 2 steps):

$$-\frac{\cot[c + dx]^3}{3bd(b + a \cot[c + dx])^3}$$

Result (type 3, 124 leaves):

$$\frac{(-6ab(a^2+b^2)\cos[c+dx] + (-6a^3b+2a^3b^3)\cos[3(c+dx)] + 2(a^2-b^2)(3a^2+b^2+(3a^2-b^2)\cos[2(c+dx)])\sin[c+dx])}{(12a(a^2+b^2)^2d(a\cos[c+dx]+b\sin[c+dx])^3)}$$

**Problem 146: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]}{(a\cos[c+dx]+b\sin[c+dx])^4} dx$$

Optimal (type 3, 231 leaves, 8 steps):

$$\frac{\operatorname{ArcTanh}[\sin[c+dx]]}{b^4 d} + \frac{a \operatorname{ArcTanh}\left[\frac{b\cos[c+dx]-a\sin[c+dx]}{\sqrt{a^2+b^2}}\right]}{2b^2(a^2+b^2)^{3/2}d} + \frac{a \operatorname{ArcTanh}\left[\frac{b\cos[c+dx]-a\sin[c+dx]}{\sqrt{a^2+b^2}}\right]}{b^4\sqrt{a^2+b^2}d} - \frac{1}{3bd(a\cos[c+dx]+b\sin[c+dx])^3} + \frac{a(b\cos[c+dx]-a\sin[c+dx])}{2b^2(a^2+b^2)d(a\cos[c+dx]+b\sin[c+dx])^2} - \frac{1}{b^3d(a\cos[c+dx]+b\sin[c+dx])}$$

Result (type 3, 478 leaves):

$$\begin{aligned} & - \frac{\sec[c+dx]^4(a\cos[c+dx]+b\sin[c+dx])}{3bd(a+b\tan[c+dx])^4} + \frac{(-2a^2-b^2)\sec[c+dx]^4(a\cos[c+dx]+b\sin[c+dx])^3}{2b^3(-ia+b)(ia+b)d(a+b\tan[c+dx])^4} - \\ & \left( a\sqrt{a^2+b^2}(2a^2+3b^2)\operatorname{ArcTanh}\left[\frac{\sqrt{a^2+b^2}(-b\cos[\frac{1}{2}(c+dx)]+a\sin[\frac{1}{2}(c+dx)])}{a^2\cos[\frac{1}{2}(c+dx)]+b^2\cos[\frac{1}{2}(c+dx)]}\right] \sec[c+dx]^4(a\cos[c+dx]+b\sin[c+dx])^4 \right) / \\ & \left( (a^4b^4+2a^2b^6+b^8)d(a+b\tan[c+dx])^4 \right) - \frac{\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]-\sin\left[\frac{1}{2}(c+dx)\right]\right] \sec[c+dx]^4(a\cos[c+dx]+b\sin[c+dx])^4}{b^4d(a+b\tan[c+dx])^4} + \\ & \frac{\operatorname{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right]+\sin\left[\frac{1}{2}(c+dx)\right]\right] \sec[c+dx]^4(a\cos[c+dx]+b\sin[c+dx])^4}{b^4d(a+b\tan[c+dx])^4} - \\ & \frac{\sec[c+dx]^3(a\cos[c+dx]+b\sin[c+dx])^2\tan[c+dx]}{2b^2d(a+b\tan[c+dx])^4} \end{aligned}$$

**Problem 169: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[c+dx]}{(a\cos[c+dx]+ia\sin[c+dx])^2} dx$$

Optimal (type 3, 46 leaves, 8 steps):

$$-\frac{\text{ArcTanh}[\text{Sin}[c + d x]]}{a^2 d} + \frac{2 i \text{Cos}[c + d x]}{a^2 d} + \frac{2 \text{Sin}[c + d x]}{a^2 d}$$

Result (type 3, 184 leaves):

$$-\frac{1}{a^2 d (-i + \text{Tan}[c + d x])^2} \text{Sec}[c + d x]^2 \left( \text{Cos}\left[\frac{1}{2}(c + d x)\right] \left( 2 i + \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) + \left( 2 + i \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - i \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) \text{Sin}\left[\frac{1}{2}(c + d x)\right] \right) \left( \text{Cos}\left[\frac{3}{2}(c + d x)\right] + i \text{Sin}\left[\frac{3}{2}(c + d x)\right] \right)$$

**Problem 171: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^3}{(a \text{Cos}[c + d x] + i a \text{Sin}[c + d x])^2} dx$$

Optimal (type 3, 56 leaves, 8 steps):

$$\frac{3 \text{ArcTanh}[\text{Sin}[c + d x]]}{2 a^2 d} - \frac{2 i \text{Sec}[c + d x]}{a^2 d} - \frac{\text{Sec}[c + d x] \text{Tan}[c + d x]}{2 a^2 d}$$

Result (type 3, 146 leaves):

$$-\frac{1}{4 a^2 d} \text{Sec}[c + d x]^2 \left( 8 i \text{Cos}[c + d x] + 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 3 \text{Cos}[2(c + d x)] \left( \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] - \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] \right) - 3 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right] + \text{Sin}\left[\frac{1}{2}(c + d x)\right]\right] + 2 \text{Sin}[c + d x] \right)$$

**Problem 173: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[c + d x]^5}{(a \text{Cos}[c + d x] + i a \text{Sin}[c + d x])^2} dx$$

Optimal (type 3, 84 leaves, 10 steps):

$$\frac{5 \text{ArcTanh}[\text{Sin}[c + d x]]}{8 a^2 d} - \frac{2 i \text{Sec}[c + d x]^3}{3 a^2 d} + \frac{5 \text{Sec}[c + d x] \text{Tan}[c + d x]}{8 a^2 d} - \frac{\text{Sec}[c + d x]^3 \text{Tan}[c + d x]}{4 a^2 d}$$

Result (type 3, 215 leaves):

$$\begin{aligned}
& -\frac{1}{192 a^2 d} \operatorname{Sec}[c+d x]^4 \left( 128 i \operatorname{Cos}[c+d x] + 45 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
& 60 \operatorname{Cos}[2(c+d x)] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) + \\
& 15 \operatorname{Cos}[4(c+d x)] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] - \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] \right) - \\
& 45 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c+d x)\right] + \operatorname{Sin}\left[\frac{1}{2}(c+d x)\right]\right] + 18 \operatorname{Sin}[c+d x] - 30 \operatorname{Sin}[3(c+d x)] \Big)
\end{aligned}$$

**Problem 179: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[c+d x]}{(a \operatorname{Cos}[c+d x] + i a \operatorname{Sin}[c+d x])^3} dx$$

Optimal (type 3, 32 leaves, 2 steps):

$$\frac{i \operatorname{Cot}[c+d x]^2}{2 a^3 d (i + \operatorname{Cot}[c+d x])^2}$$

Result (type 3, 77 leaves):

$$\frac{i \operatorname{Cos}[2(c+d x)]}{4 a^3 d} + \frac{i \operatorname{Cos}[4(c+d x)]}{8 a^3 d} + \frac{\operatorname{Sin}[2(c+d x)]}{4 a^3 d} + \frac{\operatorname{Sin}[4(c+d x)]}{8 a^3 d}$$

**Problem 185: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c+d x]^5}{(a \operatorname{Cos}[c+d x] + i a \operatorname{Sin}[c+d x])^3} dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{i (i - \operatorname{Cot}[c+d x])^4 \operatorname{Tan}[c+d x]^4}{4 a^3 d}$$

Result (type 3, 90 leaves):

$$-\frac{1}{4 a^3 d} i \operatorname{Sec}[c] \operatorname{Sec}[c+d x]^4 (3 \operatorname{Cos}[c] + 2 \operatorname{Cos}[c+2 d x] + 2 \operatorname{Cos}[3 c+2 d x] - 3 i \operatorname{Sin}[c] + 2 i \operatorname{Sin}[c+2 d x] - 2 i \operatorname{Sin}[3 c+2 d x] + i \operatorname{Sin}[3 c+4 d x])$$

**Problem 188: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Sec}[x] + \operatorname{Tan}[x]} dx$$

Optimal (type 3, 5 leaves, 3 steps):

$$\text{Log}[1 + \text{Sin}[x]]$$

Result (type 3, 16 leaves):

$$2 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]$$

**Problem 191: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[x]}{\text{Sec}[x] + \text{Tan}[x]} dx$$

Optimal (type 3, 11 leaves, 3 steps):

$$x + \frac{\text{Cos}[x]}{1 + \text{Sin}[x]}$$

Result (type 3, 25 leaves):

$$x - \frac{2 \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]}$$

**Problem 192: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Cot}[x]}{\text{Sec}[x] + \text{Tan}[x]} dx$$

Optimal (type 3, 9 leaves, 4 steps):

$$-x - \text{ArcTanh}[\text{Cos}[x]]$$

Result (type 3, 20 leaves):

$$-x - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

**Problem 193: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[x]}{\text{Sec}[x] + \text{Tan}[x]} dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\frac{\cos[x]}{1 + \sin[x]}$$

Result (type 3, 23 leaves):

$$\frac{2 \sin\left[\frac{x}{2}\right]}{\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]}$$

**Problem 199: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cot[x]}{\sec[x] - \tan[x]} dx$$

Optimal (type 3, 7 leaves, 4 steps):

$$x - \operatorname{ArcTanh}[\cos[x]]$$

Result (type 3, 18 leaves):

$$x - \log\left[\cos\left[\frac{x}{2}\right]\right] + \log\left[\sin\left[\frac{x}{2}\right]\right]$$

**Problem 200: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[x]}{\sec[x] - \tan[x]} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{\cos[x]}{1 - \sin[x]}$$

Result (type 3, 25 leaves):

$$\frac{2 \sin\left[\frac{x}{2}\right]}{\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}$$

**Problem 203: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sin[x]}{\cot[x] + \csc[x]} dx$$

Optimal (type 3, 6 leaves, 3 steps):

$$x - \sin[x]$$

Result (type 3, 14 leaves):

$$2 \left( \frac{x}{2} - \frac{\text{Sin}[x]}{2} \right)$$

**Problem 205: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[x]}{\text{Cot}[x] + \text{Csc}[x]} dx$$

Optimal (type 3, 7 leaves, 4 steps):

$$-x + \text{ArcTanh}[\text{Sin}[x]]$$

Result (type 3, 36 leaves):

$$-x - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]$$

**Problem 209: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[x]}{-\text{Cot}[x] + \text{Csc}[x]} dx$$

Optimal (type 3, 4 leaves, 3 steps):

$$x + \text{Sin}[x]$$

Result (type 3, 14 leaves):

$$2 \left( \frac{x}{2} + \frac{\text{Sin}[x]}{2} \right)$$

**Problem 211: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[x]}{-\text{Cot}[x] + \text{Csc}[x]} dx$$

Optimal (type 3, 5 leaves, 4 steps):

$$x + \text{ArcTanh}[\text{Sin}[x]]$$

Result (type 3, 46 leaves):

$$2 \left( \frac{x}{2} - \frac{1}{2} \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \frac{1}{2} \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] \right)$$

**Problem 215:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\operatorname{Csc}[c + dx] + \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 23 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\operatorname{Cos}[c+dx]}{\sqrt{2}}\right]}{\sqrt{2} d}$$

Result (type 3, 61 leaves):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\operatorname{Cos}[c] - (-i + \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right]}{\sqrt{2}}\right] + \operatorname{ArcTanh}\left[\frac{\operatorname{Cos}[c] - (i + \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{dx}{2}\right]}{\sqrt{2}}\right]}{\sqrt{2} d}$$

**Problem 218:** Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tan}[c + dx]}{\operatorname{Csc}[c + dx] + \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 29 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}[\operatorname{Sin}[c + dx]]}{2d} + \frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2d}$$

Result (type 3, 63 leaves):

$$-\frac{1}{2d} \left( \operatorname{ArcTan}[\operatorname{Sin}[c + dx]] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right)$$

**Problem 225:** Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tan}[c + dx]}{\operatorname{Csc}[c + dx] - \operatorname{Sin}[c + dx]} dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Sin}[c + dx]]}{2d} + \frac{\operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]}{2d}$$

Result (type 3, 69 leaves):

$$\frac{1}{2d} \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] - \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(c + dx)\right] + \operatorname{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx] \right)$$



### Problem 226: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[c + dx]}{\text{Csc}[c + dx] - \text{Sin}[c + dx]} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[c + dx]]}{d}$$

Result (type 3, 68 leaves):

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] - \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d} + \frac{\text{Log}\left[\text{Cos}\left[\frac{c}{2} + \frac{dx}{2}\right] + \text{Sin}\left[\frac{c}{2} + \frac{dx}{2}\right]\right]}{d}$$

### Problem 240: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[c + dx] (a \text{Sin}[c + dx] + b \text{Tan}[c + dx])^2 dx$$

Optimal (type 3, 90 leaves, 7 steps):

$$-2abx + \frac{(2a^2 - b^2) \text{ArcTanh}[\text{Sin}[c + dx]]}{2d} - \frac{3a^2 \text{Sin}[c + dx]}{2d} + \frac{ab \text{Tan}[c + dx]}{d} + \frac{(b + a \text{Cos}[c + dx])^2 \text{Sec}[c + dx] \text{Tan}[c + dx]}{2d}$$

Result (type 3, 265 leaves):

$$\begin{aligned} & -\frac{1}{4d} \text{Sec}[c + dx]^2 \left( 4abc + 4abd + 2a^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - b^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] - \right. \\ & \quad \left. 2a^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + b^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + \text{Cos}[2(c + dx)] \right. \\ & \quad \left. \left( 4ab(c + dx) + (2a^2 - b^2) \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right] - \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] + (-2a^2 + b^2) \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + dx)\right] + \text{Sin}\left[\frac{1}{2}(c + dx)\right]\right] \right) + \right. \\ & \quad \left. (a^2 - 2b^2) \text{Sin}[c + dx] - 4ab \text{Sin}[2(c + dx)] + a^2 \text{Sin}[3(c + dx)] \right) \end{aligned}$$

### Problem 241: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[c + dx]^2 (a \text{Sin}[c + dx] + b \text{Tan}[c + dx])^2 dx$$

Optimal (type 3, 99 leaves, 7 steps):

$$-a^2 x - \frac{a b \operatorname{ArcTanh}[\sin [c+d x]]}{d} + \frac{(2 a^2 - b^2) \tan [c+d x]}{3 d} + \frac{a b \sec [c+d x] \tan [c+d x]}{3 d} + \frac{(b+a \cos [c+d x])^2 \sec [c+d x]^2 \tan [c+d x]}{3 d}$$

Result (type 3, 201 leaves):

$$\begin{aligned} & \frac{1}{12 d} \sec [c+d x]^3 \left( -9 a \cos [c+d x] \left( a (c+d x) - b \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] + b \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] \right) - \right. \\ & \quad \left. 3 a \cos [3 (c+d x)] \left( a (c+d x) - b \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] + b \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] \right) + \right. \\ & \quad \left. 2 \left( 3 a^2 + b^2 + 6 a b \cos [c+d x] + (3 a^2 - b^2) \cos [2 (c+d x)] \right) \sin [c+d x] \right) \end{aligned}$$

**Problem 242: Result more than twice size of optimal antiderivative.**

$$\int \sec [c+d x]^3 (a \sin [c+d x] + b \tan [c+d x])^2 dx$$

Optimal (type 3, 125 leaves, 9 steps):

$$\begin{aligned} & -\frac{(4 a^2 + b^2) \operatorname{ArcTanh}[\sin [c+d x]]}{8 d} - \frac{2 a b \tan [c+d x]}{3 d} + \frac{(2 a^2 - b^2) \sec [c+d x] \tan [c+d x]}{8 d} + \\ & \frac{a b \sec [c+d x]^2 \tan [c+d x]}{6 d} + \frac{(b+a \cos [c+d x])^2 \sec [c+d x]^3 \tan [c+d x]}{4 d} \end{aligned}$$

Result (type 3, 336 leaves):

$$\begin{aligned} & \frac{1}{192 d} \sec [c+d x]^4 \left( 36 a^2 \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] + 9 b^2 \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] + \right. \\ & \quad \left. 12 (4 a^2 + b^2) \cos [2 (c+d x)] \left( \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] \right) + \right. \\ & \quad \left. 3 (4 a^2 + b^2) \cos [4 (c+d x)] \left( \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] - \sin \left[ \frac{1}{2} (c+d x) \right] \right] - \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] \right) - \right. \\ & \quad \left. 36 a^2 \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] - 9 b^2 \log \left[ \cos \left[ \frac{1}{2} (c+d x) \right] + \sin \left[ \frac{1}{2} (c+d x) \right] \right] + 24 a^2 \sin [c+d x] + \right. \\ & \quad \left. 42 b^2 \sin [c+d x] + 32 a b \sin [2 (c+d x)] + 24 a^2 \sin [3 (c+d x)] - 6 b^2 \sin [3 (c+d x)] - 16 a b \sin [4 (c+d x)] \right) \end{aligned}$$

**Problem 264: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c+d x]^3}{(a \sin [c+d x] + b \tan [c+d x])^3} dx$$

Optimal (type 3, 248 leaves, 6 steps):

$$\frac{b^6}{2 a^3 (a^2 - b^2)^2 d (b + a \cos [c + d x])^2} - \frac{2 b^5 (3 a^2 - b^2)}{a^3 (a^2 - b^2)^3 d (b + a \cos [c + d x])} - \frac{(a (a^2 + 3 b^2) - b (3 a^2 + b^2) \cos [c + d x]) \operatorname{Csc}[c + d x]^2}{2 (a^2 - b^2)^3 d} - \frac{(2 a + 5 b) \operatorname{Log}[1 - \cos [c + d x]]}{4 (a + b)^4 d} - \frac{(2 a - 5 b) \operatorname{Log}[1 + \cos [c + d x]]}{4 (a - b)^4 d} - \frac{b^4 (15 a^4 - 4 a^2 b^2 + b^4) \operatorname{Log}[b + a \cos [c + d x]]}{a^3 (a^2 - b^2)^4 d}$$

Result (type 3, 713 leaves):

$$\frac{b^6 (b + a \cos [c + d x]) \operatorname{Tan}[c + d x]^3}{2 a^3 (-a + b)^2 (a + b)^2 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} - \frac{2 b^5 (-3 a^2 + b^2) (b + a \cos [c + d x])^2 \operatorname{Tan}[c + d x]^3}{a^3 (-a + b)^3 (a + b)^3 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} - \frac{2 i (a^5 - 4 a^3 b^2 - 9 a b^4) (c + d x) (b + a \cos [c + d x])^3 \operatorname{Tan}[c + d x]^3}{(a - b)^4 (a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} - \frac{i (-2 a - 5 b) \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (b + a \cos [c + d x])^3 \operatorname{Tan}[c + d x]^3}{2 (a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} - \frac{i (-2 a + 5 b) \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (b + a \cos [c + d x])^3 \operatorname{Tan}[c + d x]^3}{2 (-a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} - \frac{(b + a \cos [c + d x])^3 \operatorname{Csc}[\frac{1}{2} (c + d x)]^2 \operatorname{Tan}[c + d x]^3}{8 (a + b)^3 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} + \frac{(-2 a + 5 b) (b + a \cos [c + d x])^3 \operatorname{Log}[\cos[\frac{1}{2} (c + d x)]]^2 \operatorname{Tan}[c + d x]^3}{4 (-a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} + \frac{(-15 a^4 b^4 + 4 a^2 b^6 - b^8) (b + a \cos [c + d x])^3 \operatorname{Log}[b + a \cos [c + d x]] \operatorname{Tan}[c + d x]^3}{a^3 (-a^2 + b^2)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} + \frac{(-2 a - 5 b) (b + a \cos [c + d x])^3 \operatorname{Log}[\sin[\frac{1}{2} (c + d x)]]^2 \operatorname{Tan}[c + d x]^3}{4 (a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} + \frac{(b + a \cos [c + d x])^3 \operatorname{Sec}[\frac{1}{2} (c + d x)]^2 \operatorname{Tan}[c + d x]^3}{8 (-a + b)^3 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3}$$

Problem 265: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos [c + d x]^2}{(a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} dx$$

Optimal (type 3, 232 leaves, 6 steps):

$$- \frac{b^5}{2 a^2 (a^2 - b^2)^2 d (b + a \cos [c + d x])^2} + \frac{b^4 (5 a^2 - b^2)}{a^2 (a^2 - b^2)^3 d (b + a \cos [c + d x])} + \frac{(b (3 a^2 + b^2) - a (a^2 + 3 b^2) \cos [c + d x]) \operatorname{Csc}[c + d x]^2}{2 (a^2 - b^2)^3 d} + \frac{(a + 4 b) \operatorname{Log}[1 - \cos [c + d x]]}{4 (a + b)^4 d} + \frac{(a - 4 b) \operatorname{Log}[1 + \cos [c + d x]]}{4 (a - b)^4 d} + \frac{2 b^3 (5 a^2 + b^2) \operatorname{Log}[b + a \cos [c + d x]]}{(a^2 - b^2)^4 d}$$

Result (type 3, 477 leaves):

$$\begin{aligned}
& - \frac{b^5 (b + a \cos [c + d x]) \tan [c + d x]^3}{2 a^2 (-a + b)^2 (a + b)^2 d (a \sin [c + d x] + b \tan [c + d x])^3} + \frac{b^4 (-5 a^2 + b^2) (b + a \cos [c + d x])^2 \tan [c + d x]^3}{a^2 (-a + b)^3 (a + b)^3 d (a \sin [c + d x] + b \tan [c + d x])^3} - \\
& \frac{(b + a \cos [c + d x])^3 \operatorname{Csc} \left[ \frac{1}{2} (c + d x) \right]^2 \tan [c + d x]^3}{8 (a + b)^3 d (a \sin [c + d x] + b \tan [c + d x])^3} + \frac{(a - 4 b) (b + a \cos [c + d x])^3 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] \right] \tan [c + d x]^3}{2 (-a + b)^4 d (a \sin [c + d x] + b \tan [c + d x])^3} + \\
& \frac{2 (5 a^2 b^3 + b^5) (b + a \cos [c + d x])^3 \operatorname{Log} [b + a \cos [c + d x]] \tan [c + d x]^3}{(-a^2 + b^2)^4 d (a \sin [c + d x] + b \tan [c + d x])^3} + \\
& \frac{(-a - 4 b) (b + a \cos [c + d x])^3 \operatorname{Log} \left[ \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] \tan [c + d x]^3}{2 (a + b)^4 d (a \sin [c + d x] + b \tan [c + d x])^3} - \frac{(b + a \cos [c + d x])^3 \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan [c + d x]^3}{8 (-a + b)^3 d (a \sin [c + d x] + b \tan [c + d x])^3}
\end{aligned}$$

**Problem 266: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos [c + d x]}{(a \sin [c + d x] + b \tan [c + d x])^3} dx$$

Optimal (type 3, 211 leaves, 6 steps):

$$\begin{aligned}
& \frac{b^4}{2 a (a^2 - b^2)^2 d (b + a \cos [c + d x])^2} - \frac{4 a b^3 (a^2 - b^2)^3 d (b + a \cos [c + d x])}{2 (a^2 - b^2)^3 d} - \frac{(a (a^2 + 3 b^2) - b (3 a^2 + b^2) \cos [c + d x]) \operatorname{Csc} [c + d x]^2}{2 (a^2 - b^2)^3 d} - \\
& \frac{3 b \operatorname{Log} [1 - \cos [c + d x]]}{4 (a + b)^4 d} + \frac{3 b \operatorname{Log} [1 + \cos [c + d x]]}{4 (a - b)^4 d} - \frac{6 a b^2 (a^2 + b^2) \operatorname{Log} [b + a \cos [c + d x]]}{(a^2 - b^2)^4 d}
\end{aligned}$$

Result (type 3, 458 leaves):

$$\begin{aligned}
& \frac{b^4 (b + a \cos [c + d x]) \tan [c + d x]^3}{2 a (-a + b)^2 (a + b)^2 d (a \sin [c + d x] + b \tan [c + d x])^3} + \frac{4 a b^3 (b + a \cos [c + d x])^2 \tan [c + d x]^3}{(-a + b)^3 (a + b)^3 d (a \sin [c + d x] + b \tan [c + d x])^3} - \\
& \frac{(b + a \cos [c + d x])^3 \operatorname{Csc} \left[ \frac{1}{2} (c + d x) \right]^2 \tan [c + d x]^3}{8 (a + b)^3 d (a \sin [c + d x] + b \tan [c + d x])^3} + \frac{3 b (b + a \cos [c + d x])^3 \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} (c + d x) \right] \right] \tan [c + d x]^3}{2 (-a + b)^4 d (a \sin [c + d x] + b \tan [c + d x])^3} - \\
& \frac{6 (a^3 b^2 + a b^4) (b + a \cos [c + d x])^3 \operatorname{Log} [b + a \cos [c + d x]] \tan [c + d x]^3}{(-a^2 + b^2)^4 d (a \sin [c + d x] + b \tan [c + d x])^3} - \\
& \frac{3 b (b + a \cos [c + d x])^3 \operatorname{Log} \left[ \operatorname{Sin} \left[ \frac{1}{2} (c + d x) \right] \right] \tan [c + d x]^3}{2 (a + b)^4 d (a \sin [c + d x] + b \tan [c + d x])^3} + \frac{(b + a \cos [c + d x])^3 \operatorname{Sec} \left[ \frac{1}{2} (c + d x) \right]^2 \tan [c + d x]^3}{8 (-a + b)^3 d (a \sin [c + d x] + b \tan [c + d x])^3}
\end{aligned}$$

**Problem 267: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a \sin[c + dx] + b \tan[c + dx])^3} dx$$

Optimal (type 3, 229 leaves, 5 steps):

$$\begin{aligned} & -\frac{b^3}{2(a^2 - b^2)^2 d (b + a \cos[c + dx])^2} + \frac{b^2(3a^2 + b^2)}{(a^2 - b^2)^3 d (b + a \cos[c + dx])} + \frac{(b(3a^2 + b^2) - a(a^2 + 3b^2) \cos[c + dx]) \operatorname{Csc}[c + dx]^2}{2(a^2 - b^2)^3 d} + \\ & \frac{(a - 2b) \operatorname{Log}[1 - \cos[c + dx]]}{4(a + b)^4 d} - \frac{(a + 2b) \operatorname{Log}[1 + \cos[c + dx]]}{4(a - b)^4 d} + \frac{b(3a^4 + 8a^2 b^2 + b^4) \operatorname{Log}[b + a \cos[c + dx]]}{(a^2 - b^2)^4 d} \end{aligned}$$

Result (type 3, 696 leaves):

$$\begin{aligned} & -\frac{b^3 (b + a \cos[c + dx]) \tan[c + dx]^3}{2(-a + b)^2 (a + b)^2 d (a \sin[c + dx] + b \tan[c + dx])^3} - \\ & \frac{b^2(3a^2 + b^2)(b + a \cos[c + dx])^2 \tan[c + dx]^3}{(-a + b)^3 (a + b)^3 d (a \sin[c + dx] + b \tan[c + dx])^3} - \frac{2i(3a^4 b + 8a^2 b^3 + b^5)(c + dx)(b + a \cos[c + dx])^3 \tan[c + dx]^3}{(a - b)^4 (a + b)^4 d (a \sin[c + dx] + b \tan[c + dx])^3} - \\ & \frac{i(-a - 2b) \operatorname{ArcTan}[\tan[c + dx]](b + a \cos[c + dx])^3 \tan[c + dx]^3}{2(-a + b)^4 d (a \sin[c + dx] + b \tan[c + dx])^3} - \frac{i(a - 2b) \operatorname{ArcTan}[\tan[c + dx]](b + a \cos[c + dx])^3 \tan[c + dx]^3}{2(a + b)^4 d (a \sin[c + dx] + b \tan[c + dx])^3} - \\ & \frac{(b + a \cos[c + dx])^3 \operatorname{Csc}[\frac{1}{2}(c + dx)]^2 \tan[c + dx]^3}{8(a + b)^3 d (a \sin[c + dx] + b \tan[c + dx])^3} + \frac{(-a - 2b)(b + a \cos[c + dx])^3 \operatorname{Log}[\cos[\frac{1}{2}(c + dx)]^2] \tan[c + dx]^3}{4(-a + b)^4 d (a \sin[c + dx] + b \tan[c + dx])^3} + \\ & \frac{(3a^4 b + 8a^2 b^3 + b^5)(b + a \cos[c + dx])^3 \operatorname{Log}[b + a \cos[c + dx]] \tan[c + dx]^3}{(-a^2 + b^2)^4 d (a \sin[c + dx] + b \tan[c + dx])^3} + \\ & \frac{(a - 2b)(b + a \cos[c + dx])^3 \operatorname{Log}[\sin[\frac{1}{2}(c + dx)]^2] \tan[c + dx]^3}{4(a + b)^4 d (a \sin[c + dx] + b \tan[c + dx])^3} - \frac{(b + a \cos[c + dx])^3 \operatorname{Sec}[\frac{1}{2}(c + dx)]^2 \tan[c + dx]^3}{8(-a + b)^3 d (a \sin[c + dx] + b \tan[c + dx])^3} \end{aligned}$$

**Problem 268: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[c + dx]}{(a \sin[c + dx] + b \tan[c + dx])^3} dx$$

Optimal (type 3, 231 leaves, 6 steps):

$$\frac{a b^2}{2 (a^2 - b^2)^2 d (b + a \cos [c + d x])^2} - \frac{2 a b (a^2 + b^2)}{(a^2 - b^2)^3 d (b + a \cos [c + d x])} - \frac{(a (a^2 + 3 b^2) - b (3 a^2 + b^2) \cos [c + d x]) \operatorname{Csc}[c + d x]^2}{2 (a^2 - b^2)^3 d} +$$

$$\frac{(2 a - b) \operatorname{Log}[1 - \cos [c + d x]]}{4 (a + b)^4 d} + \frac{(2 a + b) \operatorname{Log}[1 + \cos [c + d x]]}{4 (a - b)^4 d} - \frac{a (a^4 + 8 a^2 b^2 + 3 b^4) \operatorname{Log}[b + a \cos [c + d x]]}{(a^2 - b^2)^4 d}$$

Result (type 3, 703 leaves):

$$\frac{a b^2 (b + a \cos [c + d x]) \operatorname{Tan}[c + d x]^3}{2 (-a + b)^2 (a + b)^2 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} +$$

$$\frac{2 a b (-i a + b) (i a + b) (b + a \cos [c + d x])^2 \operatorname{Tan}[c + d x]^3}{(-a + b)^3 (a + b)^3 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} + \frac{2 i (a^5 + 8 a^3 b^2 + 3 a b^4) (c + d x) (b + a \cos [c + d x])^3 \operatorname{Tan}[c + d x]^3}{(a - b)^4 (a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} -$$

$$\frac{i (2 a - b) \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (b + a \cos [c + d x])^3 \operatorname{Tan}[c + d x]^3}{2 (a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} - \frac{i (2 a + b) \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (b + a \cos [c + d x])^3 \operatorname{Tan}[c + d x]^3}{2 (-a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} -$$

$$\frac{(b + a \cos [c + d x])^3 \operatorname{Csc}[\frac{1}{2} (c + d x)]^2 \operatorname{Tan}[c + d x]^3}{8 (a + b)^3 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} + \frac{(2 a + b) (b + a \cos [c + d x])^3 \operatorname{Log}[\cos [\frac{1}{2} (c + d x)]^2] \operatorname{Tan}[c + d x]^3}{4 (-a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} +$$

$$\frac{(-a^5 - 8 a^3 b^2 - 3 a b^4) (b + a \cos [c + d x])^3 \operatorname{Log}[b + a \cos [c + d x]] \operatorname{Tan}[c + d x]^3}{(-a^2 + b^2)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} +$$

$$\frac{(2 a - b) (b + a \cos [c + d x])^3 \operatorname{Log}[\sin [\frac{1}{2} (c + d x)]^2] \operatorname{Tan}[c + d x]^3}{4 (a + b)^4 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3} + \frac{(b + a \cos [c + d x])^3 \operatorname{Sec}[\frac{1}{2} (c + d x)]^2 \operatorname{Tan}[c + d x]^3}{8 (-a + b)^3 d (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^3}$$

Problem 272: Result more than twice size of optimal antiderivative.

$$\int \cos [c + d x]^m (a \sin [c + d x] + b \operatorname{Tan}[c + d x])^2 dx$$

Optimal (type 5, 264 leaves, 8 steps):

$$\frac{(a^2 - 2 b^2) \cos [c + d x]^{-1+m} \sin [c + d x]}{d m (2 + m)} - \frac{2 a b \cos [c + d x]^m \sin [c + d x]}{d (2 + 3 m + m^2)} - \frac{\cos [c + d x]^{-1+m} (b + a \cos [c + d x])^2 \sin [c + d x]}{d (2 + m)} -$$

$$\left( (a^2 (1 - m) - b^2 (2 + m)) \cos [c + d x]^{-1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (-1 + m), \frac{1 + m}{2}, \cos [c + d x]^2\right] \sin [c + d x] \right) /$$

$$\left( d (1 - m) m (2 + m) \sqrt{\sin [c + d x]^2} \right) - \frac{2 a b \cos [c + d x]^m \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{2}, \frac{2 + m}{2}, \cos [c + d x]^2\right] \sin [c + d x]}{d m (1 + m) \sqrt{\sin [c + d x]^2}}$$

Result (type 5, 890 leaves):

$$\begin{aligned}
& - \left( \left( b^2 \cos [c + d x]^{1+m} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, \frac{1}{2} (-1+m), \frac{1+m}{2}, \cos [c + d x]^2 \right] \sin [c + d x] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \right. \\
& \quad \left. (4096 d (-1+m) (b + a \cos [c + d x])^2 (\sin [c + d x]^2)^{3/2}) \right) - \\
& \left( a b \cos [c + d x]^{2+m} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, \cos [c + d x]^2 \right] \sin [c + d x] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \\
& \quad (2048 d m (b + a \cos [c + d x])^2 (\sin [c + d x]^2)^{3/2}) - \\
& \left( a^2 \cos [c + d x]^{3+m} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c + d x]^2 \right] \sin [c + d x] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \\
& \quad (2 d (1+m) (b + a \cos [c + d x])^2 (\sin [c + d x]^2)^{3/2}) - \\
& \left( 4095 b^2 \cos [c + d x]^{1+m} \operatorname{Csc} [c + d x] \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1}{2} (-1+m), \frac{1+m}{2}, \cos [c + d x]^2 \right] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \\
& \quad \left( 4096 d (-1+m) (b + a \cos [c + d x])^2 \sqrt{\sin [c + d x]^2} \right) - \\
& \left( 4095 a b \cos [c + d x]^{2+m} \operatorname{Csc} [c + d x] \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, \cos [c + d x]^2 \right] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \\
& \quad \left( 2048 d m (b + a \cos [c + d x])^2 \sqrt{\sin [c + d x]^2} \right) - \\
& \left( a^2 \cos [c + d x]^{3+m} \operatorname{Csc} [c + d x] \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c + d x]^2 \right] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \\
& \quad \left( 2 d (1+m) (b + a \cos [c + d x])^2 \sqrt{\sin [c + d x]^2} \right) + \\
& \left( 4095 b^2 \cos [c + d x]^{3+m} \operatorname{Csc} [c + d x] \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos [c + d x]^2 \right] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \\
& \quad \left( 4096 d (1+m) (b + a \cos [c + d x])^2 \sqrt{\sin [c + d x]^2} \right) + \\
& \left( 4095 a b \cos [c + d x]^{4+m} \operatorname{Csc} [c + d x] \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos [c + d x]^2 \right] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \\
& \quad \left( 2048 d (2+m) (b + a \cos [c + d x])^2 \sqrt{\sin [c + d x]^2} \right) + \\
& \left( a^2 \cos [c + d x]^{5+m} \operatorname{Csc} [c + d x] \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos [c + d x]^2 \right] (a \sin [c + d x] + b \tan [c + d x])^2 \right) / \\
& \quad \left( 2 d (3+m) (b + a \cos [c + d x])^2 \sqrt{\sin [c + d x]^2} \right)
\end{aligned}$$

**Problem 276:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos [x] \sin [x]^2}{a \cos [x] + b \sin [x]} dx$$

Optimal (type 3, 92 leaves, 7 steps):

$$-\frac{a b^2 x}{(a^2 + b^2)^2} + \frac{a x}{2(a^2 + b^2)} + \frac{a^2 b \operatorname{Log}[a \operatorname{Cos}[x] + b \operatorname{Sin}[x]]}{(a^2 + b^2)^2} - \frac{a \operatorname{Cos}[x] \operatorname{Sin}[x]}{2(a^2 + b^2)} + \frac{b \operatorname{Sin}[x]^2}{2(a^2 + b^2)}$$

Result (type 3, 153 leaves):

$$-\frac{1}{8(a^2 + b^2)^2} \left( -2 a^3 x - 6 i a^2 b x + 6 a b^2 x + 2 i b^3 x - 2 i b (-3 a^2 + b^2) \operatorname{ArcTan}[\operatorname{Tan}[x]] + 2 b (a^2 + b^2) \operatorname{Cos}[2 x] - 2 (a^2 + b^2) (a x + b \operatorname{Log}[a \operatorname{Cos}[x] + b \operatorname{Sin}[x]]) - 3 a^2 b \operatorname{Log}[(a \operatorname{Cos}[x] + b \operatorname{Sin}[x])^2] + b^3 \operatorname{Log}[(a \operatorname{Cos}[x] + b \operatorname{Sin}[x])^2] + 2 a^3 \operatorname{Sin}[2 x] + 2 a b^2 \operatorname{Sin}[2 x] \right)$$

**Problem 278: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[x]^2 \operatorname{Sin}[x]}{a \operatorname{Cos}[x] + b \operatorname{Sin}[x]} dx$$

Optimal (type 3, 93 leaves, 7 steps):

$$-\frac{a^2 b x}{(a^2 + b^2)^2} + \frac{b x}{2(a^2 + b^2)} - \frac{a b^2 \operatorname{Log}[a \operatorname{Cos}[x] + b \operatorname{Sin}[x]]}{(a^2 + b^2)^2} + \frac{b \operatorname{Cos}[x] \operatorname{Sin}[x]}{2(a^2 + b^2)} + \frac{a \operatorname{Sin}[x]^2}{2(a^2 + b^2)}$$

Result (type 3, 82 leaves):

$$\frac{1}{4(a^2 + b^2)^2} \left( 4 i a b^2 \operatorname{ArcTan}[\operatorname{Tan}[x]] - a (a^2 + b^2) \operatorname{Cos}[2 x] - 2 b \left( (a + i b)^2 x + a b \operatorname{Log}[(a \operatorname{Cos}[x] + b \operatorname{Sin}[x])^2] \right) + b (a^2 + b^2) \operatorname{Sin}[2 x] \right)$$

**Problem 280: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[x]^2 \operatorname{Sin}[x]^3}{a \operatorname{Cos}[x] + b \operatorname{Sin}[x]} dx$$

Optimal (type 3, 176 leaves, 13 steps):

$$\frac{a^2 b^3 x}{(a^2 + b^2)^3} - \frac{a^2 b x}{2(a^2 + b^2)^2} + \frac{b x}{8(a^2 + b^2)} - \frac{a^3 b^2 \operatorname{Log}[a \operatorname{Cos}[x] + b \operatorname{Sin}[x]]}{(a^2 + b^2)^3} + \frac{a^2 b \operatorname{Cos}[x] \operatorname{Sin}[x]}{2(a^2 + b^2)^2} + \frac{b \operatorname{Cos}[x] \operatorname{Sin}[x]}{8(a^2 + b^2)} - \frac{b \operatorname{Cos}[x]^3 \operatorname{Sin}[x]}{4(a^2 + b^2)} - \frac{a b^2 \operatorname{Sin}[x]^2}{2(a^2 + b^2)^2} + \frac{a \operatorname{Sin}[x]^4}{4(a^2 + b^2)}$$

Result (type 3, 178 leaves):



$$\frac{1}{32 (a^2 + b^2)^3} \left( -12 a^4 b x - 32 i a^3 b^2 x + 24 a^2 b^3 x + 4 b^5 x + 32 i a^3 b^2 \operatorname{ArcTan}[\operatorname{Tan}[x]] - 4 a (a^4 - b^4) \cos[2x] + a^5 \cos[4x] + 2 a^3 b^2 \cos[4x] + a b^4 \cos[4x] - 16 a^3 b^2 \operatorname{Log}[(a \cos[x] + b \sin[x])^2] + 8 a^4 b \sin[2x] + 8 a^2 b^3 \sin[2x] - a^4 b \sin[4x] - 2 a^2 b^3 \sin[4x] - b^5 \sin[4x] \right)$$

**Problem 282: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[x]^3 \sin[x]^2}{a \cos[x] + b \sin[x]} dx$$

Optimal (type 3, 175 leaves, 13 steps):

$$\frac{a^3 b^2 x}{(a^2 + b^2)^3} - \frac{a b^2 x}{2 (a^2 + b^2)^2} + \frac{a x}{8 (a^2 + b^2)} - \frac{b \cos[x]^4}{4 (a^2 + b^2)} + \frac{a^2 b^3 \operatorname{Log}[a \cos[x] + b \sin[x]]}{(a^2 + b^2)^3} - \frac{a b^2 \cos[x] \sin[x]}{2 (a^2 + b^2)^2} + \frac{a \cos[x] \sin[x]}{8 (a^2 + b^2)} - \frac{a \cos[x]^3 \sin[x]}{4 (a^2 + b^2)} - \frac{a^2 b \sin[x]^2}{2 (a^2 + b^2)^2}$$

Result (type 3, 287 leaves):

$$-\frac{1}{32 (a^2 + b^2)^3} \left( -4 a^5 x + 4 i a^4 b x - 24 a^3 b^2 x - 24 i a^2 b^3 x + 12 a b^4 x + 4 i b^5 x - 4 i b (a^4 - 6 a^2 b^2 + b^4) \operatorname{ArcTan}[\operatorname{Tan}[x]] + 4 b (-a^4 + b^4) \cos[2x] + a^4 b \cos[4x] + 2 a^2 b^3 \cos[4x] + b^5 \cos[4x] - 4 a^4 b \operatorname{Log}[a \cos[x] + b \sin[x]] - 8 a^2 b^3 \operatorname{Log}[a \cos[x] + b \sin[x]] - 4 b^5 \operatorname{Log}[a \cos[x] + b \sin[x]] + 2 a^4 b \operatorname{Log}[(a \cos[x] + b \sin[x])^2] - 12 a^2 b^3 \operatorname{Log}[(a \cos[x] + b \sin[x])^2] + 2 b^5 \operatorname{Log}[(a \cos[x] + b \sin[x])^2] + 8 a^3 b^2 \sin[2x] + 8 a b^4 \sin[2x] + a^5 \sin[4x] + 2 a^3 b^2 \sin[4x] + a b^4 \sin[4x] \right)$$

**Problem 284: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x] \sin[x]}{(a \cos[x] + b \sin[x])^2} dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{2 a b x}{(a^2 + b^2)^2} - \frac{(a^2 - b^2) \operatorname{Log}[a \cos[x] + b \sin[x]]}{(a^2 + b^2)^2} - \frac{b \sin[x]}{(a^2 + b^2) (a \cos[x] + b \sin[x])}$$

Result (type 3, 144 leaves):

$$\left( a \cos[x] \left( -2 i (a + i b)^2 x + (-a^2 + b^2) \operatorname{Log}[(a \cos[x] + b \sin[x])^2] \right) + b \left( 2 (a + i b) (a (-1 - i x) + b (i + x)) + (-a^2 + b^2) \operatorname{Log}[(a \cos[x] + b \sin[x])^2] \right) \sin[x] + 2 i (a^2 - b^2) \operatorname{ArcTan}[\operatorname{Tan}[x]] (a \cos[x] + b \sin[x]) \right) / \left( 2 (a^2 + b^2)^2 (a \cos[x] + b \sin[x]) \right)$$

### Problem 286: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos [x] \sin [x]^3}{(a \cos [x] + b \sin [x])^2} dx$$

Optimal (type 3, 129 leaves, 17 steps):

$$\frac{b (3 a^3 - a b^2) x}{(a^2 + b^2)^3} - \frac{a^2 (a^2 - 3 b^2) \operatorname{Log}[a \cos [x] + b \sin [x]]}{(a^2 + b^2)^3} - \frac{a b \cos [x] \sin [x]}{(a^2 + b^2)^2} - \frac{(a^2 - b^2) \sin [x]^2}{2 (a^2 + b^2)^2} - \frac{a^2 b \sin [x]}{(a^2 + b^2)^2 (a \cos [x] + b \sin [x])}$$

Result (type 3, 226 leaves):

$$\frac{1}{4 (a^2 + b^2)^3 (a \cos [x] + b \sin [x])} \left( 4 i a^2 (a^2 - 3 b^2) \operatorname{ArcTan}[\operatorname{Tan}[x]] (a \cos [x] + b \sin [x]) + \right. \\ \left. a \cos [x] \left( (a^4 - b^4) \cos [2 x] + 2 a \left( 2 (i a - b)^3 x - a (a^2 - 3 b^2) \operatorname{Log}[(a \cos [x] + b \sin [x])^2] - b (a^2 + b^2) \sin [2 x] \right) \right) - b \sin [x] \right. \\ \left. \left( (-a^4 + b^4) \cos [2 x] + 2 a \left( 2 (a^3 (1 + i x) + a b^2 (1 - 3 i x) - 3 a^2 b x + b^3 x) + a (a^2 - 3 b^2) \operatorname{Log}[(a \cos [x] + b \sin [x])^2] + b (a^2 + b^2) \sin [2 x] \right) \right) \right)$$

### Problem 290: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos [x]^3 \sin [x]}{(a \cos [x] + b \sin [x])^2} dx$$

Optimal (type 3, 128 leaves, 17 steps):

$$-\frac{a b (a^2 - 3 b^2) x}{(a^2 + b^2)^3} - \frac{b^2 (3 a^2 - b^2) \operatorname{Log}[a \cos [x] + b \sin [x]]}{(a^2 + b^2)^3} + \frac{a b \cos [x] \sin [x]}{(a^2 + b^2)^2} + \frac{(a^2 - b^2) \sin [x]^2}{2 (a^2 + b^2)^2} + \frac{a b^2 \cos [x]}{(a^2 + b^2)^2 (a \cos [x] + b \sin [x])}$$

Result (type 3, 221 leaves):

$$\frac{1}{4 (a^2 + b^2)^3 (a \cos [x] + b \sin [x])} \left( -4 i b^2 (-3 a^2 + b^2) \operatorname{ArcTan}[\operatorname{Tan}[x]] (a \cos [x] + b \sin [x]) - \right. \\ \left. a \cos [x] \left( (a^4 - b^4) \cos [2 x] + 2 b \left( 2 (a + i b)^3 x - b (-3 a^2 + b^2) \operatorname{Log}[(a \cos [x] + b \sin [x])^2] - a (a^2 + b^2) \sin [2 x] \right) \right) + b \sin [x] \right. \\ \left. \left( (-a^4 + b^4) \cos [2 x] + 2 b \left( -2 (a + i b) (a^2 x - b^2 (i + x) + a (b + 2 i b x)) + (-3 a^2 b + b^3) \operatorname{Log}[(a \cos [x] + b \sin [x])^2] + a (a^2 + b^2) \sin [2 x] \right) \right) \right)$$

### Problem 292: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos [x]^3 \sin [x]^3}{(a \cos [x] + b \sin [x])^2} dx$$

Optimal (type 3, 210 leaves, 48 steps):

$$\begin{aligned}
& - \frac{3 a b \left(a^4 - 6 a^2 b^2 + b^4\right) x}{4 \left(a^2 + b^2\right)^4} - \frac{b^2 \cos [x]^4}{4 \left(a^2 + b^2\right)^2} - \frac{3 a^2 b^2 \left(a^2 - b^2\right) \log [a \cos [x] + b \sin [x]]}{\left(a^2 + b^2\right)^4} + \\
& \frac{a b \left(5 a^2 - 3 b^2\right) \cos [x] \sin [x]}{4 \left(a^2 + b^2\right)^3} - \frac{a b \cos [x]^3 \sin [x]}{2 \left(a^2 + b^2\right)^2} - \frac{2 a^2 b^2 \sin [x]^2}{\left(a^2 + b^2\right)^3} + \frac{a^2 \sin [x]^4}{4 \left(a^2 + b^2\right)^2} - \frac{a^2 b^3 \sin [x]}{\left(a^2 + b^2\right)^3 \left(a \cos [x] + b \sin [x]\right)}
\end{aligned}$$

Result (type 3, 409 leaves):

$$\begin{aligned}
& \frac{1}{32 \left(a^2 + b^2\right)^4} \left( -12 a b \left(a^2 - 3 b^2\right) \left(3 a^2 - b^2\right) x + 6 i \left(a^6 - 15 a^4 b^2 + 15 a^2 b^4 - b^6\right) x - \right. \\
& 6 i \left(a^6 - 15 a^4 b^2 + 15 a^2 b^4 - b^6\right) \operatorname{ArcTan}[\tan [x]] - 4 \left(a^2 + b^2\right) \left(a^4 - 6 a^2 b^2 + b^4\right) \cos [2 x] + \left(a^2 - b^2\right) \left(a^2 + b^2\right)^2 \cos [4 x] + \\
& 3 \left(a^6 - 15 a^4 b^2 + 15 a^2 b^4 - b^6\right) \log \left[\left(a \cos [x] + b \sin [x]\right)^2\right] + \frac{2 b \left(a^2 + b^2\right) \left(3 a^4 - 10 a^2 b^2 + 3 b^4\right) \sin [x]}{a \cos [x] + b \sin [x]} + \\
& \frac{1}{a \cos [x] + b \sin [x]} 3 \left(a^2 + b^2\right)^2 \left(a \cos [x] \left(-2 i \left(a + i b\right)^2 x + \left(-a^2 + b^2\right) \log \left[\left(a \cos [x] + b \sin [x]\right)^2\right]\right) + \right. \\
& \left. b \left(2 \left(a + i b\right) \left(a \left(-1 - i x\right) + b \left(i + x\right)\right) + \left(-a^2 + b^2\right) \log \left[\left(a \cos [x] + b \sin [x]\right)^2\right]\right) \sin [x] + \right. \\
& \left. 2 i \left(a^2 - b^2\right) \operatorname{ArcTan}[\tan [x]] \left(a \cos [x] + b \sin [x]\right)\right) + 16 a b \left(a^4 - b^4\right) \sin [2 x] - 2 a b \left(a^2 + b^2\right)^2 \sin [4 x] \left. \right)
\end{aligned}$$

## Test results for the 397 problems in "4.7.3 (c+d x)^m trig^n trig^p.m"

Problem 32: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^4 \cot [a + b x] dx$$

Optimal (type 4, 151 leaves, 7 steps):

$$\begin{aligned}
& - \frac{i (c + d x)^5}{5 d} + \frac{(c + d x)^4 \log [1 - e^{2 i (a + b x)}]}{b} - \frac{2 i d (c + d x)^3 \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{b^2} + \\
& \frac{3 d^2 (c + d x)^2 \operatorname{PolyLog}[3, e^{2 i (a + b x)}]}{b^3} + \frac{3 i d^3 (c + d x) \operatorname{PolyLog}[4, e^{2 i (a + b x)}]}{b^4} - \frac{3 d^4 \operatorname{PolyLog}[5, e^{2 i (a + b x)}]}{2 b^5}
\end{aligned}$$

Result (type 4, 527 leaves):

$$\begin{aligned}
& \frac{2 i c^3 d \pi x}{b} - 2 i c^2 d^2 x^3 - i c d^3 x^4 - \frac{1}{5} i d^4 x^5 - \frac{4 i c^3 d x \operatorname{ArcTan}[\operatorname{Tan}[a]]}{b} + 2 c^3 d x^2 \operatorname{Cot}[a] + \frac{2 c^3 d \pi \operatorname{Log}[1 + e^{-2 i b x}]}{b^2} + \\
& \frac{6 c^2 d^2 x^2 \operatorname{Log}[1 - e^{2 i (a+b x)}]}{b} + \frac{4 c d^3 x^3 \operatorname{Log}[1 - e^{2 i (a+b x)}]}{b} + \frac{d^4 x^4 \operatorname{Log}[1 - e^{2 i (a+b x)}]}{b} + \frac{4 c^3 d x \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])}]}{b} + \\
& \frac{4 c^3 d \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])}]}{b^2} - \frac{2 c^3 d \pi \operatorname{Log}[\operatorname{Cos}[b x]]}{b^2} + \frac{c^4 \operatorname{Log}[\operatorname{Sin}[a + b x]]}{b} - \\
& \frac{4 c^3 d \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]}{b^2} - \frac{2 i d^2 x (3 c^2 + 3 c d x + d^2 x^2) \operatorname{PolyLog}[2, e^{2 i (a+b x)}]}{b^2} - \\
& \frac{2 i c^3 d \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])}]}{b^2} + \frac{3 c^2 d^2 \operatorname{PolyLog}[3, e^{2 i (a+b x)}]}{b^3} + \frac{6 c d^3 x \operatorname{PolyLog}[3, e^{2 i (a+b x)}]}{b^3} + \frac{3 d^4 x^2 \operatorname{PolyLog}[3, e^{2 i (a+b x)}]}{b^3} + \\
& \frac{3 i c d^3 \operatorname{PolyLog}[4, e^{2 i (a+b x)}]}{b^4} + \frac{3 i d^4 x \operatorname{PolyLog}[4, e^{2 i (a+b x)}]}{b^4} - \frac{3 d^4 \operatorname{PolyLog}[5, e^{2 i (a+b x)}]}{2 b^5} - 2 c^3 d e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 \operatorname{Cot}[a] \sqrt{\operatorname{Sec}[a]^2}
\end{aligned}$$

**Problem 33: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 \operatorname{Cot}[a + b x] dx$$

Optimal (type 4, 127 leaves, 6 steps):

$$\begin{aligned}
& - \frac{i (c + d x)^4}{4 d} + \frac{(c + d x)^3 \operatorname{Log}[1 - e^{2 i (a+b x)}]}{b} - \frac{3 i d (c + d x)^2 \operatorname{PolyLog}[2, e^{2 i (a+b x)}]}{2 b^2} + \\
& \frac{3 d^2 (c + d x) \operatorname{PolyLog}[3, e^{2 i (a+b x)}]}{2 b^3} + \frac{3 i d^3 \operatorname{PolyLog}[4, e^{2 i (a+b x)}]}{4 b^4}
\end{aligned}$$

Result (type 4, 410 leaves):

$$\begin{aligned}
& \frac{1}{4 b^4} \left( 6 i b^3 c^2 d \pi x - 4 i b^4 c d^2 x^3 - i b^4 d^3 x^4 - 12 i b^3 c^2 d x \operatorname{ArcTan}[\operatorname{Tan}[a]] + \right. \\
& 6 b^4 c^2 d x^2 \operatorname{Cot}[a] + 6 b^2 c^2 d \pi \operatorname{Log}[1 + e^{-2 i b x}] + 12 b^3 c d^2 x^2 \operatorname{Log}[1 - e^{2 i (a+b x)}] + 4 b^3 d^3 x^3 \operatorname{Log}[1 - e^{2 i (a+b x)}] + \\
& 12 b^3 c^2 d x \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] + 12 b^2 c^2 d \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] - \\
& 6 b^2 c^2 d \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 4 b^3 c^3 \operatorname{Log}[\operatorname{Sin}[a + b x]] - 12 b^2 c^2 d \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] - \\
& 6 i b^2 d^2 x (2 c + d x) \operatorname{PolyLog}[2, e^{2 i (a+b x)}] - 6 i b^2 c^2 d \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] + 6 b c d^2 \operatorname{PolyLog}[3, e^{2 i (a+b x)}] + \\
& \left. 6 b d^3 x \operatorname{PolyLog}[3, e^{2 i (a+b x)}] + 3 i d^3 \operatorname{PolyLog}[4, e^{2 i (a+b x)}] - 6 b^4 c^2 d e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 \operatorname{Cot}[a] \sqrt{\operatorname{Sec}[a]^2} \right)
\end{aligned}$$

**Problem 34: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Cot}[a + b x] dx$$

Optimal (type 4, 93 leaves, 5 steps):

$$-\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2 \operatorname{Log}[1 - e^{2i(a+bx)}]}{b} - \frac{id(c+dx) \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^2} + \frac{d^2 \operatorname{PolyLog}[3, e^{2i(a+bx)}]}{2b^3}$$

Result (type 4, 287 leaves):

$$\begin{aligned} & \frac{1}{6b^3} \left( 6ib^2cd\pi x - 2ib^3d^2x^3 - 12ib^2cdx \operatorname{ArcTan}[\operatorname{Tan}[a]] + 6b^3cdx^2 \operatorname{Cot}[a] + 6bcd\pi \operatorname{Log}[1 + e^{-2ibx}] + 6b^2d^2x^2 \operatorname{Log}[1 - e^{2i(a+bx)}] + \right. \\ & 12b^2cdx \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] + 12bcd \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] - 6bcd\pi \operatorname{Log}[\operatorname{Cos}[bx]] + \\ & 6b^2c^2 \operatorname{Log}[\operatorname{Sin}[a + bx]] - 12bcd \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] - 6ib^2d^2x \operatorname{PolyLog}[2, e^{2i(a+bx)}] - \\ & \left. 6ibcd \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] + 3d^2 \operatorname{PolyLog}[3, e^{2i(a+bx)}] - 6b^3cd e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 \operatorname{Cot}[a] \sqrt{\operatorname{Sec}[a]^2} \right) \end{aligned}$$

**Problem 35: Result more than twice size of optimal antiderivative.**

$$\int (c+dx) \operatorname{Cot}[a+bx] dx$$

Optimal (type 4, 65 leaves, 4 steps):

$$-\frac{i(c+dx)^2}{2d} + \frac{(c+dx) \operatorname{Log}[1 - e^{2i(a+bx)}]}{b} - \frac{id \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{2b^2}$$

Result (type 4, 180 leaves):

$$\begin{aligned} & \frac{1}{2} dx^2 \operatorname{Cot}[a] + \frac{c \operatorname{Log}[\operatorname{Sin}[a+bx]]}{b} - \\ & \left( d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} (ibx(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}] - 2(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])]) \right. \right. \\ & \left. \left. \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + \right. \right. \\ & \left. \left. ib \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] \operatorname{Tan}[a] \right) \right) / \left( 2b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) \end{aligned}$$

**Problem 39: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^4 \operatorname{Cot}[a+bx] \operatorname{Csc}[a+bx] dx$$

Optimal (type 4, 208 leaves, 10 steps):

$$\begin{aligned}
& - \frac{8 d (c+d x)^3 \operatorname{ArcTanh}\left[e^{i(a+b x)}\right]}{b^2} - \frac{(c+d x)^4 \operatorname{Csc}[a+b x]}{b} + \frac{12 i d^2 (c+d x)^2 \operatorname{PolyLog}\left[2, -e^{i(a+b x)}\right]}{b^3} - \frac{12 i d^2 (c+d x)^2 \operatorname{PolyLog}\left[2, e^{i(a+b x)}\right]}{b^3} \\
& - \frac{24 d^3 (c+d x) \operatorname{PolyLog}\left[3, -e^{i(a+b x)}\right]}{b^4} + \frac{24 d^3 (c+d x) \operatorname{PolyLog}\left[3, e^{i(a+b x)}\right]}{b^4} - \frac{24 i d^4 \operatorname{PolyLog}\left[4, -e^{i(a+b x)}\right]}{b^5} + \frac{24 i d^4 \operatorname{PolyLog}\left[4, e^{i(a+b x)}\right]}{b^5}
\end{aligned}$$

Result (type 4, 458 leaves):

$$\begin{aligned}
& - \frac{1}{b^5} \left( 8 b^3 c^3 d \operatorname{ArcTanh}\left[e^{i(a+b x)}\right] + b^4 c^4 \operatorname{Csc}[a+b x] + 4 b^4 c^3 d x \operatorname{Csc}[a+b x] + 6 b^4 c^2 d^2 x^2 \operatorname{Csc}[a+b x] + 4 b^4 c d^3 x^3 \operatorname{Csc}[a+b x] + \right. \\
& b^4 d^4 x^4 \operatorname{Csc}[a+b x] - 12 b^3 c^2 d^2 x \operatorname{Log}\left[1 - e^{i(a+b x)}\right] - 12 b^3 c d^3 x^2 \operatorname{Log}\left[1 - e^{i(a+b x)}\right] - 4 b^3 d^4 x^3 \operatorname{Log}\left[1 - e^{i(a+b x)}\right] + \\
& 12 b^3 c^2 d^2 x \operatorname{Log}\left[1 + e^{i(a+b x)}\right] + 12 b^3 c d^3 x^2 \operatorname{Log}\left[1 + e^{i(a+b x)}\right] + 4 b^3 d^4 x^3 \operatorname{Log}\left[1 + e^{i(a+b x)}\right] - 12 i b^2 d^2 (c+d x)^2 \operatorname{PolyLog}\left[2, -e^{i(a+b x)}\right] + \\
& 12 i b^2 d^2 (c+d x)^2 \operatorname{PolyLog}\left[2, e^{i(a+b x)}\right] + 24 b c d^3 \operatorname{PolyLog}\left[3, -e^{i(a+b x)}\right] + 24 b d^4 x \operatorname{PolyLog}\left[3, -e^{i(a+b x)}\right] - \\
& \left. 24 b c d^3 \operatorname{PolyLog}\left[3, e^{i(a+b x)}\right] - 24 b d^4 x \operatorname{PolyLog}\left[3, e^{i(a+b x)}\right] + 24 i d^4 \operatorname{PolyLog}\left[4, -e^{i(a+b x)}\right] - 24 i d^4 \operatorname{PolyLog}\left[4, e^{i(a+b x)}\right] \right)
\end{aligned}$$

### Problem 41: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^2 \operatorname{Cot}[a+b x] \operatorname{Csc}[a+b x] dx$$

Optimal (type 4, 90 leaves, 6 steps):

$$\begin{aligned}
& - \frac{4 d (c+d x) \operatorname{ArcTanh}\left[e^{i(a+b x)}\right]}{b^2} - \frac{(c+d x)^2 \operatorname{Csc}[a+b x]}{b} + \frac{2 i d^2 \operatorname{PolyLog}\left[2, -e^{i(a+b x)}\right]}{b^3} - \frac{2 i d^2 \operatorname{PolyLog}\left[2, e^{i(a+b x)}\right]}{b^3}
\end{aligned}$$

Result (type 4, 234 leaves):

$$\begin{aligned}
& \frac{1}{2 b^3} \left( -8 b c d \operatorname{ArcTanh}\left[\operatorname{Cos}[a] - \operatorname{Sin}[a] \operatorname{Tan}\left[\frac{b x}{2}\right]\right] - 2 b^2 (c+d x)^2 \operatorname{Csc}[a] + \right. \\
& 4 d^2 \left( 2 \operatorname{ArcTan}\left[\operatorname{Tan}[a]\right] \operatorname{ArcTanh}\left[\operatorname{Cos}[a] - \operatorname{Sin}[a] \operatorname{Tan}\left[\frac{b x}{2}\right]\right] + \frac{1}{\sqrt{\operatorname{Sec}[a]^2}} \left( (b x + \operatorname{ArcTan}\left[\operatorname{Tan}[a]\right]) \left(\operatorname{Log}\left[1 - e^{i(b x + \operatorname{ArcTan}\left[\operatorname{Tan}[a])}\right]\right) - \right. \right. \\
& \left. \left. \operatorname{Log}\left[1 + e^{i(b x + \operatorname{ArcTan}\left[\operatorname{Tan}[a])}\right]\right] + i \operatorname{PolyLog}\left[2, -e^{i(b x + \operatorname{ArcTan}\left[\operatorname{Tan}[a])}\right]\right] - i \operatorname{PolyLog}\left[2, e^{i(b x + \operatorname{ArcTan}\left[\operatorname{Tan}[a])}\right]\right] \right) \operatorname{Sec}[a] \right) + \\
& \left. b^2 (c+d x)^2 \operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{1}{2}(a+b x)\right] \operatorname{Sin}\left[\frac{b x}{2}\right] - b^2 (c+d x)^2 \operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right] \operatorname{Sin}\left[\frac{b x}{2}\right] \right)
\end{aligned}$$

### Problem 42: Result more than twice size of optimal antiderivative.

$$\int (c + dx) \cot [a + bx] \csc [a + bx] dx$$

Optimal (type 3, 30 leaves, 2 steps):

$$-\frac{d \operatorname{ArcTanh}[\cos [a + bx]]}{b^2} - \frac{(c + dx) \csc [a + bx]}{b}$$

Result (type 3, 131 leaves):

$$-\frac{dx \csc [a]}{b} - \frac{c \csc [a + bx]}{b} - \frac{d \operatorname{Log}[\cos [\frac{a}{2} + \frac{bx}{2}]]}{b^2} + \frac{d \operatorname{Log}[\sin [\frac{a}{2} + \frac{bx}{2}]]}{b^2} + \frac{dx \csc [\frac{a}{2}] \csc [\frac{a}{2} + \frac{bx}{2}] \sin [\frac{bx}{2}]}{2b} - \frac{dx \sec [\frac{a}{2}] \sec [\frac{a}{2} + \frac{bx}{2}] \sin [\frac{bx}{2}]}{2b}$$

### Problem 46: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^4 \cot [a + bx] \csc [a + bx]^2 dx$$

Optimal (type 4, 137 leaves, 7 steps):

$$-\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3 \cot [a + bx]}{b^2} - \frac{(c+dx)^4 \csc [a + bx]^2}{2b} + \frac{6d^2(c+dx)^2 \operatorname{Log}[1 - e^{2i(a+bx)}]}{b^3} - \frac{6id^3(c+dx) \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^4} + \frac{3d^4 \operatorname{PolyLog}[3, e^{2i(a+bx)}]}{b^5}$$

Result (type 4, 412 leaves):

$$\begin{aligned}
& - \frac{(c+dx)^4 \operatorname{Csc}[a+bx]^2}{2b} - \frac{1}{2b^5} d^4 e^{-ia} \operatorname{Csc}[a] \\
& \frac{(2b^2 x^2 (2b e^{2ia} x + 3i(-1 + e^{2ia}) \operatorname{Log}[1 - e^{2i(a+bx)}]) + 6b(-1 + e^{2ia}) x \operatorname{PolyLog}[2, e^{2i(a+bx)}] + 3i(-1 + e^{2ia}) \operatorname{PolyLog}[3, e^{2i(a+bx)}]) + 6c^2 d^2 \operatorname{Csc}[a](-bx \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[bx] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[bx]] \operatorname{Sin}[a])}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{2 \operatorname{Csc}[a] \operatorname{Csc}[a+bx] (c^3 d \operatorname{Sin}[bx] + 3c^2 d^2 x \operatorname{Sin}[bx] + 3c d^3 x^2 \operatorname{Sin}[bx] + d^4 x^3 \operatorname{Sin}[bx])}{b^2} - \\
& \left( 6cd^3 \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}] - 2(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] + 2 \right. \right. \\
& \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] \operatorname{Tan}[a] \right) \right) \Bigg/ \left( b^4 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)
\end{aligned}$$

**Problem 47: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^3 \operatorname{Cot}[a+bx] \operatorname{Csc}[a+bx]^2 dx$$

Optimal (type 4, 115 leaves, 6 steps):

$$-\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2 \operatorname{Cot}[a+bx]}{2b^2} - \frac{(c+dx)^3 \operatorname{Csc}[a+bx]^2}{2b} + \frac{3d^2(c+dx) \operatorname{Log}[1 - e^{2i(a+bx)}]}{b^3} - \frac{3id^3 \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{2b^4}$$

Result (type 4, 277 leaves):

$$\begin{aligned}
& - \frac{(c+dx)^3 \operatorname{Csc}[a+bx]^2}{2b} + \frac{3cd^2 \operatorname{Csc}[a](-bx \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[bx] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[bx]] \operatorname{Sin}[a])}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{3 \operatorname{Csc}[a] \operatorname{Csc}[a+bx] (c^2 d \operatorname{Sin}[bx] + 2c d^2 x \operatorname{Sin}[bx] + d^3 x^2 \operatorname{Sin}[bx])}{2b^2} - \\
& \left( 3d^3 \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}] - 2(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + \right. \right. \\
& \left. \left. i \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] \operatorname{Tan}[a] \right) \right) \Bigg/ \left( 2b^4 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)
\end{aligned}$$



### Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int (c + dx)^2 \cot [a + bx] \csc [a + bx]^2 dx$$

Optimal (type 3, 54 leaves, 3 steps):

$$-\frac{d(c+dx)\cot[a+bx]}{b^2} - \frac{(c+dx)^2 \csc[a+bx]^2}{2b} + \frac{d^2 \operatorname{Log}[\sin[a+bx]]}{b^3}$$

Result (type 3, 94 leaves):

$$\frac{1}{2b^3} \left( 2i b d^2 x - 2i d^2 \operatorname{ArcTan}[\tan[a+bx]] - 2bd^2 x \cot[a] - b^2 (c+dx)^2 \csc[a+bx]^2 + d^2 \operatorname{Log}[\sin[a+bx]^2] + 2bd(c+dx) \csc[a] \csc[a+bx] \sin[bx] \right)$$

### Problem 98: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^4 \cos [a + bx] \cot [a + bx] dx$$

Optimal (type 4, 333 leaves, 17 steps):

$$\begin{aligned} & -\frac{2(c+dx)^4 \operatorname{ArcTanh}[e^{i(a+bx)}]}{b} + \frac{24d^4 \cos[a+bx]}{b^5} - \frac{12d^2(c+dx)^2 \cos[a+bx]}{b^3} + \frac{(c+dx)^4 \cos[a+bx]}{b} + \\ & \frac{4id(c+dx)^3 \operatorname{PolyLog}[2, -e^{i(a+bx)}]}{b^2} - \frac{4id(c+dx)^3 \operatorname{PolyLog}[2, e^{i(a+bx)}]}{b^2} - \frac{12d^2(c+dx)^2 \operatorname{PolyLog}[3, -e^{i(a+bx)}]}{b^3} + \\ & \frac{12d^2(c+dx)^2 \operatorname{PolyLog}[3, e^{i(a+bx)}]}{b^3} - \frac{24id^3(c+dx) \operatorname{PolyLog}[4, -e^{i(a+bx)}]}{b^4} + \frac{24id^3(c+dx) \operatorname{PolyLog}[4, e^{i(a+bx)}]}{b^4} + \\ & \frac{24d^4 \operatorname{PolyLog}[5, -e^{i(a+bx)}]}{b^5} - \frac{24d^4 \operatorname{PolyLog}[5, e^{i(a+bx)}]}{b^5} + \frac{24d^3(c+dx) \sin[a+bx]}{b^4} - \frac{4d(c+dx)^3 \sin[a+bx]}{b^2} \end{aligned}$$

Result (type 4, 812 leaves):

$$\frac{1}{b^5} \left( -2 b^4 c^4 \operatorname{ArcTanh} \left[ e^{i(a+bx)} \right] + b^4 c^4 \cos [a+bx] - 12 b^2 c^2 d^2 \cos [a+bx] + 24 d^4 \cos [a+bx] + 4 b^4 c^3 d x \cos [a+bx] - \right. \\ \left. 24 b^2 c d^3 x \cos [a+bx] + 6 b^4 c^2 d^2 x^2 \cos [a+bx] - 12 b^2 d^4 x^2 \cos [a+bx] + 4 b^4 c d^3 x^3 \cos [a+bx] + b^4 d^4 x^4 \cos [a+bx] + \right. \\ \left. 4 b^4 c^3 d x \log \left[ 1 - e^{i(a+bx)} \right] + 6 b^4 c^2 d^2 x^2 \log \left[ 1 - e^{i(a+bx)} \right] + 4 b^4 c d^3 x^3 \log \left[ 1 - e^{i(a+bx)} \right] + b^4 d^4 x^4 \log \left[ 1 - e^{i(a+bx)} \right] - \right. \\ \left. 4 b^4 c^3 d x \log \left[ 1 + e^{i(a+bx)} \right] - 6 b^4 c^2 d^2 x^2 \log \left[ 1 + e^{i(a+bx)} \right] - 4 b^4 c d^3 x^3 \log \left[ 1 + e^{i(a+bx)} \right] - b^4 d^4 x^4 \log \left[ 1 + e^{i(a+bx)} \right] + \right. \\ \left. 4 i b^3 d (c+dx)^3 \operatorname{PolyLog} \left[ 2, -e^{i(a+bx)} \right] - 4 i b^3 d (c+dx)^3 \operatorname{PolyLog} \left[ 2, e^{i(a+bx)} \right] - 12 b^2 c^2 d^2 \operatorname{PolyLog} \left[ 3, -e^{i(a+bx)} \right] - \right. \\ \left. 24 b^2 c d^3 x \operatorname{PolyLog} \left[ 3, -e^{i(a+bx)} \right] - 12 b^2 d^4 x^2 \operatorname{PolyLog} \left[ 3, -e^{i(a+bx)} \right] + 12 b^2 c^2 d^2 \operatorname{PolyLog} \left[ 3, e^{i(a+bx)} \right] + 24 b^2 c d^3 x \operatorname{PolyLog} \left[ 3, e^{i(a+bx)} \right] + \right. \\ \left. 12 b^2 d^4 x^2 \operatorname{PolyLog} \left[ 3, e^{i(a+bx)} \right] - 24 i b c d^3 \operatorname{PolyLog} \left[ 4, -e^{i(a+bx)} \right] - 24 i b d^4 x \operatorname{PolyLog} \left[ 4, -e^{i(a+bx)} \right] + 24 i b c d^3 \operatorname{PolyLog} \left[ 4, e^{i(a+bx)} \right] + \right. \\ \left. 24 i b d^4 x \operatorname{PolyLog} \left[ 4, e^{i(a+bx)} \right] + 24 d^4 \operatorname{PolyLog} \left[ 5, -e^{i(a+bx)} \right] - 24 d^4 \operatorname{PolyLog} \left[ 5, e^{i(a+bx)} \right] - 4 b^3 c^3 d \sin [a+bx] + \right. \\ \left. 24 b c d^3 \sin [a+bx] - 12 b^3 c^2 d^2 x \sin [a+bx] + 24 b d^4 x \sin [a+bx] - 12 b^3 c d^3 x^2 \sin [a+bx] - 4 b^3 d^4 x^3 \sin [a+bx] \right)$$

**Problem 99: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^3 \cos [a+bx] \cot [a+bx] dx$$

Optimal (type 4, 254 leaves, 14 steps):

$$\frac{2(c+dx)^3 \operatorname{ArcTanh} \left[ e^{i(a+bx)} \right]}{b} - \frac{6d^2(c+dx) \cos [a+bx]}{b^3} + \frac{(c+dx)^3 \cos [a+bx]}{b} + \frac{3id(c+dx)^2 \operatorname{PolyLog} \left[ 2, -e^{i(a+bx)} \right]}{b^2} - \\ \frac{3id(c+dx)^2 \operatorname{PolyLog} \left[ 2, e^{i(a+bx)} \right]}{b^2} - \frac{6d^2(c+dx) \operatorname{PolyLog} \left[ 3, -e^{i(a+bx)} \right]}{b^3} + \frac{6d^2(c+dx) \operatorname{PolyLog} \left[ 3, e^{i(a+bx)} \right]}{b^3} - \\ \frac{6id^3 \operatorname{PolyLog} \left[ 4, -e^{i(a+bx)} \right]}{b^4} + \frac{6id^3 \operatorname{PolyLog} \left[ 4, e^{i(a+bx)} \right]}{b^4} + \frac{6d^3 \sin [a+bx]}{b^4} - \frac{3d(c+dx)^2 \sin [a+bx]}{b^2}$$

Result (type 4, 512 leaves):

$$\frac{1}{b^4} \left( -2 b^3 c^3 \operatorname{ArcTanh} \left[ e^{i(a+bx)} \right] + b^3 c^3 \cos [a+bx] - 6 b c d^2 \cos [a+bx] + 3 b^3 c^2 d x \cos [a+bx] - \right. \\ \left. 6 b d^3 x \cos [a+bx] + 3 b^3 c d^2 x^2 \cos [a+bx] + b^3 d^3 x^3 \cos [a+bx] + 3 b^3 c^2 d x \log \left[ 1 - e^{i(a+bx)} \right] + 3 b^3 c d^2 x^2 \log \left[ 1 - e^{i(a+bx)} \right] + \right. \\ \left. b^3 d^3 x^3 \log \left[ 1 - e^{i(a+bx)} \right] - 3 b^3 c^2 d x \log \left[ 1 + e^{i(a+bx)} \right] - 3 b^3 c d^2 x^2 \log \left[ 1 + e^{i(a+bx)} \right] - b^3 d^3 x^3 \log \left[ 1 + e^{i(a+bx)} \right] + \right. \\ \left. 3 i b^2 d (c+dx)^2 \operatorname{PolyLog} \left[ 2, -e^{i(a+bx)} \right] - 3 i b^2 d (c+dx)^2 \operatorname{PolyLog} \left[ 2, e^{i(a+bx)} \right] - 6 b c d^2 \operatorname{PolyLog} \left[ 3, -e^{i(a+bx)} \right] - \right. \\ \left. 6 b d^3 x \operatorname{PolyLog} \left[ 3, -e^{i(a+bx)} \right] + 6 b c d^2 \operatorname{PolyLog} \left[ 3, e^{i(a+bx)} \right] + 6 b d^3 x \operatorname{PolyLog} \left[ 3, e^{i(a+bx)} \right] - 6 i d^3 \operatorname{PolyLog} \left[ 4, -e^{i(a+bx)} \right] + \right. \\ \left. 6 i d^3 \operatorname{PolyLog} \left[ 4, e^{i(a+bx)} \right] - 3 b^2 c^2 d \sin [a+bx] + 6 d^3 \sin [a+bx] - 6 b^2 c d^2 x \sin [a+bx] - 3 b^2 d^3 x^2 \sin [a+bx] \right)$$

**Problem 105: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^4 \cot [a+bx]^2 dx$$

Optimal (type 4, 155 leaves, 8 steps):

$$-\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \operatorname{Cot}[a+bx]}{b} + \frac{4d(c+dx)^3 \operatorname{Log}[1 - e^{2i(a+bx)}]}{b^2} -$$

$$\frac{6id^2(c+dx)^2 \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^3} + \frac{6d^3(c+dx) \operatorname{PolyLog}[3, e^{2i(a+bx)}]}{b^4} + \frac{3id^4 \operatorname{PolyLog}[4, e^{2i(a+bx)}]}{b^5}$$

Result (type 4, 592 leaves):

$$-\frac{1}{5}x(5c^4 + 10c^3dx + 10c^2d^2x^2 + 5cd^3x^3 + d^4x^4) - \frac{1}{b^4}cd^3e^{-ia} \operatorname{Csc}[a]$$

$$(2b^2x^2(2be^{2ia}x + 3i(-1 + e^{2ia}) \operatorname{Log}[1 - e^{2i(a+bx)}]) + 6b(-1 + e^{2ia})x \operatorname{PolyLog}[2, e^{2i(a+bx)}] + 3i(-1 + e^{2ia}) \operatorname{PolyLog}[3, e^{2i(a+bx)}]) -$$

$$\frac{1}{b}d^4e^{ia} \operatorname{Csc}[a] \left( x^4 + (-1 + e^{-2ia})x^4 + \frac{1}{2b^4}e^{-2ia}(-1 + e^{2ia}) \right.$$

$$\left. (2b^4x^4 + 4ib^3x^3 \operatorname{Log}[1 - e^{2i(a+bx)}] + 6b^2x^2 \operatorname{PolyLog}[2, e^{2i(a+bx)}] + 6ibx \operatorname{PolyLog}[3, e^{2i(a+bx)}] - 3 \operatorname{PolyLog}[4, e^{2i(a+bx)}]) \right) +$$

$$\frac{4c^3d \operatorname{Csc}[a](-bx \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[bx] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[bx]]) \operatorname{Sin}[a]}{b^2(\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \frac{1}{b}$$

$$\operatorname{Csc}[a] \operatorname{Csc}[a+bx] (c^4 \operatorname{Sin}[bx] + 4c^3dx \operatorname{Sin}[bx] + 6c^2d^2x^2 \operatorname{Sin}[bx] + 4cd^3x^3 \operatorname{Sin}[bx] + d^4x^4 \operatorname{Sin}[bx]) -$$

$$\left( 6c^2d^2 \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right.$$

$$\left. \left( ibx(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}] - 2(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \right] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] + 2 \right.$$

$$\left. \left. \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]] + i \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] \operatorname{Tan}[a] \right) \right) \Bigg/ \left( b^3 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)$$

**Problem 106: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^3 \operatorname{Cot}[a+bx]^2 dx$$

Optimal (type 4, 127 leaves, 7 steps):

$$-\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3 \operatorname{Cot}[a+bx]}{b} +$$

$$\frac{3d(c+dx)^2 \operatorname{Log}[1 - e^{2i(a+bx)}]}{b^2} - \frac{3id^2(c+dx) \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^3} + \frac{3d^3 \operatorname{PolyLog}[3, e^{2i(a+bx)}]}{2b^4}$$

Result (type 4, 418 leaves):

$$\begin{aligned}
& -\frac{1}{4} x \left( 4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) - \frac{1}{4 b^4} d^3 e^{-i a} \operatorname{Csc}[a] \\
& \left( 2 b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( -1 + e^{2 i a} \right) \operatorname{Log}\left[1 - e^{2 i (a+b x)}\right]\right) + 6 b \left( -1 + e^{2 i a} \right) x \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right] + 3 i \left( -1 + e^{2 i a} \right) \operatorname{PolyLog}\left[3, e^{2 i (a+b x)}\right] \right) + \\
& \frac{3 c^2 d \operatorname{Csc}[a] \left( -b x \operatorname{Cos}[a] + \operatorname{Log}\left[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]\right] \operatorname{Sin}[a] \right)}{b^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} + \\
& \frac{\operatorname{Csc}[a] \operatorname{Csc}[a+b x] \left( c^3 \operatorname{Sin}[b x] + 3 c^2 d x \operatorname{Sin}[b x] + 3 c d^2 x^2 \operatorname{Sin}[b x] + d^3 x^3 \operatorname{Sin}[b x] \right)}{b} \\
& \left( 3 c d^2 \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. \left( i b x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \right) \right) - \pi \operatorname{Log}\left[1 + e^{-2 i b x}\right] - 2 \left( b x + \operatorname{ArcTan}[\operatorname{Tan}[a]] \right) \operatorname{Log}\left[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \right. \right. \\
& \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) \operatorname{Tan}[a] \right) \right) \Bigg/ \left( b^3 \sqrt{\operatorname{Sec}[a]^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right)
\end{aligned}$$

**Problem 107: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Cot}[a + b x]^2 dx$$

Optimal (type 4, 97 leaves, 6 steps):

$$-\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} - \frac{(c+dx)^2 \operatorname{Cot}[a+bx]}{b} + \frac{2d(c+dx) \operatorname{Log}[1 - e^{2i(a+bx)}]}{b^2} - \frac{i d^2 \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^3}$$

Result (type 4, 268 leaves):

$$\begin{aligned}
& -\frac{1}{3} x \left( 3 c^2 + 3 c d x + d^2 x^2 \right) + \frac{2 c d \operatorname{Csc}[a] \left( -b x \operatorname{Cos}[a] + \operatorname{Log}\left[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]\right] \operatorname{Sin}[a] \right)}{b^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} + \\
& \frac{\operatorname{Csc}[a] \operatorname{Csc}[a+b x] \left( c^2 \operatorname{Sin}[b x] + 2 c d x \operatorname{Sin}[b x] + d^2 x^2 \operatorname{Sin}[b x] \right)}{b} \\
& \left( d^2 \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. \left( i b x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \right) \right) - \pi \operatorname{Log}\left[1 + e^{-2 i b x}\right] - \right. \right. \\
& \left. \left. 2 \left( b x + \operatorname{ArcTan}[\operatorname{Tan}[a]] \right) \operatorname{Log}\left[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + \right. \right. \\
& \left. \left. i \operatorname{PolyLog}\left[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) \operatorname{Tan}[a] \right) \right) \Bigg/ \left( b^3 \sqrt{\operatorname{Sec}[a]^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right)
\end{aligned}$$

### Problem 112: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^4 \cot [a + b x]^2 \csc [a + b x] dx$$

Optimal (type 4, 416 leaves, 31 steps):

$$\begin{aligned} & -\frac{12 d^2 (c + d x)^2 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b^3} + \frac{(c + d x)^4 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{2 d (c + d x)^3 \csc [a + b x]}{b^2} \\ & - \frac{(c + d x)^4 \cot [a + b x] \csc [a + b x]}{2 b} + \frac{12 i d^3 (c + d x) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{b^4} - \frac{2 i d (c + d x)^3 \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{b^2} \\ & - \frac{12 i d^3 (c + d x) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{b^4} + \frac{2 i d (c + d x)^3 \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{b^2} - \frac{12 d^4 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right]}{b^5} + \\ & - \frac{6 d^2 (c + d x)^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right]}{b^3} + \frac{12 d^4 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right]}{b^5} - \frac{6 d^2 (c + d x)^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right]}{b^3} + \\ & - \frac{12 i d^3 (c + d x) \operatorname{PolyLog}\left[4, -e^{i(a+bx)}\right]}{b^4} - \frac{12 i d^3 (c + d x) \operatorname{PolyLog}\left[4, e^{i(a+bx)}\right]}{b^4} - \frac{12 d^4 \operatorname{PolyLog}\left[5, -e^{i(a+bx)}\right]}{b^5} + \frac{12 d^4 \operatorname{PolyLog}\left[5, e^{i(a+bx)}\right]}{b^5} \end{aligned}$$

Result (type 4, 966 leaves):

$$\begin{aligned} & \frac{1}{2 b^5} \\ & \left( -b^4 c^4 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 12 b^2 c^2 d^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - 4 b^4 c^3 d x \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 24 b^2 c d^3 x \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - 6 b^4 c^2 d^2 x^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + \right. \\ & 12 b^2 d^4 x^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - 4 b^4 c d^3 x^3 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - b^4 d^4 x^4 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + b^4 c^4 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 12 b^2 c^2 d^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + \\ & 4 b^4 c^3 d x \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 24 b^2 c d^3 x \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + 6 b^4 c^2 d^2 x^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 12 b^2 d^4 x^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + \\ & 4 b^4 c d^3 x^3 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + b^4 d^4 x^4 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 4 i b d (c + d x) \left(-6 d^2 + b^2 (c + d x)^2\right) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] + \\ & 4 i b d (c + d x) \left(-6 d^2 + b^2 (c + d x)^2\right) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] + 12 b^2 c^2 d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] - 24 d^4 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] + \\ & 24 b^2 c d^3 x \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] + 12 b^2 d^4 x^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] - 12 b^2 c^2 d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] + 24 d^4 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] - \\ & 24 b^2 c d^3 x \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] - 12 b^2 d^4 x^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] + 24 i b c d^3 \operatorname{PolyLog}\left[4, -e^{i(a+bx)}\right] + 24 i b d^4 x \operatorname{PolyLog}\left[4, -e^{i(a+bx)}\right] - \\ & 24 i b c d^3 \operatorname{PolyLog}\left[4, e^{i(a+bx)}\right] - 24 i b d^4 x \operatorname{PolyLog}\left[4, e^{i(a+bx)}\right] - 24 d^4 \operatorname{PolyLog}\left[5, -e^{i(a+bx)}\right] + 24 d^4 \operatorname{PolyLog}\left[5, e^{i(a+bx)}\right] \left. \right) - \\ & \frac{1}{2 b^2} \csc [a + b x]^2 \left( b^4 \cos [a + b x] + 4 b c^3 d x \cos [a + b x] + 6 b c^2 d^2 x^2 \cos [a + b x] + 4 b c d^3 x^3 \cos [a + b x] + \right. \\ & \left. b d^4 x^4 \cos [a + b x] + 4 c^3 d \sin [a + b x] + 12 c^2 d^2 x \sin [a + b x] + 12 c d^3 x^2 \sin [a + b x] + 4 d^4 x^3 \sin [a + b x] \right) \end{aligned}$$

### Problem 114: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \cot [a + b x]^2 \csc [a + b x] dx$$

Optimal (type 4, 179 leaves, 17 steps):

$$\frac{(c+dx)^2 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{d^2 \operatorname{ArcTanh}\left[\operatorname{Cos}[a+bx]\right]}{b^3} - \frac{d(c+dx) \operatorname{Csc}[a+bx]}{b^2} - \frac{(c+dx)^2 \operatorname{Cot}[a+bx] \operatorname{Csc}[a+bx]}{2b} - \frac{i d(c+dx) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{b^2} + \frac{i d(c+dx) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{b^2} + \frac{d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right]}{b^3} - \frac{d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right]}{b^3}$$

Result (type 4, 471 leaves):

$$\begin{aligned} & -\frac{d(c+dx) \operatorname{Csc}[a]}{b^2} + \frac{(-c^2 - 2cdx - d^2x^2) \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} + \\ & \frac{1}{2b^3} \left( -b^2c^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 2d^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - 2b^2cdx \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - b^2d^2x^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + b^2c^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - \right. \\ & \quad \left. 2d^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + 2b^2cdx \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + b^2d^2x^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 2ibd(c+dx) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] + \right. \\ & \quad \left. 2ibd(c+dx) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] + 2d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] - 2d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] \right) + \\ & \frac{(c^2 + 2cdx + d^2x^2) \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} + \frac{\operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right] \left(-cd \operatorname{Sin}\left[\frac{bx}{2}\right] - d^2x \operatorname{Sin}\left[\frac{bx}{2}\right]\right)}{2b^2} + \\ & \frac{\operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right] \left(cd \operatorname{Sin}\left[\frac{bx}{2}\right] + d^2x \operatorname{Sin}\left[\frac{bx}{2}\right]\right)}{2b^2} \end{aligned}$$

**Problem 115: Result more than twice size of optimal antiderivative.**

$$\int (c+dx) \operatorname{Cot}[a+bx]^2 \operatorname{Csc}[a+bx] dx$$

Optimal (type 4, 108 leaves, 12 steps):

$$\frac{(c+dx) \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{d \operatorname{Csc}[a+bx]}{2b^2} - \frac{(c+dx) \operatorname{Cot}[a+bx] \operatorname{Csc}[a+bx]}{2b} - \frac{i d \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{2b^2} + \frac{i d \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{2b^2}$$

Result (type 4, 260 leaves):

$$\begin{aligned} & -\frac{d \operatorname{Cot}\left[\frac{1}{2}(a+bx)\right]}{4b^2} - \frac{c \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{8b} - \frac{dx \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{8b} + \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} - \frac{c \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \\ & \frac{ad \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]}{2b^2} - \frac{d \left( (a+bx) \left( \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - \operatorname{Log}\left[1 + e^{i(a+bx)}\right] \right) + i \left( \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] - \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] \right) \right)}{2b^2} + \\ & \frac{c \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{8b} + \frac{dx \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{8b} - \frac{d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{4b^2} \end{aligned}$$

### Problem 130: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^{5/2} \cos [a + bx]^2 \sin [a + bx]^3 dx$$

Optimal (type 4, 615 leaves, 26 steps):

$$\begin{aligned} & \frac{15 d^2 \sqrt{c+dx} \cos [a+bx]}{32 b^3} - \frac{(c+dx)^{5/2} \cos [a+bx]}{8 b} + \frac{5 d^2 \sqrt{c+dx} \cos [3a+3bx]}{576 b^3} - \frac{(c+dx)^{5/2} \cos [3a+3bx]}{48 b} - \\ & \frac{3 d^2 \sqrt{c+dx} \cos [5a+5bx]}{1600 b^3} + \frac{(c+dx)^{5/2} \cos [5a+5bx]}{80 b} - \frac{15 d^{5/2} \sqrt{\frac{\pi}{2}} \cos \left[ a - \frac{bc}{d} \right] \operatorname{FresnelC} \left[ \frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}} \right]}{32 b^{7/2}} - \\ & \frac{5 d^{5/2} \sqrt{\frac{\pi}{6}} \cos \left[ 3a - \frac{3bc}{d} \right] \operatorname{FresnelC} \left[ \frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}}{\sqrt{d}} \right]}{576 b^{7/2}} + \frac{3 d^{5/2} \sqrt{\frac{\pi}{10}} \cos \left[ 5a - \frac{5bc}{d} \right] \operatorname{FresnelC} \left[ \frac{\sqrt{b} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}}{\sqrt{d}} \right]}{1600 b^{7/2}} - \\ & \frac{3 d^{5/2} \sqrt{\frac{\pi}{10}} \operatorname{FresnelS} \left[ \frac{\sqrt{b} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}}{\sqrt{d}} \right] \sin \left[ 5a - \frac{5bc}{d} \right]}{1600 b^{7/2}} + \frac{5 d^{5/2} \sqrt{\frac{\pi}{6}} \operatorname{FresnelS} \left[ \frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}}{\sqrt{d}} \right] \sin \left[ 3a - \frac{3bc}{d} \right]}{576 b^{7/2}} + \\ & \frac{15 d^{5/2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ \frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}} \right] \sin \left[ a - \frac{bc}{d} \right]}{32 b^{7/2}} + \frac{5 d (c+dx)^{3/2} \sin [a+bx]}{16 b^2} + \frac{5 d (c+dx)^{3/2} \sin [3a+3bx]}{288 b^2} - \frac{d (c+dx)^{3/2} \sin [5a+5bx]}{160 b^2} \end{aligned}$$

Result (type 4, 4921 leaves):

$$\begin{aligned} & \frac{1}{160 \sqrt{5} b \sqrt{\frac{b}{d}}} c^2 \left( 2 \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos [5(a+bx)] - \right. \\ & \left. \sqrt{2\pi} \cos \left[ 5a - \frac{5bc}{d} \right] \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] + \sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] \sin \left[ 5a - \frac{5bc}{d} \right] \right) - \\ & \frac{1}{96 \sqrt{3} b \sqrt{\frac{b}{d}}} c^2 \left( 2 \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos [3(a+bx)] - \sqrt{2\pi} \cos \left[ 3a - \frac{3bc}{d} \right] \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] + \right. \end{aligned}$$

$$\begin{aligned}
& \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \operatorname{Sin}\left[3a - \frac{3bc}{d}\right]\right] - \frac{1}{16b\sqrt{\frac{b}{d}}} \\
& c^2 \left( 2\sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}[a+bx] - \sqrt{2\pi} \operatorname{Cos}\left[a - \frac{bc}{d}\right] \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] + \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \operatorname{Sin}\left[a - \frac{bc}{d}\right] \right) - \\
& \frac{1}{16b^3} c \sqrt{\frac{b}{d}} d \left( \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \left( 3d \operatorname{Cos}\left[a - \frac{bc}{d}\right] - 2bc \operatorname{Sin}\left[a - \frac{bc}{d}\right] \right) + \right. \\
& \left. \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \left( 2bc \operatorname{Cos}\left[a - \frac{bc}{d}\right] + 3d \operatorname{Sin}\left[a - \frac{bc}{d}\right] \right) + 2\sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( 2bx \operatorname{Cos}[a+bx] - 3 \operatorname{Sin}[a+bx] \right) \right) + \frac{1}{64b^5} \\
& \left( \frac{b}{d} \right)^{3/2} d^2 \left( \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \left( (4b^2c^2 - 15d^2) \operatorname{Cos}\left[a - \frac{bc}{d}\right] + 12bcd \operatorname{Sin}\left[a - \frac{bc}{d}\right] \right) - \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \right. \\
& \left. \left( -12bcd \operatorname{Cos}\left[a - \frac{bc}{d}\right] + (4b^2c^2 - 15d^2) \operatorname{Sin}\left[a - \frac{bc}{d}\right] \right) - 2\sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( d(-15 + 4b^2x^2) \operatorname{Cos}[a+bx] + 2b(c - 5dx) \operatorname{Sin}[a+bx] \right) \right) - \\
& \frac{1}{96\sqrt{3}b^3} c \sqrt{\frac{b}{d}} d \left( \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \left( d \operatorname{Cos}\left[3a - \frac{3bc}{d}\right] - 2bc \operatorname{Sin}\left[3a - \frac{3bc}{d}\right] \right) + \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right. \\
& \left. \left( 2bc \operatorname{Cos}\left[3a - \frac{3bc}{d}\right] + d \operatorname{Sin}\left[3a - \frac{3bc}{d}\right] \right) + 2\sqrt{3} \sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( 2bx \operatorname{Cos}[3(a+bx)] - \operatorname{Sin}[3(a+bx)] \right) \right) + \frac{1}{800\sqrt{5}b^3} \\
& c \sqrt{\frac{b}{d}} d \left( \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \left( 3d \operatorname{Cos}\left[5a - \frac{5bc}{d}\right] - 10bc \operatorname{Sin}\left[5a - \frac{5bc}{d}\right] \right) + \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right. \\
& \left. \left( 10bc \operatorname{Cos}\left[5a - \frac{5bc}{d}\right] + 3d \operatorname{Sin}\left[5a - \frac{5bc}{d}\right] \right) + 2\sqrt{5} \sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( 10bx \operatorname{Cos}[5(a+bx)] - 3 \operatorname{Sin}[5(a+bx)] \right) \right) +
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{16} d^2 \left( \text{Sin}[3 a] \left( \frac{c^2 \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Cos}\left[\frac{3 b (c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right]\right) \text{Sin}\left[\frac{3 b c}{d}\right]}{3 \sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} + \right. \right. \\
& \frac{c^2 \text{Cos}\left[\frac{3 b c}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Sin}\left[\frac{3 b (c+d x)}{d}\right] \right)}{3 \sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} - \frac{1}{9 \sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2 c \text{Cos}\left[\frac{3 b c}{d}\right] \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Cos}\left[\frac{3 b (c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] \right) + 3 \sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \text{Sin}\left[\frac{3 b (c+d x)}{d}\right] \right) - \right. \\
& \frac{1}{9 \sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2 c \text{Sin}\left[\frac{3 b c}{d}\right] \left( -3 \sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \text{Cos}\left[\frac{3 b (c+d x)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] + \right. \right. \\
& \left. \left. \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Sin}\left[\frac{3 b (c+d x)}{d}\right] \right) \right) + \frac{1}{27 \sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \text{Sin}\left[\frac{3 b c}{d}\right] \left( -9 \sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+d x)^{5/2} \text{Cos}\left[\frac{3 b (c+d x)}{d}\right] + \right. \\
& \left. \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Cos}\left[\frac{3 b (c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] \right) + 3 \sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \text{Sin}\left[\frac{3 b (c+d x)}{d}\right] \right) \right) \right) + \\
& \frac{1}{27 \sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \text{Cos}\left[\frac{3 b c}{d}\right] \left( 9 \sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+d x)^{5/2} \text{Sin}\left[\frac{3 b (c+d x)}{d}\right] - \frac{5}{2} \left( -3 \sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \text{Cos}\left[\frac{3 b (c+d x)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Sin}\left[\frac{3 b (c+d x)}{d}\right] \right) \right) \right) \right) \right) + \\
& \left. \text{Cos}[3 a] \left( \frac{c^2 \text{Cos}\left[\frac{3 b c}{d}\right] \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Cos}\left[\frac{3 b (c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] \right)}{3 \sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{c^2 \operatorname{Sin}\left[\frac{3bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right)}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} + \frac{1}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Sin}\left[\frac{3bc}{d}\right] \\
& \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) - \\
& \frac{1}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Cos}\left[\frac{3bc}{d}\right] \left( -3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \right. \right. \\
& \left. \left. \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) + \frac{1}{27\sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{3bc}{d}\right] \left( -9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) - \\
& \frac{1}{27\sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{3bc}{d}\right] \left( 9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] - \frac{5}{2} \left( -3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) \right) \right) - \\
& \frac{1}{16} d^2 \left( \operatorname{Sin}[5a] \left( \frac{c^2 \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) \operatorname{Sin}\left[\frac{5bc}{d}\right]}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} + \right. \right. \\
& \left. \frac{c^2 \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} - \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 2c \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + \right. \\
& \quad \left. 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) - \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Sin}\left[\frac{5bc}{d}\right] \\
& \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{5}{2} \right. \\
& \quad \left. \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( 25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] - \frac{5}{2} \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \right. \right. \\
& \quad \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) \right) + \\
& \operatorname{Cos}[5a] \left( \frac{c^2 \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} - \right. \\
& \quad \left. \frac{c^2 \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} + \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} \right)
\end{aligned}$$

$$\begin{aligned}
& 2c \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + \right. \\
& \left. 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) - \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Cos}\left[\frac{5bc}{d}\right] \\
& \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{5}{2} \right. \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) - \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( 25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] - \frac{5}{2} \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) \right) \right) \right)
\end{aligned}$$

**Problem 135: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^{5/2} \operatorname{Cos}[a+bx]^2 \operatorname{Sin}[a+bx]^3 dx$$

Optimal (type 4, 615 leaves, 26 steps):

$$\begin{aligned}
& \frac{15 d^2 \sqrt{c+d x} \operatorname{Cos}[a+b x]}{32 b^3} - \frac{(c+d x)^{5/2} \operatorname{Cos}[a+b x]}{8 b} + \frac{5 d^2 \sqrt{c+d x} \operatorname{Cos}[3 a+3 b x]}{576 b^3} - \frac{(c+d x)^{5/2} \operatorname{Cos}[3 a+3 b x]}{48 b} \\
& \frac{3 d^2 \sqrt{c+d x} \operatorname{Cos}[5 a+5 b x]}{1600 b^3} + \frac{(c+d x)^{5/2} \operatorname{Cos}[5 a+5 b x]}{80 b} - \frac{15 d^{5/2} \sqrt{\frac{\pi}{2}} \operatorname{Cos}\left[a-\frac{b c}{d}\right] \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right]}{32 b^{7/2}} \\
& \frac{5 d^{5/2} \sqrt{\frac{\pi}{6}} \operatorname{Cos}\left[3 a-\frac{3 b c}{d}\right] \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right]}{576 b^{7/2}} + \frac{3 d^{5/2} \sqrt{\frac{\pi}{10}} \operatorname{Cos}\left[5 a-\frac{5 b c}{d}\right] \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right]}{1600 b^{7/2}} \\
& \frac{3 d^{5/2} \sqrt{\frac{\pi}{10}} \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sin}\left[5 a-\frac{5 b c}{d}\right]}{1600 b^{7/2}} + \frac{5 d^{5/2} \sqrt{\frac{\pi}{6}} \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sin}\left[3 a-\frac{3 b c}{d}\right]}{576 b^{7/2}} \\
& \frac{15 d^{5/2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sin}\left[a-\frac{b c}{d}\right]}{32 b^{7/2}} + \frac{5 d(c+d x)^{3/2} \operatorname{Sin}[a+b x]}{16 b^2} + \frac{5 d(c+d x)^{3/2} \operatorname{Sin}[3 a+3 b x]}{288 b^2} - \frac{d(c+d x)^{3/2} \operatorname{Sin}[5 a+5 b x]}{160 b^2}
\end{aligned}$$

Result (type 4, 4921 leaves):

$$\begin{aligned}
& \frac{1}{160 \sqrt{5} b \sqrt{\frac{b}{d}}} c^2 \left( 2 \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}[5(a+b x)] - \right. \\
& \left. \sqrt{2 \pi} \operatorname{Cos}\left[5 a-\frac{5 b c}{d}\right] \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}\right] + \sqrt{2 \pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}\right] \operatorname{Sin}\left[5 a-\frac{5 b c}{d}\right] \right) - \\
& \frac{1}{96 \sqrt{3} b \sqrt{\frac{b}{d}}} c^2 \left( 2 \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}[3(a+b x)] - \sqrt{2 \pi} \operatorname{Cos}\left[3 a-\frac{3 b c}{d}\right] \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] + \right. \\
& \left. \sqrt{2 \pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}\right] \operatorname{Sin}\left[3 a-\frac{3 b c}{d}\right] \right) - \frac{1}{16 b \sqrt{\frac{b}{d}}} \\
& c^2 \left( 2 \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}[a+b x] - \sqrt{2 \pi} \operatorname{Cos}\left[a-\frac{b c}{d}\right] \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+d x}\right] + \sqrt{2 \pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+d x}\right] \operatorname{Sin}\left[a-\frac{b c}{d}\right] \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{16 b^3} c \sqrt{\frac{b}{d}} d \left( \sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx} \right] \left( 3 d \operatorname{Cos} \left[ a - \frac{bc}{d} \right] - 2 bc \operatorname{Sin} \left[ a - \frac{bc}{d} \right] \right) + \right. \\
& \left. \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx} \right] \left( 2 bc \operatorname{Cos} \left[ a - \frac{bc}{d} \right] + 3 d \operatorname{Sin} \left[ a - \frac{bc}{d} \right] \right) + 2 \sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( 2 bx \operatorname{Cos} [a+bx] - 3 \operatorname{Sin} [a+bx] \right) \right) + \frac{1}{64 b^5} \\
& \left( \frac{b}{d} \right)^{3/2} d^2 \left( \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx} \right] \left( (4 b^2 c^2 - 15 d^2) \operatorname{Cos} \left[ a - \frac{bc}{d} \right] + 12 bcd \operatorname{Sin} \left[ a - \frac{bc}{d} \right] \right) - \sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx} \right] \right. \\
& \left. \left( -12 bcd \operatorname{Cos} \left[ a - \frac{bc}{d} \right] + (4 b^2 c^2 - 15 d^2) \operatorname{Sin} \left[ a - \frac{bc}{d} \right] \right) - 2 \sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( d (-15 + 4 b^2 x^2) \operatorname{Cos} [a+bx] + 2 b (c - 5 dx) \operatorname{Sin} [a+bx] \right) \right) - \\
& \frac{1}{96 \sqrt{3} b^3} c \sqrt{\frac{b}{d}} d \left( \sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \left( d \operatorname{Cos} \left[ 3a - \frac{3bc}{d} \right] - 2 bc \operatorname{Sin} \left[ 3a - \frac{3bc}{d} \right] \right) + \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right. \\
& \left. \left( 2 bc \operatorname{Cos} \left[ 3a - \frac{3bc}{d} \right] + d \operatorname{Sin} \left[ 3a - \frac{3bc}{d} \right] \right) + 2 \sqrt{3} \sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( 2 bx \operatorname{Cos} [3(a+bx)] - \operatorname{Sin} [3(a+bx)] \right) \right) + \frac{1}{800 \sqrt{5} b^3} \\
& c \sqrt{\frac{b}{d}} d \left( \sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] \left( 3 d \operatorname{Cos} \left[ 5a - \frac{5bc}{d} \right] - 10 bc \operatorname{Sin} \left[ 5a - \frac{5bc}{d} \right] \right) + \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] \right. \\
& \left. \left( 10 bc \operatorname{Cos} \left[ 5a - \frac{5bc}{d} \right] + 3 d \operatorname{Sin} \left[ 5a - \frac{5bc}{d} \right] \right) + 2 \sqrt{5} \sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( 10 bx \operatorname{Cos} [5(a+bx)] - 3 \operatorname{Sin} [5(a+bx)] \right) \right) + \\
& \frac{1}{16} d^2 \left( \operatorname{Sin} [3a] \left( \frac{c^2 \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right) \operatorname{Sin} \left[ \frac{3bc}{d} \right]}{3 \sqrt{3} \left( \frac{b}{d} \right)^{3/2} d^3} + \right. \right. \\
& \left. \frac{c^2 \operatorname{Cos} \left[ \frac{3bc}{d} \right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right)}{3 \sqrt{3} \left( \frac{b}{d} \right)^{3/2} d^3} - \frac{1}{9 \sqrt{3} \left( \frac{b}{d} \right)^{5/2} d^3} 2 c \operatorname{Cos} \left[ \frac{3bc}{d} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{3b(c+dx)}{d}\right] \right) - \\
& \frac{1}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \sin\left[\frac{3bc}{d}\right] \left( -3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{3b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \right. \right. \\
& \left. \left. \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{3b(c+dx)}{d}\right] \right) \right) + \frac{1}{27\sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \sin\left[\frac{3bc}{d}\right] \left( -9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \cos\left[\frac{3b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{3b(c+dx)}{d}\right] \right) \right) \right) + \\
& \frac{1}{27\sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \cos\left[\frac{3bc}{d}\right] \left( 9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \sin\left[\frac{3b(c+dx)}{d}\right] - \frac{5}{2} \left( -3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{3b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{3b(c+dx)}{d}\right] \right) \right) \right) \right) \right) + \\
& \operatorname{Cos}[3a] \left( \frac{c^2 \cos\left[\frac{3bc}{d}\right] \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right)}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} - \right. \\
& \left. \frac{c^2 \sin\left[\frac{3bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{3b(c+dx)}{d}\right] \right)}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} + \frac{1}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \sin\left[\frac{3bc}{d}\right] \right) - \\
& \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{3b(c+dx)}{d}\right] \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{9\sqrt{3}\left(\frac{b}{d}\right)^{5/2}d^3} 2c \operatorname{Cos}\left[\frac{3bc}{d}\right] \left( -3\sqrt{3}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right] + \right. \right. \\
& \left. \left. \sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) + \frac{1}{27\sqrt{3}\left(\frac{b}{d}\right)^{7/2}d^3} \operatorname{Cos}\left[\frac{3bc}{d}\right] \left( -9\sqrt{3}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right] \right) + 3\sqrt{3}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) - \\
& \frac{1}{27\sqrt{3}\left(\frac{b}{d}\right)^{7/2}d^3} \operatorname{Sin}\left[\frac{3bc}{d}\right] \left( 9\sqrt{3}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] - \frac{5}{2} \left( -3\sqrt{3}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right] + \sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) \right) \right) - \\
& \frac{1}{16} d^2 \left( \operatorname{Sin}[5a] \left( \frac{c^2 \left( -\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right] \right) \operatorname{Sin}\left[\frac{5bc}{d}\right]}{5\sqrt{5}\left(\frac{b}{d}\right)^{3/2}d^3} + \right. \right. \\
& \left. \frac{c^2 \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right] + \sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right)}{5\sqrt{5}\left(\frac{b}{d}\right)^{3/2}d^3} - \frac{1}{25\sqrt{5}\left(\frac{b}{d}\right)^{5/2}d^3} \right. \\
& \left. 2c \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right] \right) + \right. \\
& \left. \left. 5\sqrt{5}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) - \frac{1}{25\sqrt{5}\left(\frac{b}{d}\right)^{5/2}d^3} 2c \operatorname{Sin}\left[\frac{5bc}{d}\right] \right)
\end{aligned}$$



$$\begin{aligned}
& \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{5}{2} \right. \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( 25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] - \frac{5}{2} \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) \right) + \\
& \operatorname{Cos}[5a] \left( \frac{c^2 \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} - \right. \\
& \left. \frac{c^2 \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} + \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} \right. \\
& \left. 2c \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + \right. \right. \\
& \left. \left. 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) - \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Cos}\left[\frac{5bc}{d}\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{5b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \cos\left[\frac{5bc}{d}\right] \left( -25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \cos\left[\frac{5b(c+dx)}{d}\right] + \frac{5}{2} \right. \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{5b(c+dx)}{d}\right] \right) \right) - \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \sin\left[\frac{5bc}{d}\right] \left( 25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \sin\left[\frac{5b(c+dx)}{d}\right] - \frac{5}{2} \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{5b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) \right) \right)
\end{aligned}$$

**Problem 156: Result unnecessarily involves imaginary or complex numbers.**

$$\int (c+dx)^3 \cos[ax+bx]^3 \sin[ax+bx]^3 dx$$

Optimal (type 3, 181 leaves, 10 steps):

$$\begin{aligned}
& \frac{9d^2(c+dx)\cos[2a+2bx]}{128b^3} - \frac{3(c+dx)^3\cos[2a+2bx]}{64b} - \frac{d^2(c+dx)\cos[6a+6bx]}{1152b^3} + \frac{(c+dx)^3\cos[6a+6bx]}{192b} - \\
& \frac{9d^3\sin[2a+2bx]}{256b^4} + \frac{9d(c+dx)^2\sin[2a+2bx]}{128b^2} + \frac{d^3\sin[6a+6bx]}{6912b^4} - \frac{d(c+dx)^2\sin[6a+6bx]}{384b^2}
\end{aligned}$$

Result (type 3, 174 leaves):

$$\begin{aligned}
& \frac{1}{6912b^4} \left( -162b(c+dx) \left( -3d^2 + 2b^2(c+dx)^2 \right) \cos[2(a+bx)] + 6b(c+dx) \left( -d^2 + 6b^2(c+dx)^2 \right) \cos[6(a+bx)] - \right. \\
& \left. 2d \left( 121d^2 - 234b^2(c+dx)^2 + \left( -d^2 + 18b^2(c+dx)^2 \right) \cos[4(a+bx)] \right) \sin[2(a+bx)] \right) \\
& \left( \cos[6(a+bx)] - i \sin[6(a+bx)] \right) \left( \cos[6(a+bx)] + i \sin[6(a+bx)] \right)
\end{aligned}$$

### Problem 162: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\cos[a + bx]^3 \sin[a + bx]^3}{(c + dx)^4} dx$$

Optimal (type 4, 287 leaves, 14 steps):

$$\begin{aligned} & -\frac{b \cos[2a + 2bx]}{32d^2(c + dx)^2} + \frac{b \cos[6a + 6bx]}{32d^2(c + dx)^2} - \frac{b^3 \cos\left[2a - \frac{2bc}{d}\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right]}{8d^4} + \\ & \frac{9b^3 \cos\left[6a - \frac{6bc}{d}\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right]}{8d^4} - \frac{\sin[2a + 2bx]}{32d(c + dx)^3} + \frac{b^2 \sin[2a + 2bx]}{16d^3(c + dx)} + \frac{\sin[6a + 6bx]}{96d(c + dx)^3} - \\ & \frac{3b^2 \sin[6a + 6bx]}{16d^3(c + dx)} + \frac{b^3 \sin\left[2a - \frac{2bc}{d}\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right]}{8d^4} - \frac{9b^3 \sin\left[6a - \frac{6bc}{d}\right] \operatorname{SinIntegral}\left[\frac{6bc}{d} + 6bx\right]}{8d^4} \end{aligned}$$

Result (type 4, 3285 leaves):

$$\begin{aligned} & \frac{1}{8(c + dx)^3} \left( \frac{\cos[6a + 6bx]}{24d^4} - \frac{\sin[6a + 6bx]}{24d^4} \right) \\ & \left( -18ib^2c^2d + 3b^3cd^2 + id^3 - 36ib^2cd^2x + 3bd^3x - 18ib^2d^3x^2 + 6ib^2c^2d \cos[4a + 4bx] - 3b^3cd^2 \cos[4a + 4bx] - \right. \\ & 3id^3 \cos[4a + 4bx] + 12ib^2cd^2x \cos[4a + 4bx] - 3bd^3x \cos[4a + 4bx] + 6ib^2d^3x^2 \cos[4a + 4bx] - 6ib^2c^2d \cos[8a + 8bx] - \\ & 3b^3cd^2 \cos[8a + 8bx] + 3id^3 \cos[8a + 8bx] - 12ib^2cd^2x \cos[8a + 8bx] - 3bd^3x \cos[8a + 8bx] - 6ib^2d^3x^2 \cos[8a + 8bx] + \\ & 18ib^2c^2d \cos[12a + 12bx] + 3b^3cd^2 \cos[12a + 12bx] - id^3 \cos[12a + 12bx] + 36ib^2cd^2x \cos[12a + 12bx] + \\ & 3bd^3x \cos[12a + 12bx] + 18ib^2d^3x^2 \cos[12a + 12bx] - 12b^3c^3 \cos\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] - \\ & 36b^3c^2dx \cos\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] - 36b^3cd^2x^2 \cos\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] - \\ & 12b^3d^3x^3 \cos\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] - 12b^3c^3 \cos\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] - \\ & 36b^3c^2dx \cos\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] - 36b^3cd^2x^2 \cos\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] - \\ & 12b^3d^3x^3 \cos\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] + 108b^3c^3 \cos\left[12a - \frac{6bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] + \\ & 324b^3c^2dx \cos\left[12a - \frac{6bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] + 324b^3cd^2x^2 \cos\left[12a - \frac{6bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] + \\ & 108b^3d^3x^3 \cos\left[12a - \frac{6bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] + 108b^3c^3 \cos\left[\frac{6bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] + \\ & 324b^3c^2dx \cos\left[\frac{6bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] + 324b^3cd^2x^2 \cos\left[\frac{6bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] + \end{aligned}$$

$$\begin{aligned}
& 108 b^3 d^3 x^3 \operatorname{Cos}\left[\frac{6bc}{d} + 6bx\right] \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] - 6b^2 c^2 d \operatorname{Sin}[4a + 4bx] - 3ibcd^2 \operatorname{Sin}[4a + 4bx] + 3d^3 \operatorname{Sin}[4a + 4bx] - \\
& 12b^2 c d^2 x \operatorname{Sin}[4a + 4bx] - 3ibd^3 x \operatorname{Sin}[4a + 4bx] - 6b^2 d^3 x^2 \operatorname{Sin}[4a + 4bx] + 108ib^3 c^3 \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] \operatorname{Sin}\left[12a - \frac{6bc}{d} + 6bx\right] + \\
& 324ib^3 c^2 d x \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] \operatorname{Sin}\left[12a - \frac{6bc}{d} + 6bx\right] + 324ib^3 c d^2 x^2 \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] \operatorname{Sin}\left[12a - \frac{6bc}{d} + 6bx\right] + \\
& 108ib^3 d^3 x^3 \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] \operatorname{Sin}\left[12a - \frac{6bc}{d} + 6bx\right] - 12ib^3 c^3 \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] \operatorname{Sin}\left[8a - \frac{2bc}{d} + 6bx\right] - \\
& 36ib^3 c^2 d x \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] \operatorname{Sin}\left[8a - \frac{2bc}{d} + 6bx\right] - 36ib^3 c d^2 x^2 \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] \operatorname{Sin}\left[8a - \frac{2bc}{d} + 6bx\right] - \\
& 12ib^3 d^3 x^3 \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] \operatorname{Sin}\left[8a - \frac{2bc}{d} + 6bx\right] - 12ib^3 c^3 \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] \operatorname{Sin}\left[4a + \frac{2bc}{d} + 6bx\right] - \\
& 36ib^3 c^2 d x \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] \operatorname{Sin}\left[4a + \frac{2bc}{d} + 6bx\right] - 36ib^3 c d^2 x^2 \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] \operatorname{Sin}\left[4a + \frac{2bc}{d} + 6bx\right] - \\
& 12ib^3 d^3 x^3 \operatorname{CosIntegral}\left[\frac{2bc}{d} + 2bx\right] \operatorname{Sin}\left[4a + \frac{2bc}{d} + 6bx\right] + 108ib^3 c^3 \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] \operatorname{Sin}\left[\frac{6bc}{d} + 6bx\right] + \\
& 324ib^3 c^2 d x \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] \operatorname{Sin}\left[\frac{6bc}{d} + 6bx\right] + 324ib^3 c d^2 x^2 \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] \operatorname{Sin}\left[\frac{6bc}{d} + 6bx\right] + \\
& 108ib^3 d^3 x^3 \operatorname{CosIntegral}\left[\frac{6bc}{d} + 6bx\right] \operatorname{Sin}\left[\frac{6bc}{d} + 6bx\right] + 6b^2 c^2 d \operatorname{Sin}[8a + 8bx] - 3ibcd^2 \operatorname{Sin}[8a + 8bx] - \\
& 3d^3 \operatorname{Sin}[8a + 8bx] + 12b^2 c d^2 x \operatorname{Sin}[8a + 8bx] - 3ibd^3 x \operatorname{Sin}[8a + 8bx] + 6b^2 d^3 x^2 \operatorname{Sin}[8a + 8bx] - 18b^2 c^2 d \operatorname{Sin}[12a + 12bx] + \\
& 3ibcd^2 \operatorname{Sin}[12a + 12bx] + d^3 \operatorname{Sin}[12a + 12bx] - 36b^2 c d^2 x \operatorname{Sin}[12a + 12bx] + 3ibd^3 x \operatorname{Sin}[12a + 12bx] - 18b^2 d^3 x^2 \operatorname{Sin}[12a + 12bx] - \\
& 12ib^3 c^3 \operatorname{Cos}\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] - 36ib^3 c^2 d x \operatorname{Cos}\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] - \\
& 36ib^3 c d^2 x^2 \operatorname{Cos}\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] - 12ib^3 d^3 x^3 \operatorname{Cos}\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + \\
& 12ib^3 c^3 \operatorname{Cos}\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + 36ib^3 c^2 d x \operatorname{Cos}\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + \\
& 36ib^3 c d^2 x^2 \operatorname{Cos}\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + 12ib^3 d^3 x^3 \operatorname{Cos}\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + \\
& 12b^3 c^3 \operatorname{Sin}\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + 36b^3 c^2 d x \operatorname{Sin}\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + \\
& 36b^3 c d^2 x^2 \operatorname{Sin}\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + 12b^3 d^3 x^3 \operatorname{Sin}\left[8a - \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] - \\
& 12b^3 c^3 \operatorname{Sin}\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] - 36b^3 c^2 d x \operatorname{Sin}\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] - \\
& 36b^3 c d^2 x^2 \operatorname{Sin}\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] - 12b^3 d^3 x^3 \operatorname{Sin}\left[4a + \frac{2bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{2bc}{d} + 2bx\right] + \\
& 108ib^3 c^3 \operatorname{Cos}\left[12a - \frac{6bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{6bc}{d} + 6bx\right] + 324ib^3 c^2 d x \operatorname{Cos}\left[12a - \frac{6bc}{d} + 6bx\right] \operatorname{SinIntegral}\left[\frac{6bc}{d} + 6bx\right] +
\end{aligned}$$

$$\begin{aligned}
& 324 i b^3 c d^2 x^2 \operatorname{Cos}\left[12 a - \frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] + 108 i b^3 d^3 x^3 \operatorname{Cos}\left[12 a - \frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] - \\
& 108 i b^3 c^3 \operatorname{Cos}\left[\frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] - 324 i b^3 c^2 d x \operatorname{Cos}\left[\frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] - \\
& 324 i b^3 c d^2 x^2 \operatorname{Cos}\left[\frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] - 108 i b^3 d^3 x^3 \operatorname{Cos}\left[\frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] - \\
& 108 b^3 c^3 \operatorname{Sin}\left[12 a - \frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] - 324 b^3 c^2 d x \operatorname{Sin}\left[12 a - \frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] - \\
& 324 b^3 c d^2 x^2 \operatorname{Sin}\left[12 a - \frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] - 108 b^3 d^3 x^3 \operatorname{Sin}\left[12 a - \frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] + \\
& 108 b^3 c^3 \operatorname{Sin}\left[\frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] + 324 b^3 c^2 d x \operatorname{Sin}\left[\frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] + \\
& 324 b^3 c d^2 x^2 \operatorname{Sin}\left[\frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right] + 108 b^3 d^3 x^3 \operatorname{Sin}\left[\frac{6 b c}{d} + 6 b x\right] \operatorname{SinIntegral}\left[\frac{6 b c}{d} + 6 b x\right]
\end{aligned}$$

**Problem 164: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^4 \operatorname{Cos}[a + b x]^2 \operatorname{Cot}[a + b x] dx$$

Optimal (type 4, 307 leaves, 13 steps):

$$\begin{aligned}
& -\frac{3 c d^3 x}{2 b^3} - \frac{3 d^4 x^2}{4 b^3} + \frac{(c + d x)^4}{4 b} - \frac{i (c + d x)^5}{5 d} + \frac{(c + d x)^4 \operatorname{Log}[1 - e^{2 i (a + b x)}]}{b} - \\
& \frac{2 i d (c + d x)^3 \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{b^2} + \frac{3 d^2 (c + d x)^2 \operatorname{PolyLog}[3, e^{2 i (a + b x)}]}{b^3} + \\
& \frac{3 i d^3 (c + d x) \operatorname{PolyLog}[4, e^{2 i (a + b x)}]}{b^4} - \frac{3 d^4 \operatorname{PolyLog}[5, e^{2 i (a + b x)}]}{2 b^5} + \frac{3 d^3 (c + d x) \operatorname{Cos}[a + b x] \operatorname{Sin}[a + b x]}{2 b^4} - \\
& \frac{d (c + d x)^3 \operatorname{Cos}[a + b x] \operatorname{Sin}[a + b x]}{b^2} - \frac{3 d^4 \operatorname{Sin}[a + b x]^2}{4 b^5} + \frac{3 d^2 (c + d x)^2 \operatorname{Sin}[a + b x]^2}{2 b^3} - \frac{(c + d x)^4 \operatorname{Sin}[a + b x]^2}{2 b}
\end{aligned}$$

Result (type 4, 2486 leaves):

$$\begin{aligned}
& -\frac{1}{2b^3}c^2d^2e^{-ia}\operatorname{Csc}[a] \\
& \left(2b^2x^2\left(2be^{2ia}x+3i(-1+e^{2ia})\operatorname{Log}\left[1-e^{2i(a+bx)}\right]\right)+6b(-1+e^{2ia})x\operatorname{PolyLog}\left[2,e^{2i(a+bx)}\right]+3i(-1+e^{2ia})\operatorname{PolyLog}\left[3,e^{2i(a+bx)}\right]\right)- \\
& cd^3e^{ia}\operatorname{Csc}[a]\left(x^4+(-1+e^{-2ia})x^4+\frac{1}{2b^4}e^{-2ia}(-1+e^{2ia})\right. \\
& \left.(2b^4x^4+4ib^3x^3\operatorname{Log}\left[1-e^{2i(a+bx)}\right]+6b^2x^2\operatorname{PolyLog}\left[2,e^{2i(a+bx)}\right]+6ibx\operatorname{PolyLog}\left[3,e^{2i(a+bx)}\right]-3\operatorname{PolyLog}\left[4,e^{2i(a+bx)}\right]\right)- \\
& \frac{1}{5}d^4e^{ia}\operatorname{Csc}[a]\left(x^5+(-1+e^{-2ia})x^5+\frac{1}{4b^5}e^{-2ia}(-1+e^{2ia})\left(4b^5x^5+10ib^4x^4\operatorname{Log}\left[1-e^{2i(a+bx)}\right]+20b^3x^3\operatorname{PolyLog}\left[2,e^{2i(a+bx)}\right]+30ib^2x^2\operatorname{PolyLog}\left[3,e^{2i(a+bx)}\right]-30bx\operatorname{PolyLog}\left[4,e^{2i(a+bx)}\right]-15i\operatorname{PolyLog}\left[5,e^{2i(a+bx)}\right]\right)\right)+ \\
& \frac{c^4\operatorname{Csc}[a]\left(-bx\operatorname{Cos}[a]+\operatorname{Log}\left[\operatorname{Cos}[bx]\operatorname{Sin}[a]+\operatorname{Cos}[a]\operatorname{Sin}[bx]\right]\operatorname{Sin}[a]\right)}{b\left(\operatorname{Cos}[a]^2+\operatorname{Sin}[a]^2\right)}+\operatorname{Csc}[a]\left(\frac{\operatorname{Cos}\left[2a+2bx\right]}{160b^5}-\frac{i\operatorname{Sin}\left[2a+2bx\right]}{160b^5}\right) \\
& \left(80b^5c^4x\operatorname{Cos}\left[a+2bx\right]+160b^5c^3d^2x^2\operatorname{Cos}\left[a+2bx\right]+160b^5c^2d^2x^3\operatorname{Cos}\left[a+2bx\right]+80b^5cd^3x^4\operatorname{Cos}\left[a+2bx\right]+16b^5d^4x^5\operatorname{Cos}\left[a+2bx\right]+80b^5c^4x\operatorname{Cos}\left[3a+2bx\right]+160b^5c^3d^2x^2\operatorname{Cos}\left[3a+2bx\right]+160b^5c^2d^2x^3\operatorname{Cos}\left[3a+2bx\right]+80b^5cd^3x^4\operatorname{Cos}\left[3a+2bx\right]+16b^5d^4x^5\operatorname{Cos}\left[3a+2bx\right]+10ib^4c^4\operatorname{Cos}\left[3a+4bx\right]-20b^3c^3d\operatorname{Cos}\left[3a+4bx\right]-30ib^2c^2d^2\operatorname{Cos}\left[3a+4bx\right]+30b^2cd^3\operatorname{Cos}\left[3a+4bx\right]+15id^4\operatorname{Cos}\left[3a+4bx\right]+40ib^4c^3d\operatorname{Cos}\left[3a+4bx\right]-60b^3c^2d^2x\operatorname{Cos}\left[3a+4bx\right]-60ib^2c^2d^3x\operatorname{Cos}\left[3a+4bx\right]+30bd^4x\operatorname{Cos}\left[3a+4bx\right]+60ib^4c^2d^2x^2\operatorname{Cos}\left[3a+4bx\right]-60b^3cd^3x^2\operatorname{Cos}\left[3a+4bx\right]-30ib^2d^4x^2\operatorname{Cos}\left[3a+4bx\right]+40ib^4cd^3x^3\operatorname{Cos}\left[3a+4bx\right]-20b^3d^4x^3\operatorname{Cos}\left[3a+4bx\right]+10ib^4d^4x^4\operatorname{Cos}\left[3a+4bx\right]-10ib^4c^4\operatorname{Cos}\left[5a+4bx\right]+20b^3c^3d\operatorname{Cos}\left[5a+4bx\right]+30ib^2c^2d^2\operatorname{Cos}\left[5a+4bx\right]-30b^2cd^3\operatorname{Cos}\left[5a+4bx\right]-15id^4\operatorname{Cos}\left[5a+4bx\right]-40ib^4c^3d\operatorname{Cos}\left[5a+4bx\right]+60b^3c^2d^2x\operatorname{Cos}\left[5a+4bx\right]+60ib^2cd^3x\operatorname{Cos}\left[5a+4bx\right]-30bd^4x\operatorname{Cos}\left[5a+4bx\right]-60ib^4c^2d^2x^2\operatorname{Cos}\left[5a+4bx\right]+60b^3cd^3x^2\operatorname{Cos}\left[5a+4bx\right]+30ib^2d^4x^2\operatorname{Cos}\left[5a+4bx\right]-40ib^4cd^3x^3\operatorname{Cos}\left[5a+4bx\right]+20b^3d^4x^3\operatorname{Cos}\left[5a+4bx\right]-10ib^4d^4x^4\operatorname{Cos}\left[5a+4bx\right]+20b^4c^4\operatorname{Sin}\left[a\right]-40ib^3c^3d\operatorname{Sin}\left[a\right]-60b^2c^2d^2\operatorname{Sin}\left[a\right]+60ibcd^3\operatorname{Sin}\left[a\right]+30d^4\operatorname{Sin}\left[a\right]+80b^4c^3d\operatorname{Sin}\left[a\right]-120ib^3c^2d^2x\operatorname{Sin}\left[a\right]-120b^2cd^3x\operatorname{Sin}\left[a\right]+60ibd^4x\operatorname{Sin}\left[a\right]+120b^4c^2d^2x^2\operatorname{Sin}\left[a\right]-120ib^3cd^3x^2\operatorname{Sin}\left[a\right]-60b^2d^4x^2\operatorname{Sin}\left[a\right]+80b^4cd^3x^3\operatorname{Sin}\left[a\right]-40ib^3d^4x^3\operatorname{Sin}\left[a\right]+20b^4d^4x^4\operatorname{Sin}\left[a\right]+80ib^5c^4x\operatorname{Sin}\left[a+2bx\right]+160ib^5c^3d^2x^2\operatorname{Sin}\left[a+2bx\right]+160ib^5c^2d^2x^3\operatorname{Sin}\left[a+2bx\right]+80ib^5cd^3x^4\operatorname{Sin}\left[a+2bx\right]+16ib^5d^4x^5\operatorname{Sin}\left[a+2bx\right]+80ib^5c^4x\operatorname{Sin}\left[3a+2bx\right]+160ib^5c^3d^2x^2\operatorname{Sin}\left[3a+2bx\right]+160ib^5c^2d^2x^3\operatorname{Sin}\left[3a+2bx\right]+80ib^5cd^3x^4\operatorname{Sin}\left[3a+2bx\right]+16ib^5d^4x^5\operatorname{Sin}\left[3a+2bx\right]-10b^4c^4\operatorname{Sin}\left[3a+4bx\right]-20ib^3c^3d\operatorname{Sin}\left[3a+4bx\right]+30b^2c^2d^2\operatorname{Sin}\left[3a+4bx\right]+30ibcd^3\operatorname{Sin}\left[3a+4bx\right]-15d^4\operatorname{Sin}\left[3a+4bx\right]-40b^4c^3d\operatorname{Sin}\left[3a+4bx\right]-60ib^3c^2d^2x\operatorname{Sin}\left[3a+4bx\right]+60b^2cd^3x\operatorname{Sin}\left[3a+4bx\right]+30ibd^4x\operatorname{Sin}\left[3a+4bx\right]-60b^4c^2d^2x^2\operatorname{Sin}\left[3a+4bx\right]-60ib^3cd^3x^2\operatorname{Sin}\left[3a+4bx\right]+30b^2d^4x^2\operatorname{Sin}\left[3a+4bx\right]-40b^4cd^3x^3\operatorname{Sin}\left[3a+4bx\right]-20ib^3d^4x^3\operatorname{Sin}\left[3a+4bx\right]-10b^4d^4x^4\operatorname{Sin}\left[3a+4bx\right]+10b^4c^4\operatorname{Sin}\left[5a+4bx\right]+20ib^3c^3d\operatorname{Sin}\left[5a+4bx\right]-30b^2c^2d^2\operatorname{Sin}\left[5a+4bx\right]-30ibcd^3\operatorname{Sin}\left[5a+4bx\right]+15d^4\operatorname{Sin}\left[5a+4bx\right]+40b^4c^3d\operatorname{Sin}\left[5a+4bx\right]+60ib^3c^2d^2x\operatorname{Sin}\left[5a+4bx\right]-60b^2cd^3x\operatorname{Sin}\left[5a+4bx\right]-30ibd^4x\operatorname{Sin}\left[5a+4bx\right]+60b^4c^2d^2x^2\operatorname{Sin}\left[5a+4bx\right]+60ib^3cd^3x^2\operatorname{Sin}\left[5a+4bx\right]-30b^2d^4x^2\operatorname{Sin}\left[5a+4bx\right]+40b^4cd^3x^3\operatorname{Sin}\left[5a+4bx\right]+20ib^3d^4x^3\operatorname{Sin}\left[5a+4bx\right]+10b^4d^4x^4\operatorname{Sin}\left[5a+4bx\right])-\left(2c^3d\operatorname{Csc}[a]\operatorname{Sec}[a]\left(b^2e^{i\operatorname{ArcTan}[\operatorname{Tan}[a]]}x^2+\frac{1}{\sqrt{1+\operatorname{Tan}[a]^2}}\right)\right. \\
& \left.(ibx(-\pi+2\operatorname{ArcTan}[\operatorname{Tan}[a]])-\pi\operatorname{Log}\left[1+e^{-2ibx}\right]-2(bx+\operatorname{ArcTan}[\operatorname{Tan}[a]])\operatorname{Log}\left[1-e^{2i(bx+\operatorname{ArcTan}[\operatorname{Tan}[a]])}\right]+\pi\operatorname{Log}[\operatorname{Cos}[bx]]+2\operatorname{ArcTan}[\operatorname{Tan}[a]]\operatorname{Log}[\operatorname{Sin}[bx+\operatorname{ArcTan}[\operatorname{Tan}[a]]]]+i\operatorname{PolyLog}\left[2,e^{2i(bx+\operatorname{ArcTan}[\operatorname{Tan}[a]])}\right]\right)\operatorname{Tan}[a]\left.\right)/\left(b^2\sqrt{\operatorname{Sec}[a]^2(\operatorname{Cos}[a]^2+\operatorname{Sin}[a]^2)}\right)
\end{aligned}$$

### Problem 165: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \cos [a + b x]^2 \cot [a + b x] dx$$

Optimal (type 4, 246 leaves, 12 steps):

$$\begin{aligned} & -\frac{3 d^3 x}{8 b^3} + \frac{(c + d x)^3}{4 b} - \frac{i (c + d x)^4}{4 d} + \frac{(c + d x)^3 \operatorname{Log}[1 - e^{2 i (a + b x)}]}{b} - \frac{3 i d (c + d x)^2 \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{2 b^2} + \\ & \frac{3 d^2 (c + d x) \operatorname{PolyLog}[3, e^{2 i (a + b x)}]}{2 b^3} + \frac{3 i d^3 \operatorname{PolyLog}[4, e^{2 i (a + b x)}]}{4 b^4} + \frac{3 d^3 \cos [a + b x] \sin [a + b x]}{8 b^4} - \\ & \frac{3 d (c + d x)^2 \cos [a + b x] \sin [a + b x]}{4 b^2} + \frac{3 d^2 (c + d x) \sin [a + b x]^2}{4 b^3} - \frac{(c + d x)^3 \sin [a + b x]^2}{2 b} \end{aligned}$$

Result (type 4, 1712 leaves):

$$\begin{aligned}
& -\frac{1}{4b^3}c d^2 e^{-ia} \operatorname{Csc}[a] \\
& \left( 2b^2 x^2 \left( 2b e^{2ia} x + 3i(-1 + e^{2ia}) \operatorname{Log}[1 - e^{2i(a+bx)}] \right) + 6b(-1 + e^{2ia}) x \operatorname{PolyLog}[2, e^{2i(a+bx)}] + 3i(-1 + e^{2ia}) \operatorname{PolyLog}[3, e^{2i(a+bx)}] \right) - \\
& \frac{1}{4}d^3 e^{ia} \operatorname{Csc}[a] \left( x^4 + (-1 + e^{-2ia}) x^4 + \frac{1}{2b^4} e^{-2ia} (-1 + e^{2ia}) \right. \\
& \left. (2b^4 x^4 + 4ib^3 x^3 \operatorname{Log}[1 - e^{2i(a+bx)}] + 6b^2 x^2 \operatorname{PolyLog}[2, e^{2i(a+bx)}] + 6ibx \operatorname{PolyLog}[3, e^{2i(a+bx)}] - 3 \operatorname{PolyLog}[4, e^{2i(a+bx)}]) \right) + \\
& \frac{c^3 \operatorname{Csc}[a] (-bx \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[bx] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[bx]]) \operatorname{Sin}[a]}{b(\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \operatorname{Csc}[a] \left( \frac{\operatorname{Cos}[2a + 2bx]}{64b^4} - \frac{i \operatorname{Sin}[2a + 2bx]}{64b^4} \right) \\
& (32b^4 c^3 x \operatorname{Cos}[a + 2bx] + 48b^4 c^2 d x^2 \operatorname{Cos}[a + 2bx] + 32b^4 c d^2 x^3 \operatorname{Cos}[a + 2bx] + 8b^4 d^3 x^4 \operatorname{Cos}[a + 2bx] + 32b^4 c^3 x \operatorname{Cos}[3a + 2bx] + \\
& 48b^4 c^2 d x^2 \operatorname{Cos}[3a + 2bx] + 32b^4 c d^2 x^3 \operatorname{Cos}[3a + 2bx] + 8b^4 d^3 x^4 \operatorname{Cos}[3a + 2bx] + 4ib^3 c^3 \operatorname{Cos}[3a + 4bx] - 6b^2 c^2 d \operatorname{Cos}[3a + 4bx] - \\
& 6ibc d^2 \operatorname{Cos}[3a + 4bx] + 3d^3 \operatorname{Cos}[3a + 4bx] + 12ib^3 c^2 d x \operatorname{Cos}[3a + 4bx] - 12b^2 c d^2 x \operatorname{Cos}[3a + 4bx] - 6ibd^3 x \operatorname{Cos}[3a + 4bx] + \\
& 12ib^3 c d^2 x^2 \operatorname{Cos}[3a + 4bx] - 6b^2 d^3 x^2 \operatorname{Cos}[3a + 4bx] + 4ib^3 d^3 x^3 \operatorname{Cos}[3a + 4bx] - 4ib^3 c^3 \operatorname{Cos}[5a + 4bx] + 6b^2 c^2 d \operatorname{Cos}[5a + 4bx] + \\
& 6ibc d^2 \operatorname{Cos}[5a + 4bx] - 3d^3 \operatorname{Cos}[5a + 4bx] - 12ib^3 c^2 d x \operatorname{Cos}[5a + 4bx] + 12b^2 c d^2 x \operatorname{Cos}[5a + 4bx] + 6ibd^3 x \operatorname{Cos}[5a + 4bx] - \\
& 12ib^3 c d^2 x^2 \operatorname{Cos}[5a + 4bx] + 6b^2 d^3 x^2 \operatorname{Cos}[5a + 4bx] - 4ib^3 d^3 x^3 \operatorname{Cos}[5a + 4bx] + 8b^3 c^3 \operatorname{Sin}[a] - 12ib^2 c^2 d \operatorname{Sin}[a] - \\
& 12bc d^2 \operatorname{Sin}[a] + 6id^3 \operatorname{Sin}[a] + 24b^3 c^2 d x \operatorname{Sin}[a] - 24ib^2 c d^2 x \operatorname{Sin}[a] - 12bd^3 x \operatorname{Sin}[a] + 24b^3 c d^2 x^2 \operatorname{Sin}[a] - 12ib^2 d^3 x^2 \operatorname{Sin}[a] + \\
& 8b^3 d^3 x^3 \operatorname{Sin}[a] + 32ib^4 c^3 x \operatorname{Sin}[a + 2bx] + 48ib^4 c^2 d x^2 \operatorname{Sin}[a + 2bx] + 32ib^4 c d^2 x^3 \operatorname{Sin}[a + 2bx] + 8ib^4 d^3 x^4 \operatorname{Sin}[a + 2bx] + \\
& 32ib^4 c^3 x \operatorname{Sin}[3a + 2bx] + 48ib^4 c^2 d x^2 \operatorname{Sin}[3a + 2bx] + 32ib^4 c d^2 x^3 \operatorname{Sin}[3a + 2bx] + 8ib^4 d^3 x^4 \operatorname{Sin}[3a + 2bx] - \\
& 4b^3 c^3 \operatorname{Sin}[3a + 4bx] - 6ib^2 c^2 d \operatorname{Sin}[3a + 4bx] + 6bc d^2 \operatorname{Sin}[3a + 4bx] + 3id^3 \operatorname{Sin}[3a + 4bx] - 12b^3 c^2 d x \operatorname{Sin}[3a + 4bx] - \\
& 12ib^2 c d^2 x \operatorname{Sin}[3a + 4bx] + 6bd^3 x \operatorname{Sin}[3a + 4bx] - 12b^3 c d^2 x^2 \operatorname{Sin}[3a + 4bx] - 6ib^2 d^3 x^2 \operatorname{Sin}[3a + 4bx] - 4b^3 d^3 x^3 \operatorname{Sin}[3a + 4bx] + \\
& 4b^3 c^3 \operatorname{Sin}[5a + 4bx] + 6ib^2 c^2 d \operatorname{Sin}[5a + 4bx] - 6bc d^2 \operatorname{Sin}[5a + 4bx] - 3id^3 \operatorname{Sin}[5a + 4bx] + 12b^3 c^2 d x \operatorname{Sin}[5a + 4bx] + \\
& 12ib^2 c d^2 x \operatorname{Sin}[5a + 4bx] - 6bd^3 x \operatorname{Sin}[5a + 4bx] + 12b^3 c d^2 x^2 \operatorname{Sin}[5a + 4bx] + 6ib^2 d^3 x^2 \operatorname{Sin}[5a + 4bx] + 4b^3 d^3 x^3 \operatorname{Sin}[5a + 4bx]) - \\
& \left( 3c^2 d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} (ibx(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}]) - \right. \right. \\
& \left. \left. 2(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + \right. \right. \\
& \left. \left. ib \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]])}] \operatorname{Tan}[a] \right) \right) / \left( 2b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)
\end{aligned}$$

**Problem 166: Result more than twice size of optimal antiderivative.**

$$\int (c + dx)^2 \operatorname{Cos}[a + bx]^2 \operatorname{Cot}[a + bx] dx$$

Optimal (type 4, 181 leaves, 9 steps):

$$\begin{aligned}
& \frac{cdx}{2b} + \frac{d^2 x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2 \operatorname{Log}[1 - e^{2i(a+bx)}]}{b} - \frac{id(c+dx) \operatorname{PolyLog}[2, e^{2i(a+bx)}]}{b^2} + \\
& \frac{d^2 \operatorname{PolyLog}[3, e^{2i(a+bx)}]}{2b^3} - \frac{d(c+dx) \operatorname{Cos}[a+bx] \operatorname{Sin}[a+bx]}{2b^2} + \frac{d^2 \operatorname{Sin}[a+bx]^2}{4b^3} - \frac{(c+dx)^2 \operatorname{Sin}[a+bx]^2}{2b}
\end{aligned}$$



Result (type 4, 511 leaves):

$$\frac{1}{3} x \left( 3 c^2 + 3 c d x + d^2 x^2 \right) \operatorname{Cot}[a] - \frac{1}{12 b^3} d^2 e^{-i a} \operatorname{Csc}[a]$$

$$\frac{\left( 2 b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( -1 + e^{2 i a} \right) \operatorname{Log}\left[ 1 - e^{2 i (a+b x)} \right] \right) + 6 b \left( -1 + e^{2 i a} \right) x \operatorname{PolyLog}\left[ 2, e^{2 i (a+b x)} \right] + 3 i \left( -1 + e^{2 i a} \right) \operatorname{PolyLog}\left[ 3, e^{2 i (a+b x)} \right] \right) + c^2 \operatorname{Csc}[a] \left( -b x \operatorname{Cos}[a] + \operatorname{Log}\left[ \operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x] \right] \operatorname{Sin}[a] \right)}{b \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} + \frac{1}{8 b^3}$$

$$\operatorname{Cos}[2 b x] \left( 2 b^2 c^2 \operatorname{Cos}[2 a] - d^2 \operatorname{Cos}[2 a] + 4 b^2 c d x \operatorname{Cos}[2 a] + 2 b^2 d^2 x^2 \operatorname{Cos}[2 a] - 2 b c d \operatorname{Sin}[2 a] - 2 b d^2 x \operatorname{Sin}[2 a] \right) - \frac{1}{8 b^3}$$

$$\left( 2 b c d \operatorname{Cos}[2 a] + 2 b d^2 x \operatorname{Cos}[2 a] + 2 b^2 c^2 \operatorname{Sin}[2 a] - d^2 \operatorname{Sin}[2 a] + 4 b^2 c d x \operatorname{Sin}[2 a] + 2 b^2 d^2 x^2 \operatorname{Sin}[2 a] \right) \operatorname{Sin}[2 b x] -$$

$$\left( c d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right) \right.$$

$$\left. \left( i b x \left( -\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \right) - \pi \operatorname{Log}\left[ 1 + e^{-2 i b x} \right] - 2 \left( b x + \operatorname{ArcTan}[\operatorname{Tan}[a]] \right) \operatorname{Log}\left[ 1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])} \right] + \pi \operatorname{Log}\left[ \operatorname{Cos}[b x] \right] + 2 \right.$$

$$\left. \left. \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}\left[ \operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]] \right] + i \operatorname{PolyLog}\left[ 2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])} \right] \right) \operatorname{Tan}[a] \right) \left. \right) / \left( b^2 \sqrt{\operatorname{Sec}[a]^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right)$$

Problem 171: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^4 \operatorname{Cos}[a + b x] \operatorname{Cot}[a + b x]^2 dx$$

Optimal (type 4, 299 leaves, 16 steps):

$$-\frac{8 d (c + d x)^3 \operatorname{ArcTanh}\left[ e^{i (a+b x)} \right]}{b^2} + \frac{24 d^3 (c + d x) \operatorname{Cos}[a + b x]}{b^4} - \frac{4 d (c + d x)^3 \operatorname{Cos}[a + b x]}{b^2}$$

$$\frac{(c + d x)^4 \operatorname{Csc}[a + b x]}{b} + \frac{12 i d^2 (c + d x)^2 \operatorname{PolyLog}\left[ 2, -e^{i (a+b x)} \right]}{b^3} - \frac{12 i d^2 (c + d x)^2 \operatorname{PolyLog}\left[ 2, e^{i (a+b x)} \right]}{b^3}$$

$$\frac{24 d^3 (c + d x) \operatorname{PolyLog}\left[ 3, -e^{i (a+b x)} \right]}{b^4} + \frac{24 d^3 (c + d x) \operatorname{PolyLog}\left[ 3, e^{i (a+b x)} \right]}{b^4} - \frac{24 i d^4 \operatorname{PolyLog}\left[ 4, -e^{i (a+b x)} \right]}{b^5} +$$

$$\frac{24 i d^4 \operatorname{PolyLog}\left[ 4, e^{i (a+b x)} \right]}{b^5} - \frac{24 d^4 \operatorname{Sin}[a + b x]}{b^5} + \frac{12 d^2 (c + d x)^2 \operatorname{Sin}[a + b x]}{b^3} - \frac{(c + d x)^4 \operatorname{Sin}[a + b x]}{b}$$

Result (type 4, 833 leaves):

$$\frac{1}{2b^5} \text{Csc}[a+bx] \left( -3b^4c^4 + 12b^2c^2d^2 - 24d^4 - 12b^4c^3dx + 24b^2cd^3x - 18b^4c^2d^2x^2 + 12b^2d^4x^2 - 12b^4cd^3x^3 - \right. \\ \left. 3b^4d^4x^4 + b^4c^4 \cos[2(a+bx)] - 12b^2c^2d^2 \cos[2(a+bx)] + 24d^4 \cos[2(a+bx)] + 4b^4c^3dx \cos[2(a+bx)] - \right. \\ \left. 24b^2cd^3x \cos[2(a+bx)] + 6b^4c^2d^2x^2 \cos[2(a+bx)] - 12b^2d^4x^2 \cos[2(a+bx)] + 4b^4cd^3x^3 \cos[2(a+bx)] + \right. \\ \left. b^4d^4x^4 \cos[2(a+bx)] - 16b^3c^3d \text{ArcTanh}[e^{i(a+bx)}] \sin[a+bx] + 24b^3c^2d^2x \log[1 - e^{i(a+bx)}] \sin[a+bx] + \right. \\ \left. 24b^3cd^3x^2 \log[1 - e^{i(a+bx)}] \sin[a+bx] + 8b^3d^4x^3 \log[1 - e^{i(a+bx)}] \sin[a+bx] - 24b^3c^2d^2x \log[1 + e^{i(a+bx)}] \sin[a+bx] - \right. \\ \left. 24b^3cd^3x^2 \log[1 + e^{i(a+bx)}] \sin[a+bx] - 8b^3d^4x^3 \log[1 + e^{i(a+bx)}] \sin[a+bx] + 24ib^2d^2(c+dx)^2 \text{PolyLog}[2, -e^{i(a+bx)}] \sin[a+bx] - \right. \\ \left. 24ib^2d^2(c+dx)^2 \text{PolyLog}[2, e^{i(a+bx)}] \sin[a+bx] - 48bcd^3 \text{PolyLog}[3, -e^{i(a+bx)}] \sin[a+bx] - \right. \\ \left. 48bd^4x \text{PolyLog}[3, -e^{i(a+bx)}] \sin[a+bx] + 48bcd^3 \text{PolyLog}[3, e^{i(a+bx)}] \sin[a+bx] + 48bd^4x \text{PolyLog}[3, e^{i(a+bx)}] \sin[a+bx] - \right. \\ \left. 48id^4 \text{PolyLog}[4, -e^{i(a+bx)}] \sin[a+bx] + 48id^4 \text{PolyLog}[4, e^{i(a+bx)}] \sin[a+bx] - 4b^3c^3d \sin[2(a+bx)] + 24bcd^3 \sin[2(a+bx)] - \right. \\ \left. 12b^3c^2d^2x \sin[2(a+bx)] + 24bd^4x \sin[2(a+bx)] - 12b^3cd^3x^2 \sin[2(a+bx)] - 4b^3d^4x^3 \sin[2(a+bx)] \right)$$

**Problem 172: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^3 \cos[a+bx] \cot[a+bx]^2 dx$$

Optimal (type 4, 216 leaves, 13 steps):

$$\frac{6d(c+dx)^2 \text{ArcTanh}[e^{i(a+bx)}]}{b^2} + \frac{6d^3 \cos[a+bx]}{b^4} - \frac{3d(c+dx)^2 \cos[a+bx]}{b^2} - \frac{(c+dx)^3 \text{Csc}[a+bx]}{b} + \frac{6id^2(c+dx) \text{PolyLog}[2, -e^{i(a+bx)}]}{b^3} - \\ \frac{6id^2(c+dx) \text{PolyLog}[2, e^{i(a+bx)}]}{b^3} - \frac{6d^3 \text{PolyLog}[3, -e^{i(a+bx)}]}{b^4} + \frac{6d^3 \text{PolyLog}[3, e^{i(a+bx)}]}{b^4} + \frac{6d^2(c+dx) \sin[a+bx]}{b^3} - \frac{(c+dx)^3 \sin[a+bx]}{b}$$

Result (type 4, 506 leaves):

$$\frac{1}{2b^4} \text{Csc}[a+bx] \left( -3b^3c^3 + 6bcd^2 - 9b^3c^2dx + 6bd^3x - 9b^3cd^2x^2 - 3b^3d^3x^3 + b^3c^3 \cos[2(a+bx)] - 6bcd^2 \cos[2(a+bx)] + 3b^3c^2dx \cos[2(a+bx)] - \right. \\ \left. 6bd^3x \cos[2(a+bx)] + 3b^3cd^2x^2 \cos[2(a+bx)] + b^3d^3x^3 \cos[2(a+bx)] - 12b^2c^2d \text{ArcTanh}[e^{i(a+bx)}] \sin[a+bx] + \right. \\ \left. 12b^2cd^2x \log[1 - e^{i(a+bx)}] \sin[a+bx] + 6b^2d^3x^2 \log[1 - e^{i(a+bx)}] \sin[a+bx] - 12b^2cd^2x \log[1 + e^{i(a+bx)}] \sin[a+bx] - \right. \\ \left. 6b^2d^3x^2 \log[1 + e^{i(a+bx)}] \sin[a+bx] + 12ib^2d^2(c+dx) \text{PolyLog}[2, -e^{i(a+bx)}] \sin[a+bx] - \right. \\ \left. 12ib^2d^2(c+dx) \text{PolyLog}[2, e^{i(a+bx)}] \sin[a+bx] - 12d^3 \text{PolyLog}[3, -e^{i(a+bx)}] \sin[a+bx] + 12d^3 \text{PolyLog}[3, e^{i(a+bx)}] \sin[a+bx] - \right. \\ \left. 3b^2c^2d \sin[2(a+bx)] + 6d^3 \sin[2(a+bx)] - 6b^2cd^2x \sin[2(a+bx)] - 3b^2d^3x^2 \sin[2(a+bx)] \right)$$

**Problem 173: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^2 \cos[a+bx] \cot[a+bx]^2 dx$$

Optimal (type 4, 139 leaves, 10 steps):

$$\begin{aligned}
& - \frac{4 d (c + d x) \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b^2} - \frac{2 d (c + d x) \operatorname{Cos}[a + b x]}{b^2} - \frac{(c + d x)^2 \operatorname{Csc}[a + b x]}{b} + \\
& \frac{2 i d^2 \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{b^3} - \frac{2 i d^2 \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{b^3} + \frac{2 d^2 \operatorname{Sin}[a + b x]}{b^3} - \frac{(c + d x)^2 \operatorname{Sin}[a + b x]}{b}
\end{aligned}$$

Result (type 4, 485 leaves):

$$\begin{aligned}
& - \frac{(c + d x)^2 \operatorname{Csc}[a]}{b} - \frac{1}{b^3} \operatorname{Cos}[b x] \left( 2 b c d \operatorname{Cos}[a] + 2 b d^2 x \operatorname{Cos}[a] + b^2 c^2 \operatorname{Sin}[a] - 2 d^2 \operatorname{Sin}[a] + 2 b^2 c d x \operatorname{Sin}[a] + b^2 d^2 x^2 \operatorname{Sin}[a] \right) + \\
& \frac{4 i c d \operatorname{ArcTan}\left[\frac{i \operatorname{Cos}[a] - i \operatorname{Sin}[a] \operatorname{Tan}\left[\frac{bx}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{b^2 \sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} + \frac{\operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right] \left( -c^2 \operatorname{Sin}\left[\frac{bx}{2}\right] - 2 c d x \operatorname{Sin}\left[\frac{bx}{2}\right] - d^2 x^2 \operatorname{Sin}\left[\frac{bx}{2}\right] \right)}{2 b} + \\
& \frac{\operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right] \left( c^2 \operatorname{Sin}\left[\frac{bx}{2}\right] + 2 c d x \operatorname{Sin}\left[\frac{bx}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{bx}{2}\right] \right)}{2 b} - \frac{1}{b^3} \\
& \left( b^2 c^2 \operatorname{Cos}[a] - 2 d^2 \operatorname{Cos}[a] + 2 b^2 c d x \operatorname{Cos}[a] + b^2 d^2 x^2 \operatorname{Cos}[a] - 2 b c d \operatorname{Sin}[a] - 2 b d^2 x \operatorname{Sin}[a] \right) \operatorname{Sin}[b x] + \\
& \frac{1}{b^3} d^2 \left( - \frac{2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{ArcTanh}\left[\frac{-\operatorname{Cos}[a] + \operatorname{Sin}[a] \operatorname{Tan}\left[\frac{bx}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \\
& \left. \left( (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \left( \operatorname{Log}\left[1 - e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] - \operatorname{Log}\left[1 + e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) \right) + \right. \\
& \left. \left. i \left( \operatorname{PolyLog}\left[2, -e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] - \operatorname{PolyLog}\left[2, e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) \right) \operatorname{Sec}[a] \right)
\end{aligned}$$

**Problem 178: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^4 \operatorname{Cot}[a + b x]^3 dx$$

Optimal (type 4, 302 leaves, 15 steps):

$$\begin{aligned}
& - \frac{2 i d (c+d x)^3}{b^2} - \frac{(c+d x)^4}{2 b} + \frac{i (c+d x)^5}{5 d} - \frac{2 d (c+d x)^3 \operatorname{Cot}[a+b x]}{b^2} - \frac{(c+d x)^4 \operatorname{Cot}[a+b x]^2}{2 b} + \frac{6 d^2 (c+d x)^2 \operatorname{Log}\left[1-e^{2 i (a+b x)}\right]}{b^3} - \\
& \frac{(c+d x)^4 \operatorname{Log}\left[1-e^{2 i (a+b x)}\right]}{b} - \frac{6 i d^3 (c+d x) \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right]}{b^4} + \frac{2 i d (c+d x)^3 \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right]}{b^2} + \\
& \frac{3 d^4 \operatorname{PolyLog}\left[3, e^{2 i (a+b x)}\right]}{b^5} - \frac{3 d^2 (c+d x)^2 \operatorname{PolyLog}\left[3, e^{2 i (a+b x)}\right]}{b^3} - \frac{3 i d^3 (c+d x) \operatorname{PolyLog}\left[4, e^{2 i (a+b x)}\right]}{b^4} + \frac{3 d^4 \operatorname{PolyLog}\left[5, e^{2 i (a+b x)}\right]}{2 b^5}
\end{aligned}$$

Result (type 4, 1101 leaves):

$$\begin{aligned}
& -\frac{1}{5} x (5 c^4 + 10 c^3 d x + 10 c^2 d^2 x^2 + 5 c d^3 x^3 + d^4 x^4) \operatorname{Cot}[a] - \frac{(c + d x)^4 \operatorname{Csc}[a + b x]^2}{2 b} + \frac{1}{2 b^3} c^2 d^2 e^{-i a} \operatorname{Csc}[a] \\
& \left( 2 b^2 x^2 (2 b e^{2 i a} x + 3 i (-1 + e^{2 i a}) \operatorname{Log}[1 - e^{2 i (a+b x)}]) + 6 b (-1 + e^{2 i a}) x \operatorname{PolyLog}[2, e^{2 i (a+b x)}] + 3 i (-1 + e^{2 i a}) \operatorname{PolyLog}[3, e^{2 i (a+b x)}] \right) - \\
& \frac{1}{2 b^5} d^4 e^{-i a} \operatorname{Csc}[a] \left( 2 b^2 x^2 (2 b e^{2 i a} x + 3 i (-1 + e^{2 i a}) \operatorname{Log}[1 - e^{2 i (a+b x)}]) + 6 b (-1 + e^{2 i a}) x \operatorname{PolyLog}[2, e^{2 i (a+b x)}] + \right. \\
& \left. 3 i (-1 + e^{2 i a}) \operatorname{PolyLog}[3, e^{2 i (a+b x)}] \right) + c d^3 e^{i a} \operatorname{Csc}[a] \left( x^4 + (-1 + e^{-2 i a}) x^4 + \frac{1}{2 b^4} e^{-2 i a} (-1 + e^{2 i a}) \right. \\
& \left. (2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log}[1 - e^{2 i (a+b x)}]) + 6 b^2 x^2 \operatorname{PolyLog}[2, e^{2 i (a+b x)}] + 6 i b x \operatorname{PolyLog}[3, e^{2 i (a+b x)}] - 3 \operatorname{PolyLog}[4, e^{2 i (a+b x)}] \right) + \\
& \frac{1}{5} d^4 e^{i a} \operatorname{Csc}[a] \left( x^5 + (-1 + e^{-2 i a}) x^5 + \frac{1}{4 b^5} e^{-2 i a} (-1 + e^{2 i a}) (4 b^5 x^5 + 10 i b^4 x^4 \operatorname{Log}[1 - e^{2 i (a+b x)}] + 20 b^3 x^3 \operatorname{PolyLog}[2, e^{2 i (a+b x)}] + \right. \\
& \left. 30 i b^2 x^2 \operatorname{PolyLog}[3, e^{2 i (a+b x)}] - 30 b x \operatorname{PolyLog}[4, e^{2 i (a+b x)}] - 15 i \operatorname{PolyLog}[5, e^{2 i (a+b x)}] \right) - \\
& \frac{c^4 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{6 c^2 d^2 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{2 \operatorname{Csc}[a] \operatorname{Csc}[a + b x] (c^3 d \operatorname{Sin}[b x] + 3 c^2 d^2 x \operatorname{Sin}[b x] + 3 c d^3 x^2 \operatorname{Sin}[b x] + d^4 x^3 \operatorname{Sin}[b x])}{b^2} + \\
& \left( 2 c^3 d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]) + \right. \right. \\
& \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]) \operatorname{Tan}[a] \right) \right) / \\
& \left( b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) - \left( 6 c d^3 \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]) + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \right. \right. \\
& \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]) \operatorname{Tan}[a] \right) \right) / \left( b^4 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)
\end{aligned}$$

**Problem 179: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 \operatorname{Cot}[a + b x]^3 dx$$

Optimal (type 4, 256 leaves, 13 steps):

$$\begin{aligned}
 & -\frac{3 \, i \, d \, (c + d x)^2}{2 b^2} - \frac{(c + d x)^3}{2 b} + \frac{i \, (c + d x)^4}{4 d} - \frac{3 d \, (c + d x)^2 \operatorname{Cot}[a + b x]}{2 b^2} - \frac{(c + d x)^3 \operatorname{Cot}[a + b x]^2}{2 b} + \\
 & \frac{3 d^2 (c + d x) \operatorname{Log}[1 - e^{2 i (a + b x)}]}{b^3} - \frac{(c + d x)^3 \operatorname{Log}[1 - e^{2 i (a + b x)}]}{b} - \frac{3 \, i \, d^3 \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{2 b^4} + \\
 & \frac{3 \, i \, d \, (c + d x)^2 \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{2 b^2} - \frac{3 d^2 (c + d x) \operatorname{PolyLog}[3, e^{2 i (a + b x)}]}{2 b^3} - \frac{3 \, i \, d^3 \operatorname{PolyLog}[4, e^{2 i (a + b x)}]}{4 b^4}
 \end{aligned}$$

Result (type 4, 788 leaves):

$$\begin{aligned}
 & -\frac{1}{4} x (4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3) \operatorname{Cot}[a] - \frac{(c + d x)^3 \operatorname{Csc}[a + b x]^2}{2 b} + \frac{1}{4 b^3} c d^2 e^{-i a} \operatorname{Csc}[a] \\
 & (2 b^2 x^2 (2 b e^{2 i a} x + 3 i (-1 + e^{2 i a}) \operatorname{Log}[1 - e^{2 i (a + b x)}]) + 6 b (-1 + e^{2 i a}) x \operatorname{PolyLog}[2, e^{2 i (a + b x)}] + 3 i (-1 + e^{2 i a}) \operatorname{PolyLog}[3, e^{2 i (a + b x)}]) + \\
 & \frac{1}{4} d^3 e^{i a} \operatorname{Csc}[a] \left( x^4 + (-1 + e^{-2 i a}) x^4 + \frac{1}{2 b^4} e^{-2 i a} (-1 + e^{2 i a}) \right. \\
 & \left. (2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log}[1 - e^{2 i (a + b x)}] + 6 b^2 x^2 \operatorname{PolyLog}[2, e^{2 i (a + b x)}] + 6 i b x \operatorname{PolyLog}[3, e^{2 i (a + b x)}] - 3 \operatorname{PolyLog}[4, e^{2 i (a + b x)}]) \right) - \\
 & \frac{c^3 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
 & \frac{3 c d^2 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
 & \frac{3 \operatorname{Csc}[a] \operatorname{Csc}[a + b x] (c^2 d \operatorname{Sin}[b x] + 2 c d^2 x \operatorname{Sin}[b x] + d^3 x^2 \operatorname{Sin}[b x])}{2 b^2} + \\
 & \left( 3 c^2 d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
 & \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]) + \right. \right. \\
 & \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]) \operatorname{Tan}[a] \right) \right) / \\
 & \left( 2 b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) - \left( 3 d^3 \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
 & \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]) + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\right. \right. \\
 & \left. \left. \operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]) \operatorname{Tan}[a] \right) \right) / \left( 2 b^4 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)
 \end{aligned}$$

### Problem 180: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Cot}[a + b x]^3 dx$$

Optimal (type 4, 168 leaves, 9 steps):

$$\begin{aligned} & -\frac{c d x}{b} - \frac{d^2 x^2}{2 b} + \frac{i (c + d x)^3}{3 d} - \frac{d (c + d x) \operatorname{Cot}[a + b x]}{b^2} - \frac{(c + d x)^2 \operatorname{Cot}[a + b x]^2}{2 b} \\ & - \frac{(c + d x)^2 \operatorname{Log}[1 - e^{2 i (a + b x)}]}{b} + \frac{d^2 \operatorname{Log}[\operatorname{Sin}[a + b x]]}{b^3} + \frac{i d (c + d x) \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{b^2} - \frac{d^2 \operatorname{PolyLog}[3, e^{2 i (a + b x)}]}{2 b^3} \end{aligned}$$

Result (type 4, 446 leaves):

$$\begin{aligned} & -\frac{1}{3} x (3 c^2 + 3 c d x + d^2 x^2) \operatorname{Cot}[a] - \frac{(c + d x)^2 \operatorname{Csc}[a + b x]^2}{2 b} + \frac{1}{12 b^3} d^2 e^{-i a} \operatorname{Csc}[a] \\ & (2 b^2 x^2 (2 b e^{2 i a} x + 3 i (-1 + e^{2 i a}) \operatorname{Log}[1 - e^{2 i (a + b x)}]) + 6 b (-1 + e^{2 i a}) x \operatorname{PolyLog}[2, e^{2 i (a + b x)}] + 3 i (-1 + e^{2 i a}) \operatorname{PolyLog}[3, e^{2 i (a + b x)}]) - \\ & \frac{c^2 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\ & \frac{d^2 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\ & \frac{\operatorname{Csc}[a] \operatorname{Csc}[a + b x] (c d \operatorname{Sin}[b x] + d^2 x \operatorname{Sin}[b x])}{b^2} + \left( c d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\ & \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \right. \right. \\ & \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \operatorname{Tan}[a]) \right) \right) \Bigg/ \left( b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) \end{aligned}$$

### Problem 181: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Cot}[a + b x]^3 dx$$

Optimal (type 4, 109 leaves, 7 steps):

$$-\frac{d x}{2 b} + \frac{i (c + d x)^2}{2 d} - \frac{d \operatorname{Cot}[a + b x]}{2 b^2} - \frac{(c + d x) \operatorname{Cot}[a + b x]^2}{2 b} - \frac{(c + d x) \operatorname{Log}[1 - e^{2 i (a + b x)}]}{b} + \frac{i d \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{2 b^2}$$

Result (type 4, 234 leaves):

$$\begin{aligned}
& -\frac{1}{2} d x^2 \operatorname{Cot}[a] - \frac{c \operatorname{Csc}[a+b x]^2}{2 b} - \frac{d x \operatorname{Csc}[a+b x]^2}{2 b} - \frac{c \operatorname{Log}[\operatorname{Sin}[a+b x]]}{b} + \\
& \frac{d \operatorname{Csc}[a] \operatorname{Csc}[a+b x] \operatorname{Sin}[b x]}{2 b^2} + \left( d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1+\operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\right. \right. \\
& \left. \left. \operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] \operatorname{Tan}[a] \right) \right) / \left( 2 b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)
\end{aligned}$$

**Problem 190: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^{5/2} \operatorname{Cos}[a+b x]^3 \operatorname{Sin}[a+b x]^2 d x$$

Optimal (type 4, 615 leaves, 26 steps):

$$\begin{aligned}
& \frac{5 d (c+d x)^{3/2} \operatorname{Cos}[a+b x]}{16 b^2} - \frac{5 d (c+d x)^{3/2} \operatorname{Cos}[3 a+3 b x]}{288 b^2} - \frac{d (c+d x)^{3/2} \operatorname{Cos}[5 a+5 b x]}{160 b^2} + \frac{15 d^{5/2} \sqrt{\frac{\pi}{2}} \operatorname{Cos}\left[a - \frac{b c}{d}\right] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right]}{32 b^{7/2}} - \\
& \frac{5 d^{5/2} \sqrt{\frac{\pi}{6}} \operatorname{Cos}\left[3 a - \frac{3 b c}{d}\right] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right]}{576 b^{7/2}} - \frac{3 d^{5/2} \sqrt{\frac{\pi}{10}} \operatorname{Cos}\left[5 a - \frac{5 b c}{d}\right] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right]}{1600 b^{7/2}} - \\
& \frac{3 d^{5/2} \sqrt{\frac{\pi}{10}} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sin}\left[5 a - \frac{5 b c}{d}\right]}{1600 b^{7/2}} - \frac{5 d^{5/2} \sqrt{\frac{\pi}{6}} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sin}\left[3 a - \frac{3 b c}{d}\right]}{576 b^{7/2}} + \\
& \frac{15 d^{5/2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sin}\left[a - \frac{b c}{d}\right]}{32 b^{7/2}} - \frac{15 d^2 \sqrt{c+d x} \operatorname{Sin}[a+b x]}{32 b^3} + \frac{(c+d x)^{5/2} \operatorname{Sin}[a+b x]}{8 b} + \\
& \frac{5 d^2 \sqrt{c+d x} \operatorname{Sin}[3 a+3 b x]}{576 b^3} - \frac{(c+d x)^{5/2} \operatorname{Sin}[3 a+3 b x]}{48 b} + \frac{3 d^2 \sqrt{c+d x} \operatorname{Sin}[5 a+5 b x]}{1600 b^3} - \frac{(c+d x)^{5/2} \operatorname{Sin}[5 a+5 b x]}{80 b}
\end{aligned}$$

Result (type 4, 4926 leaves):

$$\frac{1}{16 b \sqrt{\frac{b}{d}}}$$



$$\begin{aligned}
& c^2 \left( -\sqrt{2\pi} \cos\left[a - \frac{bc}{d}\right] \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] - \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \sin\left[a - \frac{bc}{d}\right] + 2\sqrt{\frac{b}{d}} \sqrt{c+dx} \sin[a+bx] \right) + \\
& \frac{1}{16b^3} cd \left( \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \left(-3d \cos\left[a - \frac{bc}{d}\right] + 2bc \sin\left[a - \frac{bc}{d}\right]\right) + \right. \\
& \left. \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \left(2bc \cos\left[a - \frac{bc}{d}\right] + 3d \sin\left[a - \frac{bc}{d}\right]\right) + 2b\sqrt{c+dx} \left(3 \cos[a+bx] + 2bx \sin[a+bx]\right) \right) + \frac{1}{64b^5} \\
& \left(\frac{b}{d}\right)^{3/2} d^2 \left( -\sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \left((4b^2c^2 - 15d^2) \cos\left[a - \frac{bc}{d}\right] + 12bcd \sin\left[a - \frac{bc}{d}\right]\right) - \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \right. \\
& \left. \left(-12bcd \cos\left[a - \frac{bc}{d}\right] + (4b^2c^2 - 15d^2) \sin\left[a - \frac{bc}{d}\right]\right) + 2\sqrt{\frac{b}{d}} d \sqrt{c+dx} \left(-2b(c-5dx) \cos[a+bx] + d(-15+4b^2x^2) \sin[a+bx]\right) \right) - \\
& \frac{1}{96\sqrt{3}b\sqrt{\frac{b}{d}}} c^2 \left( -\sqrt{2\pi} \cos\left[3a - \frac{3bc}{d}\right] \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] - \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \sin\left[3a - \frac{3bc}{d}\right] + \right. \\
& \left. 2\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin[3(a+bx)] \right) - \frac{1}{96\sqrt{3}b^3} \\
& cd \left( \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \left(-d \cos\left[3a - \frac{3bc}{d}\right] + 2bc \sin\left[3a - \frac{3bc}{d}\right]\right) + \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right. \\
& \left. \left(2bc \cos\left[3a - \frac{3bc}{d}\right] + d \sin\left[3a - \frac{3bc}{d}\right]\right) + 2\sqrt{3}b\sqrt{c+dx} \left(\cos[3(a+bx)] + 2bx \sin[3(a+bx)]\right) \right) - \frac{1}{160\sqrt{5}b\sqrt{\frac{b}{d}}} \\
& c^2 \left( -\sqrt{2\pi} \cos\left[5a - \frac{5bc}{d}\right] \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] - \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \sin\left[5a - \frac{5bc}{d}\right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 2\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}[5(a+bx)] \Big) - \frac{1}{800\sqrt{5}b^3} \\
& cd \left( \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] \left( -3d \operatorname{Cos} \left[ 5a - \frac{5bc}{d} \right] + 10bc \operatorname{Sin} \left[ 5a - \frac{5bc}{d} \right] \right) + \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] \right. \\
& \left. \left( 10bc \operatorname{Cos} \left[ 5a - \frac{5bc}{d} \right] + 3d \operatorname{Sin} \left[ 5a - \frac{5bc}{d} \right] \right) + 2\sqrt{5} b \sqrt{c+dx} \left( 3 \operatorname{Cos} [5(a+bx)] + 10bx \operatorname{Sin} [5(a+bx)] \right) \right) - \\
& \frac{1}{16} d^2 \operatorname{Cos} [3a] \left( \frac{c^2 \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right) \operatorname{Sin} \left[ \frac{3bc}{d} \right]}{3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} d^3} + \right. \\
& \left. \frac{c^2 \operatorname{Cos} \left[ \frac{3bc}{d} \right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right)}{3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} d^3} - \frac{1}{9\sqrt{3} \left( \frac{b}{d} \right)^{5/2} d^3} 2c \operatorname{Cos} \left[ \frac{3bc}{d} \right] \right. \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right) + 3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right) - \\
& \frac{1}{9\sqrt{3} \left( \frac{b}{d} \right)^{5/2} d^3} 2c \operatorname{Sin} \left[ \frac{3bc}{d} \right] \left( -3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] + \right. \right. \\
& \left. \left. \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right) \right) + \frac{1}{27\sqrt{3} \left( \frac{b}{d} \right)^{7/2} d^3} \operatorname{Sin} \left[ \frac{3bc}{d} \right] \left( -9\sqrt{3} \left( \frac{b}{d} \right)^{5/2} (c+dx)^{5/2} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right) + 3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right) \right) + \\
& \left. \frac{1}{27\sqrt{3} \left( \frac{b}{d} \right)^{7/2} d^3} \operatorname{Cos} \left[ \frac{3bc}{d} \right] \left( 9\sqrt{3} \left( \frac{b}{d} \right)^{5/2} (c+dx)^{5/2} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] - \frac{5}{2} \left( -3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right) \right) \right) \right) \right) - \\
\operatorname{Sin}[3a] & \left( \frac{c^2 \operatorname{Cos} \left[ \frac{3bc}{d} \right] \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right)}{3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} d^3} - \right. \\
& \frac{c^2 \operatorname{Sin} \left[ \frac{3bc}{d} \right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right)}{3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} d^3} + \frac{1}{9\sqrt{3} \left( \frac{b}{d} \right)^{5/2} d^3} 2c \operatorname{Sin} \left[ \frac{3bc}{d} \right] \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right) + 3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right) \right) - \\
& \frac{1}{9\sqrt{3} \left( \frac{b}{d} \right)^{5/2} d^3} 2c \operatorname{Cos} \left[ \frac{3bc}{d} \right] \left( -3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] + \right. \right. \\
& \left. \left. \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right) \right) + \frac{1}{27\sqrt{3} \left( \frac{b}{d} \right)^{7/2} d^3} \operatorname{Cos} \left[ \frac{3bc}{d} \right] \left( -9\sqrt{3} \left( \frac{b}{d} \right)^{5/2} (c+dx)^{5/2} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right) + 3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right) \right) - \\
& \frac{1}{27\sqrt{3} \left( \frac{b}{d} \right)^{7/2} d^3} \operatorname{Sin} \left[ \frac{3bc}{d} \right] \left( 9\sqrt{3} \left( \frac{b}{d} \right)^{5/2} (c+dx)^{5/2} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] - \frac{5}{2} \left( -3\sqrt{3} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Cos} \left[ \frac{3b(c+dx)}{d} \right] + \right. \\
& \left. \left. \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{3b(c+dx)}{d} \right] \right) \right) \right) \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{16} d^2 \left( \text{Cos}[5 a] \left( \frac{c^2 \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Cos}\left[\frac{5 b (c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}\right]\right) \text{Sin}\left[\frac{5 b c}{d}\right]}{5 \sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} + \right. \right. \\
& \left. \frac{c^2 \text{Cos}\left[\frac{5 b c}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Sin}\left[\frac{5 b (c+d x)}{d}\right]\right)}{5 \sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} - \frac{1}{25 \sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} \right. \\
& \left. \left. 2 c \text{Cos}\left[\frac{5 b c}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Cos}\left[\frac{5 b (c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}\right]\right) + \right. \right. \right. \\
& \left. \left. \left. 5 \sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \text{Sin}\left[\frac{5 b (c+d x)}{d}\right] \right) - \frac{1}{25 \sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} 2 c \text{Sin}\left[\frac{5 b c}{d}\right] \right. \right. \\
& \left. \left. \left. \left( -5 \sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \text{Cos}\left[\frac{5 b (c+d x)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Sin}\left[\frac{5 b (c+d x)}{d}\right] \right) \right) \right) \right) + \right. \\
& \left. \frac{1}{125 \sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \text{Sin}\left[\frac{5 b c}{d}\right] \left( -25 \sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+d x)^{5/2} \text{Cos}\left[\frac{5 b (c+d x)}{d}\right] + \frac{5}{2} \right. \right. \\
& \left. \left. \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Cos}\left[\frac{5 b (c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}\right]\right) + 5 \sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \text{Sin}\left[\frac{5 b (c+d x)}{d}\right] \right) \right) \right) + \right. \\
& \left. \frac{1}{125 \sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \text{Cos}\left[\frac{5 b c}{d}\right] \left( 25 \sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+d x)^{5/2} \text{Sin}\left[\frac{5 b (c+d x)}{d}\right] - \frac{5}{2} \left( -5 \sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \text{Cos}\left[\frac{5 b (c+d x)}{d}\right] + \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+d x}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+d x} \text{Sin}\left[\frac{5 b (c+d x)}{d}\right] \right) \right) \right) \right) \right) \right) \right) \right) - \left. \right) \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Sin}[5a] \left( \frac{c^2 \text{Cos}\left[\frac{5bc}{d}\right] \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \text{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} \right) \\
& \frac{c^2 \text{Sin}\left[\frac{5bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \text{Sin}\left[\frac{5b(c+dx)}{d}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} + \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} \\
& 2c \text{Sin}\left[\frac{5bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \text{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) \right) + \\
& 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \text{Sin}\left[\frac{5b(c+dx)}{d}\right] \left) - \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \text{Cos}\left[\frac{5bc}{d}\right] \right. \\
& \left. \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \text{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \text{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \text{Cos}\left[\frac{5bc}{d}\right] \left( -25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \text{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{5}{2} \right. \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \text{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \text{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) - \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \text{Sin}\left[\frac{5bc}{d}\right] \left( 25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \text{Sin}\left[\frac{5b(c+dx)}{d}\right] - \frac{5}{2} \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \text{Cos}\left[\frac{5b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \text{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) \right) \right) \right)
\end{aligned}$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^{5/2} \cos[ax + bx]^3 \sin[ax + bx]^2 dx$$

Optimal (type 4, 615 leaves, 26 steps):

$$\begin{aligned} & \frac{5d(c+dx)^{3/2} \cos[ax+bx]}{16b^2} - \frac{5d(c+dx)^{3/2} \cos[3a+3bx]}{288b^2} - \frac{d(c+dx)^{3/2} \cos[5a+5bx]}{160b^2} + \frac{15d^{5/2} \sqrt{\frac{\pi}{2}} \cos\left[a - \frac{bc}{d}\right] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right]}{32b^{7/2}} - \\ & \frac{5d^{5/2} \sqrt{\frac{\pi}{6}} \cos\left[3a - \frac{3bc}{d}\right] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right]}{576b^{7/2}} - \frac{3d^{5/2} \sqrt{\frac{\pi}{10}} \cos\left[5a - \frac{5bc}{d}\right] \operatorname{FresnelS}\left[\frac{\sqrt{b} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right]}{1600b^{7/2}} - \\ & \frac{3d^{5/2} \sqrt{\frac{\pi}{10}} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right] \sin\left[5a - \frac{5bc}{d}\right]}{1600b^{7/2}} - \frac{5d^{5/2} \sqrt{\frac{\pi}{6}} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right] \sin\left[3a - \frac{3bc}{d}\right]}{576b^{7/2}} + \\ & \frac{15d^{5/2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{\sqrt{b} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}}{\sqrt{d}}\right] \sin\left[a - \frac{bc}{d}\right]}{32b^{7/2}} - \frac{15d^2 \sqrt{c+dx} \sin[ax+bx]}{32b^3} + \frac{(c+dx)^{5/2} \sin[ax+bx]}{8b} + \\ & \frac{5d^2 \sqrt{c+dx} \sin[3a+3bx]}{576b^3} - \frac{(c+dx)^{5/2} \sin[3a+3bx]}{48b} + \frac{3d^2 \sqrt{c+dx} \sin[5a+5bx]}{1600b^3} - \frac{(c+dx)^{5/2} \sin[5a+5bx]}{80b} \end{aligned}$$

Result (type 4, 4926 leaves):

$$\begin{aligned} & \frac{1}{16b \sqrt{\frac{b}{d}}} \\ & c^2 \left( -\sqrt{2\pi} \cos\left[a - \frac{bc}{d}\right] \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] - \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \sin\left[a - \frac{bc}{d}\right] + 2 \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin[ax+bx] \right) + \\ & \frac{1}{16b^3} c d \left( \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \left( -3d \cos\left[a - \frac{bc}{d}\right] + 2bc \sin\left[a - \frac{bc}{d}\right] \right) + \right. \\ & \left. \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx}\right] \left( 2bc \cos\left[a - \frac{bc}{d}\right] + 3d \sin\left[a - \frac{bc}{d}\right] \right) + 2b \sqrt{c+dx} (3 \cos[ax+bx] + 2bx \sin[ax+bx]) \right) + \frac{1}{64b^5} \end{aligned}$$

$$\begin{aligned}
& \left( \frac{b}{d} \right)^{3/2} d^2 \left( -\sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx} \right] \left( (4b^2c^2 - 15d^2) \cos \left[ a - \frac{bc}{d} \right] + 12bcd \sin \left[ a - \frac{bc}{d} \right] \right) - \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{2}{\pi}} \sqrt{c+dx} \right] \right. \\
& \quad \left. \left( -12bcd \cos \left[ a - \frac{bc}{d} \right] + (4b^2c^2 - 15d^2) \sin \left[ a - \frac{bc}{d} \right] \right) + 2 \sqrt{\frac{b}{d}} d \sqrt{c+dx} \left( -2b(c-5dx) \cos[a+bx] + d(-15+4b^2x^2) \sin[a+bx] \right) \right) - \\
& \quad \frac{1}{96\sqrt{3}b\sqrt{\frac{b}{d}}} c^2 \left( -\sqrt{2\pi} \cos \left[ 3a - \frac{3bc}{d} \right] \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] - \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \sin \left[ 3a - \frac{3bc}{d} \right] + \right. \\
& \quad \left. 2\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin[3(a+bx)] \right) - \frac{1}{96\sqrt{3}b^3} \\
& \quad cd \left( \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \left( -d \cos \left[ 3a - \frac{3bc}{d} \right] + 2bc \sin \left[ 3a - \frac{3bc}{d} \right] \right) + \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx} \right] \right. \\
& \quad \left. \left( 2bc \cos \left[ 3a - \frac{3bc}{d} \right] + d \sin \left[ 3a - \frac{3bc}{d} \right] \right) + 2\sqrt{3}b\sqrt{c+dx} \left( \cos[3(a+bx)] + 2bx \sin[3(a+bx)] \right) \right) - \frac{1}{160\sqrt{5}b\sqrt{\frac{b}{d}}} \\
& \quad c^2 \left( -\sqrt{2\pi} \cos \left[ 5a - \frac{5bc}{d} \right] \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] - \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] \sin \left[ 5a - \frac{5bc}{d} \right] + \right. \\
& \quad \left. 2\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin[5(a+bx)] \right) - \frac{1}{800\sqrt{5}b^3} \\
& \quad cd \left( \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelC} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] \left( -3d \cos \left[ 5a - \frac{5bc}{d} \right] + 10bc \sin \left[ 5a - \frac{5bc}{d} \right] \right) + \sqrt{\frac{b}{d}} \sqrt{2\pi} \operatorname{FresnelS} \left[ \sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx} \right] \right. \\
& \quad \left. \left( 10bc \cos \left[ 5a - \frac{5bc}{d} \right] + 3d \sin \left[ 5a - \frac{5bc}{d} \right] \right) + 2\sqrt{5}b\sqrt{c+dx} \left( 3 \cos[5(a+bx)] + 10bx \sin[5(a+bx)] \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{16} d^2 \left( \cos[3a] \left( \frac{c^2 \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right]\right) \sin\left[\frac{3bc}{d}\right]}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} + \right. \right. \\
& \frac{c^2 \cos\left[\frac{3bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{3b(c+dx)}{d}\right] \right)}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} - \frac{1}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \cos\left[\frac{3bc}{d}\right] \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{3b(c+dx)}{d}\right] \right) - \right. \\
& \frac{1}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \sin\left[\frac{3bc}{d}\right] \left( -3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{3b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \right. \right. \\
& \left. \left. \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{3b(c+dx)}{d}\right] \right) \right) + \frac{1}{27\sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \sin\left[\frac{3bc}{d}\right] \left( -9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \cos\left[\frac{3b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{3b(c+dx)}{d}\right] \right) \right) \right) + \\
& \frac{1}{27\sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \cos\left[\frac{3bc}{d}\right] \left( 9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \sin\left[\frac{3b(c+dx)}{d}\right] - \frac{5}{2} \left( -3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{3b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{3b(c+dx)}{d}\right] \right) \right) \right) \right) \right) - \\
& \left. \sin[3a] \left( \frac{c^2 \cos\left[\frac{3bc}{d}\right] \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right)}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} - \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{c^2 \operatorname{Sin}\left[\frac{3bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right)}{3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} d^3} + \frac{1}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Sin}\left[\frac{3bc}{d}\right] \\
& \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) - \\
& \frac{1}{9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Cos}\left[\frac{3bc}{d}\right] \left( -3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \right. \right. \\
& \left. \left. \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) + \frac{1}{27\sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{3bc}{d}\right] \left( -9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] \right) + 3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) - \\
& \frac{1}{27\sqrt{3} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{3bc}{d}\right] \left( 9\sqrt{3} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] - \frac{5}{2} \left( -3\sqrt{3} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{3b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{6}{\pi}} \sqrt{c+dx}\right] + \sqrt{3} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{3b(c+dx)}{d}\right] \right) \right) \right) \right) - \\
& \frac{1}{16} d^2 \operatorname{Cos}[5a] \left( \frac{c^2 \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) \operatorname{Sin}\left[\frac{5bc}{d}\right]}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} + \right. \\
& \left. \frac{c^2 \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} - \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} \right)
\end{aligned}$$

$$\begin{aligned}
& 2c \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + \right. \\
& \left. 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) - \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Sin}\left[\frac{5bc}{d}\right] \\
& \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{5}{2} \right. \\
& \left. \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( 25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] - \frac{5}{2} \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \right. \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) \right) - \\
& \operatorname{Sin}[5a] \left( \frac{c^2 \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} - \right. \\
& \left. \frac{c^2 \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right)}{5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} d^3} + \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} \right)
\end{aligned}$$

$$\begin{aligned}
& 2c \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + \right. \\
& \quad \left. 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) - \frac{1}{25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} d^3} 2c \operatorname{Cos}\left[\frac{5bc}{d}\right] \\
& \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{5bc}{d}\right] \left( -25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \frac{5}{2} \right. \\
& \quad \left. \left( -\frac{3}{2} \left( -\sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] \right) + 5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) - \\
& \frac{1}{125\sqrt{5} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{5bc}{d}\right] \left( 25\sqrt{5} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] - \frac{5}{2} \left( -5\sqrt{5} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{5b(c+dx)}{d}\right] + \right. \right. \\
& \quad \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{b}{d}} \sqrt{\frac{10}{\pi}} \sqrt{c+dx}\right] + \sqrt{5} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{5b(c+dx)}{d}\right] \right) \right) \right) \right) \right) \right)
\end{aligned}$$

**Problem 196: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^{5/2} \operatorname{Cos}[a+bx]^3 \operatorname{Sin}[a+bx]^3 dx$$

Optimal (type 4, 407 leaves, 18 steps):

$$\begin{aligned}
& \frac{45 d^2 \sqrt{c+d x} \operatorname{Cos}[2 a+2 b x]}{1024 b^3} - \frac{3(c+d x)^{5/2} \operatorname{Cos}[2 a+2 b x]}{64 b} - \frac{5 d^2 \sqrt{c+d x} \operatorname{Cos}[6 a+6 b x]}{9216 b^3} + \\
& \frac{(c+d x)^{5/2} \operatorname{Cos}[6 a+6 b x]}{192 b} + \frac{5 d^{5/2} \sqrt{\frac{\pi}{3}} \operatorname{Cos}\left[6 a - \frac{6 b c}{d}\right] \operatorname{FresnelC}\left[\frac{2 \sqrt{b} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right]}{18432 b^{7/2}} - \\
& \frac{45 d^{5/2} \sqrt{\pi} \operatorname{Cos}\left[2 a - \frac{2 b c}{d}\right] \operatorname{FresnelC}\left[\frac{2 \sqrt{b} \sqrt{c+d x}}{\sqrt{d} \sqrt{\pi}}\right]}{2048 b^{7/2}} - \frac{5 d^{5/2} \sqrt{\frac{\pi}{3}} \operatorname{FresnelS}\left[\frac{2 \sqrt{b} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sin}\left[6 a - \frac{6 b c}{d}\right]}{18432 b^{7/2}} + \\
& \frac{45 d^{5/2} \sqrt{\pi} \operatorname{FresnelS}\left[\frac{2 \sqrt{b} \sqrt{c+d x}}{\sqrt{d} \sqrt{\pi}}\right] \operatorname{Sin}\left[2 a - \frac{2 b c}{d}\right]}{2048 b^{7/2}} + \frac{15 d(c+d x)^{3/2} \operatorname{Sin}[2 a+2 b x]}{256 b^2} - \frac{5 d(c+d x)^{3/2} \operatorname{Sin}[6 a+6 b x]}{2304 b^2}
\end{aligned}$$

Result (type 4, 6763 leaves):

$$\begin{aligned}
& \frac{1}{8} \left( \frac{3}{4} c^2 \operatorname{Sin}[2 a] \left( \frac{\operatorname{Cos}\left[\frac{b c}{d}\right] \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{2 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2 \sqrt{\frac{b}{d}} \sqrt{c+d x}}{\sqrt{\pi}}\right] \right) \operatorname{Sin}\left[\frac{b c}{d}\right]}{\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d} \right) + \right. \\
& \left. \frac{\operatorname{Cos}\left[\frac{2 b c}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2 \sqrt{\frac{b}{d}} \sqrt{c+d x}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Sin}\left[\frac{2 b(c+d x)}{d}\right] \right)}{2 \sqrt{2} \left(\frac{b}{d}\right)^{3/2} d} \right) + \frac{3}{4} c^2 (\operatorname{Cos}[a] - \operatorname{Sin}[a]) (\operatorname{Cos}[a] + \operatorname{Sin}[a]) \\
& \left( \frac{1}{2 \sqrt{2} \left(\frac{b}{d}\right)^{3/2} d} \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{2 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2 \sqrt{\frac{b}{d}} \sqrt{c+d x}}{\sqrt{\pi}}\right] \right) (\operatorname{Cos}\left[\frac{b c}{d}\right] - \operatorname{Sin}\left[\frac{b c}{d}\right]) (\operatorname{Cos}\left[\frac{b c}{d}\right] + \operatorname{Sin}\left[\frac{b c}{d}\right]) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sin\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d} \right\} + \\
& \frac{3}{2} cd \sin[2a] \left( -\frac{c \cos\left[\frac{bc}{d}\right] \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) \sin\left[\frac{bc}{d}\right]}{\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^2} \right) - \\
& \frac{c \cos\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^2} + \frac{1}{4\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^2} \\
& \cos\left[\frac{2bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) + 2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{2b(c+dx)}{d}\right] \right) + \\
& \frac{1}{2\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^2} \cos\left[\frac{bc}{d}\right] \sin\left[\frac{bc}{d}\right] \left( -2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \right. \\
& \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) + \frac{3}{2} cd (\cos[a] - \sin[a]) (\cos[a] + \sin[a])
\end{aligned}$$

$$\begin{aligned}
 & \left( -\frac{1}{2\sqrt{2}} \left(\frac{b}{d}\right)^{3/2} d^2 \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \right. \\
 & \left. \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) + \frac{c \sin\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^2} - \frac{1}{4\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^2} \right. \\
 & \left. \sin\left[\frac{2bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) + 2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right. \\
 & \left. \frac{1}{4\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^2} \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) \right. \\
 & \left. \left( -2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) \right) \right) + \\
 & \frac{3}{4} d^2 \sin[2a] \left( \frac{c^2 \cos\left[\frac{bc}{d}\right] \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) \sin\left[\frac{bc}{d}\right]}{\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^3} + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{c^2 \operatorname{Cos}\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}d^3} - \frac{1}{2\sqrt{2}\left(\frac{b}{d}\right)^{5/2}d^3} \\
& c \operatorname{Cos}\left[\frac{2bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Cos}\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) + \right. \\
& \left. 2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] \right) - \frac{1}{\sqrt{2}\left(\frac{b}{d}\right)^{5/2}d^3} c \operatorname{Cos}\left[\frac{bc}{d}\right] \operatorname{Sin}\left[\frac{bc}{d}\right] \\
& \left( -2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Cos}\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{4\sqrt{2}\left(\frac{b}{d}\right)^{7/2}d^3} \operatorname{Cos}\left[\frac{bc}{d}\right] \operatorname{Sin}\left[\frac{bc}{d}\right] \left( -4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \operatorname{Cos}\left[\frac{2b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Cos}\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) + 2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}d^3} \operatorname{Cos}\left[\frac{2bc}{d}\right] \left( 4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \frac{5}{2} \left( -2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) \right) \right) \right) + \\
& \frac{3}{4} d^2 (\cos[a] - \sin[a]) (\cos[a] + \sin[a]) \left( \frac{1}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^3} c^2 \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] \right) \right. \\
& \left. \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) - \frac{c^2 \sin\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^3} \right) + \\
& \frac{1}{2\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^3} c \sin\left[\frac{2bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] \right) + \right. \\
& \left. 2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{2b(c+dx)}{d}\right] \right) - \frac{1}{2\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^3} c \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) \\
& \left. \left( -2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) \right) +
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}d^3} \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) \left( -4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + 2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) - \\
& \frac{1}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}d^3} \sin\left[\frac{2bc}{d}\right] \left( 4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \sin\left[\frac{2b(c+dx)}{d}\right] - \right. \\
& \left. \frac{5}{2} \left( -2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) \right) \right) - \\
& \frac{1}{4} c^2 \sin[6a] \left( \frac{\left( -\sqrt{6}\sqrt{\frac{b}{d}}\sqrt{c+dx} \cos\left[\frac{6b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right] \right) \sin\left[\frac{6bc}{d}\right]}{6\sqrt{6}\left(\frac{b}{d}\right)^{3/2}d} + \right. \\
& \left. \frac{\cos\left[\frac{6bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right] + \sqrt{6}\sqrt{\frac{b}{d}}\sqrt{c+dx} \sin\left[\frac{6b(c+dx)}{d}\right] \right)}{6\sqrt{6}\left(\frac{b}{d}\right)^{3/2}d} \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} c^2 \cos[6a] \left( \frac{\cos\left[\frac{6bc}{d}\right] \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{6b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right]\right)}{6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} d} - \right. \\
& \left. \frac{\sin\left[\frac{6bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{6b(c+dx)}{d}\right]\right)}{6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} d} \right) - \\
& \frac{1}{2} cd \cos[6a] \left( - \frac{c \cos\left[\frac{6bc}{d}\right] \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{6b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right]\right)}{6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^2} + \right. \\
& \left. \frac{c \sin\left[\frac{6bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{6b(c+dx)}{d}\right]\right)}{6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^2} - \frac{1}{36\sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^2} \right. \\
& \left. \sin\left[\frac{6bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{6b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right]\right) + \right. \\
& \left. 6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{6b(c+dx)}{d}\right] \right) + \frac{1}{36\sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^2} \cos\left[\frac{6bc}{d}\right] \right. \\
& \left. \left( -6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{6b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{6b(c+dx)}{d}\right]\right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} c d \operatorname{Sin}[6 a] \left( \frac{c \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right]\right) \operatorname{Sin}\left[\frac{6 b c}{d}\right]}{6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^2} - \right. \\
& \frac{c \operatorname{Cos}\left[\frac{6 b c}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Sin}\left[\frac{6 b(c+d x)}{d}\right]\right)}{6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^2} + \frac{1}{36 \sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^2} \\
& \operatorname{Cos}\left[\frac{6 b c}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right]\right) + \right. \\
& \left. 6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \operatorname{Sin}\left[\frac{6 b(c+d x)}{d}\right]\right) + \frac{1}{36 \sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^2} \operatorname{Sin}\left[\frac{6 b c}{d}\right] \\
& \left. \left( -6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Sin}\left[\frac{6 b(c+d x)}{d}\right]\right) \right) \right) \right) - \\
& \frac{1}{4} d^2 \operatorname{Sin}[6 a] \left( \frac{c^2 \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right]\right) \operatorname{Sin}\left[\frac{6 b c}{d}\right]}{6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^3} + \right. \\
& \frac{c^2 \operatorname{Cos}\left[\frac{6 b c}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Sin}\left[\frac{6 b(c+d x)}{d}\right]\right)}{6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^3} - \frac{1}{18 \sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^3} \\
& \left. c \operatorname{Cos}\left[\frac{6 b c}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right]\right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \Bigg) - \frac{1}{18\sqrt{6} \left( \frac{b}{d} \right)^{5/2} d^3} c \operatorname{Sin} \left[ \frac{6bc}{d} \right] \\
& \left( -6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \right) \right) + \\
& \frac{1}{216\sqrt{6} \left( \frac{b}{d} \right)^{7/2} d^3} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \left( -36\sqrt{6} \left( \frac{b}{d} \right)^{5/2} (c+dx)^{5/2} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] \right) + 6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \right) \right) + \\
& \frac{1}{216\sqrt{6} \left( \frac{b}{d} \right)^{7/2} d^3} \operatorname{Cos} \left[ \frac{6bc}{d} \right] \left( 36\sqrt{6} \left( \frac{b}{d} \right)^{5/2} (c+dx)^{5/2} \operatorname{Sin} \left[ \frac{6bc}{d} \right] - \frac{5}{2} \left( -6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \right) \right) \right) \right) \Bigg) - \\
& \frac{1}{4} d^2 \operatorname{Cos} [6a] \left( \frac{c^2 \operatorname{Cos} \left[ \frac{6bc}{d} \right] \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] \right)}{6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} d^3} - \right. \\
& \left. \frac{c^2 \operatorname{Sin} \left[ \frac{6bc}{d} \right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \right)}{6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} d^3} + \frac{1}{18\sqrt{6} \left( \frac{b}{d} \right)^{5/2} d^3} \right. \\
& \left. c \operatorname{Sin} \left[ \frac{6bc}{d} \right] \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{6bc}{d}\right] - \frac{1}{18\sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^3} c \operatorname{Cos}\left[\frac{6bc}{d}\right] \\
& \left( -6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{6bc}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{6bc}{d}\right] \right) \right) + \\
& \frac{1}{216\sqrt{6} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{6bc}{d}\right] \left( -36\sqrt{6} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{6bc}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{6bc}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] \right) + 6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{6bc}{d}\right] \right) \right) - \\
& \frac{1}{216\sqrt{6} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{6bc}{d}\right] \left( 36\sqrt{6} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{6bc}{d}\right] - \frac{5}{2} \right. \\
& \left. \left( -6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{6bc}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{6bc}{d}\right] \right) \right) \right) \right)
\end{aligned}$$

**Problem 201: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^{5/2} \operatorname{Cos}[a+bx]^3 \operatorname{Sin}[a+bx]^3 dx$$

Optimal (type 4, 407 leaves, 18 steps):

$$\begin{aligned}
& \frac{45 d^2 \sqrt{c+d x} \operatorname{Cos}[2 a+2 b x]}{1024 b^3} - \frac{3(c+d x)^{5/2} \operatorname{Cos}[2 a+2 b x]}{64 b} - \frac{5 d^2 \sqrt{c+d x} \operatorname{Cos}[6 a+6 b x]}{9216 b^3} + \\
& \frac{(c+d x)^{5/2} \operatorname{Cos}[6 a+6 b x]}{192 b} + \frac{5 d^{5/2} \sqrt{\frac{\pi}{3}} \operatorname{Cos}\left[6 a - \frac{6 b c}{d}\right] \operatorname{FresnelC}\left[\frac{2 \sqrt{b} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right]}{18432 b^{7/2}} - \\
& \frac{45 d^{5/2} \sqrt{\pi} \operatorname{Cos}\left[2 a - \frac{2 b c}{d}\right] \operatorname{FresnelC}\left[\frac{2 \sqrt{b} \sqrt{c+d x}}{\sqrt{d} \sqrt{\pi}}\right]}{2048 b^{7/2}} - \frac{5 d^{5/2} \sqrt{\frac{\pi}{3}} \operatorname{FresnelS}\left[\frac{2 \sqrt{b} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}}{\sqrt{d}}\right] \operatorname{Sin}\left[6 a - \frac{6 b c}{d}\right]}{18432 b^{7/2}} + \\
& \frac{45 d^{5/2} \sqrt{\pi} \operatorname{FresnelS}\left[\frac{2 \sqrt{b} \sqrt{c+d x}}{\sqrt{d} \sqrt{\pi}}\right] \operatorname{Sin}\left[2 a - \frac{2 b c}{d}\right]}{2048 b^{7/2}} + \frac{15 d(c+d x)^{3/2} \operatorname{Sin}[2 a+2 b x]}{256 b^2} - \frac{5 d(c+d x)^{3/2} \operatorname{Sin}[6 a+6 b x]}{2304 b^2}
\end{aligned}$$

Result (type 4, 6763 leaves):

$$\begin{aligned}
& \frac{1}{8} \left( \frac{3}{4} c^2 \operatorname{Sin}[2 a] \left( \frac{\operatorname{Cos}\left[\frac{b c}{d}\right] \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{2 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2 \sqrt{\frac{b}{d}} \sqrt{c+d x}}{\sqrt{\pi}}\right] \right) \operatorname{Sin}\left[\frac{b c}{d}\right]}{\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d} \right) + \right. \\
& \left. \frac{\operatorname{Cos}\left[\frac{2 b c}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2 \sqrt{\frac{b}{d}} \sqrt{c+d x}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Sin}\left[\frac{2 b(c+d x)}{d}\right] \right)}{2 \sqrt{2} \left(\frac{b}{d}\right)^{3/2} d} \right) + \frac{3}{4} c^2 (\operatorname{Cos}[a] - \operatorname{Sin}[a]) (\operatorname{Cos}[a] + \operatorname{Sin}[a]) \\
& \left( \frac{1}{2 \sqrt{2} \left(\frac{b}{d}\right)^{3/2} d} \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{2 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2 \sqrt{\frac{b}{d}} \sqrt{c+d x}}{\sqrt{\pi}}\right] \right) \left( \operatorname{Cos}\left[\frac{b c}{d}\right] - \operatorname{Sin}\left[\frac{b c}{d}\right] \right) \left( \operatorname{Cos}\left[\frac{b c}{d}\right] + \operatorname{Sin}\left[\frac{b c}{d}\right] \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sin\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d} \right\} + \\
& \frac{3}{2} cd \sin[2a] \left( -\frac{c \cos\left[\frac{bc}{d}\right] \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) \sin\left[\frac{bc}{d}\right]}{\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^2} \right) - \\
& \frac{c \cos\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^2} + \frac{1}{4\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^2} \\
& \cos\left[\frac{2bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) + 2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{2b(c+dx)}{d}\right] \right) + \\
& \frac{1}{2\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^2} \cos\left[\frac{bc}{d}\right] \sin\left[\frac{bc}{d}\right] \left( -2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \right. \\
& \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) + \frac{3}{2} cd (\cos[a] - \sin[a]) (\cos[a] + \sin[a])
\end{aligned}$$

$$\begin{aligned}
 & \left( -\frac{1}{2\sqrt{2}} \left(\frac{b}{d}\right)^{3/2} d^2 \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \right. \\
 & \left. \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) + \frac{c \sin\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^2} - \frac{1}{4\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^2} \right. \\
 & \left. \sin\left[\frac{2bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) + 2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right. \\
 & \left. \frac{1}{4\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^2} \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) \right. \\
 & \left. \left( -2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) \right) \right) + \\
 & \frac{3}{4} d^2 \sin[2a] \left( \frac{c^2 \cos\left[\frac{bc}{d}\right] \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) \sin\left[\frac{bc}{d}\right]}{\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^3} + \right.
 \end{aligned}$$



$$\begin{aligned}
& \frac{c^2 \operatorname{Cos}\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}d^3} - \frac{1}{2\sqrt{2}\left(\frac{b}{d}\right)^{5/2}d^3} \\
& c \operatorname{Cos}\left[\frac{2bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Cos}\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) + \right. \\
& \left. 2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] \right) - \frac{1}{\sqrt{2}\left(\frac{b}{d}\right)^{5/2}d^3} c \operatorname{Cos}\left[\frac{bc}{d}\right] \operatorname{Sin}\left[\frac{bc}{d}\right] \\
& \left( -2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Cos}\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{4\sqrt{2}\left(\frac{b}{d}\right)^{7/2}d^3} \operatorname{Cos}\left[\frac{bc}{d}\right] \operatorname{Sin}\left[\frac{bc}{d}\right] \left( -4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \operatorname{Cos}\left[\frac{2b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \operatorname{Cos}\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] \right) + 2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] \right) \right) + \\
& \frac{1}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}d^3} \operatorname{Cos}\left[\frac{2bc}{d}\right] \left( 4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \operatorname{Sin}\left[\frac{2b(c+dx)}{d}\right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \frac{5}{2} \left( -2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) \right) \right) \right) + \\
& \frac{3}{4} d^2 (\cos[a] - \sin[a]) (\cos[a] + \sin[a]) \left( \frac{1}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^3} c^2 \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] \right) \right. \\
& \left. \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) - \frac{c^2 \sin\left[\frac{2bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right)}{2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} d^3} \right) + \\
& \frac{1}{2\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^3} c \sin\left[\frac{2bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] \right) + \right. \\
& \left. 2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{2b(c+dx)}{d}\right] \right) - \frac{1}{2\sqrt{2} \left(\frac{b}{d}\right)^{5/2} d^3} c \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) \\
& \left. \left( -2\sqrt{2} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}}\right] + \sqrt{2} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}d^3} \left( \cos\left[\frac{bc}{d}\right] - \sin\left[\frac{bc}{d}\right] \right) \left( \cos\left[\frac{bc}{d}\right] + \sin\left[\frac{bc}{d}\right] \right) \left( -4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \cos\left[\frac{2b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + 2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) - \right. \\
& \left. \frac{1}{8\sqrt{2}\left(\frac{b}{d}\right)^{7/2}d^3} \sin\left[\frac{2bc}{d}\right] \left( 4\sqrt{2}\left(\frac{b}{d}\right)^{5/2}(c+dx)^{5/2} \sin\left[\frac{2b(c+dx)}{d}\right] - \right. \right. \\
& \left. \left. \frac{5}{2} \left( -2\sqrt{2}\left(\frac{b}{d}\right)^{3/2}(c+dx)^{3/2} \cos\left[\frac{2b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right] + \sqrt{2}\sqrt{\frac{b}{d}}\sqrt{c+dx} \sin\left[\frac{2b(c+dx)}{d}\right] \right) \right) \right) \right) - \right. \\
& \left. \frac{1}{4} c^2 \sin[6a] \left( \frac{\left( -\sqrt{6}\sqrt{\frac{b}{d}}\sqrt{c+dx} \cos\left[\frac{6b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right] \right) \sin\left[\frac{6bc}{d}\right]}{6\sqrt{6}\left(\frac{b}{d}\right)^{3/2}d} + \right. \right. \\
& \left. \left. \frac{\cos\left[\frac{6bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right] + \sqrt{6}\sqrt{\frac{b}{d}}\sqrt{c+dx} \sin\left[\frac{6b(c+dx)}{d}\right] \right)}{6\sqrt{6}\left(\frac{b}{d}\right)^{3/2}d} \right) \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} c^2 \cos[6a] \left( \frac{\cos\left[\frac{6bc}{d}\right] \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{6b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right]\right)}{6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} d} - \right. \\
& \left. \frac{\sin\left[\frac{6bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{6b(c+dx)}{d}\right]\right)}{6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} d} \right) - \\
& \frac{1}{2} cd \cos[6a] \left( - \frac{c \cos\left[\frac{6bc}{d}\right] \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{6b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right]\right)}{6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^2} + \right. \\
& \left. \frac{c \sin\left[\frac{6bc}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{6b(c+dx)}{d}\right]\right)}{6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^2} - \frac{1}{36\sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^2} \right) \\
& \sin\left[\frac{6bc}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \cos\left[\frac{6b(c+dx)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right]\right) + \right. \\
& \left. 6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \sin\left[\frac{6b(c+dx)}{d}\right] \right) + \frac{1}{36\sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^2} \cos\left[\frac{6bc}{d}\right] \\
& \left( -6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \cos\left[\frac{6b(c+dx)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \sin\left[\frac{6b(c+dx)}{d}\right] \right) \right) \left. \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} c d \operatorname{Sin}[6 a] \left( \frac{c \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right]\right) \operatorname{Sin}\left[\frac{6 b c}{d}\right]}{6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^2} - \right. \\
& \frac{c \operatorname{Cos}\left[\frac{6 b c}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Sin}\left[\frac{6 b(c+d x)}{d}\right]\right)}{6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^2} + \frac{1}{36 \sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^2} \\
& \operatorname{Cos}\left[\frac{6 b c}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right]\right) + \right. \\
& \left. 6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \operatorname{Sin}\left[\frac{6 b(c+d x)}{d}\right]\right) + \frac{1}{36 \sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^2} \operatorname{Sin}\left[\frac{6 b c}{d}\right] \\
& \left. \left( -6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+d x)^{3/2} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Sin}\left[\frac{6 b(c+d x)}{d}\right]\right) \right) \right) \right) \\
& \frac{1}{4} d^2 \operatorname{Sin}[6 a] \left( \frac{c^2 \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right]\right) \operatorname{Sin}\left[\frac{6 b c}{d}\right]}{6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^3} + \right. \\
& \frac{c^2 \operatorname{Cos}\left[\frac{6 b c}{d}\right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Sin}\left[\frac{6 b(c+d x)}{d}\right]\right)}{6 \sqrt{6} \left(\frac{b}{d}\right)^{3/2} d^3} - \frac{1}{18 \sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^3} \\
& \left. c \operatorname{Cos}\left[\frac{6 b c}{d}\right] \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+d x} \operatorname{Cos}\left[\frac{6 b(c+d x)}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2 \sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+d x}\right]\right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \Bigg) - \frac{1}{18\sqrt{6} \left( \frac{b}{d} \right)^{5/2} d^3} c \operatorname{Sin} \left[ \frac{6bc}{d} \right] \\
& \left( -6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \right) \right) \Bigg) + \\
& \frac{1}{216\sqrt{6} \left( \frac{b}{d} \right)^{7/2} d^3} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \left( -36\sqrt{6} \left( \frac{b}{d} \right)^{5/2} (c+dx)^{5/2} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] \right) + 6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \right) \right) \Bigg) + \\
& \frac{1}{216\sqrt{6} \left( \frac{b}{d} \right)^{7/2} d^3} \operatorname{Cos} \left[ \frac{6bc}{d} \right] \left( 36\sqrt{6} \left( \frac{b}{d} \right)^{5/2} (c+dx)^{5/2} \operatorname{Sin} \left[ \frac{6bc}{d} \right] - \frac{5}{2} \left( -6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} (c+dx)^{3/2} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \right. \\
& \left. \left. \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \right) \right) \right) \right) \Bigg) - \\
& \frac{1}{4} d^2 \operatorname{Cos} [6a] \left( \frac{c^2 \operatorname{Cos} \left[ \frac{6bc}{d} \right] \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] \right)}{6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} d^3} - \right. \\
& \left. \frac{c^2 \operatorname{Sin} \left[ \frac{6bc}{d} \right] \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin} \left[ \frac{6bc}{d} \right] \right)}{6\sqrt{6} \left( \frac{b}{d} \right)^{3/2} d^3} + \frac{1}{18\sqrt{6} \left( \frac{b}{d} \right)^{5/2} d^3} \right. \\
& \left. c \operatorname{Sin} \left[ \frac{6bc}{d} \right] \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos} \left[ \frac{6bc}{d} \right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[ 2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx} \right] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{6bc}{d}\right] - \frac{1}{18\sqrt{6} \left(\frac{b}{d}\right)^{5/2} d^3} c \operatorname{Cos}\left[\frac{6bc}{d}\right] \\
& \left( -6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{6bc}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{6bc}{d}\right] \right) \right) + \\
& \frac{1}{216\sqrt{6} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Cos}\left[\frac{6bc}{d}\right] \left( -36\sqrt{6} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Cos}\left[\frac{6bc}{d}\right] + \right. \\
& \left. \frac{5}{2} \left( -\frac{3}{2} \left( -\sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Cos}\left[\frac{6bc}{d}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] \right) + 6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Sin}\left[\frac{6bc}{d}\right] \right) \right) - \\
& \frac{1}{216\sqrt{6} \left(\frac{b}{d}\right)^{7/2} d^3} \operatorname{Sin}\left[\frac{6bc}{d}\right] \left( 36\sqrt{6} \left(\frac{b}{d}\right)^{5/2} (c+dx)^{5/2} \operatorname{Sin}\left[\frac{6bc}{d}\right] - \frac{5}{2} \right. \\
& \left. \left( -6\sqrt{6} \left(\frac{b}{d}\right)^{3/2} (c+dx)^{3/2} \operatorname{Cos}\left[\frac{6bc}{d}\right] + \frac{3}{2} \left( -\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[2\sqrt{\frac{b}{d}} \sqrt{\frac{3}{\pi}} \sqrt{c+dx}\right] + \sqrt{6} \sqrt{\frac{b}{d}} \sqrt{c+dx} \operatorname{Sin}\left[\frac{6bc}{d}\right] \right) \right) \right) \right)
\end{aligned}$$

**Problem 209: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^4 \operatorname{Tan}[a+bx] dx$$

Optimal (type 4, 158 leaves, 7 steps):

$$\begin{aligned}
& \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \operatorname{Log}[1+e^{2i(a+bx)}]}{b} + \frac{2id(c+dx)^3 \operatorname{PolyLog}[2, -e^{2i(a+bx)}]}{b^2} - \\
& \frac{3d^2(c+dx)^2 \operatorname{PolyLog}[3, -e^{2i(a+bx)}]}{b^3} - \frac{3id^3(c+dx) \operatorname{PolyLog}[4, -e^{2i(a+bx)}]}{b^4} + \frac{3d^4 \operatorname{PolyLog}[5, -e^{2i(a+bx)}]}{2b^5}
\end{aligned}$$

Result (type 4, 722 leaves):

$$\begin{aligned}
& \frac{1}{2 b^3} c^2 d^2 e^{-i a} \\
& \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log}\left[1 + e^{2 i (a+b x)}\right]\right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right] \right) \\
& \operatorname{Sec}[a] - i c d^3 e^{i a} \left( -x^4 + \left( 1 + e^{-2 i a} \right) x^4 - \frac{1}{2 b^4} e^{-2 i a} \left( 1 + e^{2 i a} \right) \right. \\
& \left. \left( 2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log}\left[1 + e^{2 i (a+b x)}\right] + 6 b^2 x^2 \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right] + 6 i b x \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right] - 3 \operatorname{PolyLog}\left[4, -e^{2 i (a+b x)}\right] \right) \right) \operatorname{Sec}[a] - \\
& \frac{1}{5} i d^4 e^{i a} \left( -x^5 + \left( 1 + e^{-2 i a} \right) x^5 - \frac{1}{4 b^5} e^{-2 i a} \left( 1 + e^{2 i a} \right) \left( 4 b^5 x^5 + 10 i b^4 x^4 \operatorname{Log}\left[1 + e^{2 i (a+b x)}\right] + 20 b^3 x^3 \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right] + \right. \right. \\
& \left. \left. 30 i b^2 x^2 \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right] - 30 b x \operatorname{PolyLog}\left[4, -e^{2 i (a+b x)}\right] - 15 i \operatorname{PolyLog}\left[5, -e^{2 i (a+b x)}\right] \right) \right) \operatorname{Sec}[a] - \\
& \frac{c^4 \operatorname{Sec}[a] \left( \operatorname{Cos}[a] \operatorname{Log}\left[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]\right] + b x \operatorname{Sin}[a] \right)}{b \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} - \left( 2 c^3 d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right. \right. \\
& \left. \left. \operatorname{Cot}[a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) - \pi \operatorname{Log}\left[1 + e^{-2 i b x}\right] - 2 \left( b x - \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) \operatorname{Log}\left[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) + \right. \right. \\
& \left. \left. \pi \operatorname{Log}\left[\operatorname{Cos}[b x]\right] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}\left[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]\right] + i \operatorname{PolyLog}\left[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \right) \operatorname{Sec}[a] \right) / \\
& \left( b^2 \sqrt{\operatorname{Csc}[a]^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right) + \frac{1}{5} x \left( 5 c^4 + 10 c^3 d x + 10 c^2 d^2 x^2 + 5 c d^3 x^3 + d^4 x^4 \right) \operatorname{Tan}[a]
\end{aligned}$$

### Problem 210: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \operatorname{Tan}[a + b x] dx$$

Optimal (type 4, 132 leaves, 6 steps):

$$\frac{i (c + d x)^4}{4 d} - \frac{(c + d x)^3 \operatorname{Log}\left[1 + e^{2 i (a+b x)}\right]}{b} + \frac{3 i d (c + d x)^2 \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right]}{2 b^2} - \\
\frac{3 d^2 (c + d x) \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right]}{2 b^3} - \frac{3 i d^3 \operatorname{PolyLog}\left[4, -e^{2 i (a+b x)}\right]}{4 b^4}$$

Result (type 4, 533 leaves):



$$\frac{1}{4 b^3} c d^2 e^{-i a} \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] \right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] \right) \\ \operatorname{Sec} [a] - \frac{1}{4} i d^3 e^{i a} \left( -x^4 + \left( 1 + e^{-2 i a} \right) x^4 - \frac{1}{2 b^4} e^{-2 i a} \left( 1 + e^{2 i a} \right) \right. \\ \left. \left( 2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] + 6 b^2 x^2 \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] + 6 i b x \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] - 3 \operatorname{PolyLog} \left[ 4, -e^{2 i (a+b x)} \right] \right) \right) \operatorname{Sec} [a] - \\ \frac{c^3 \operatorname{Sec} [a] \left( \operatorname{Cos} [a] \operatorname{Log} \left[ \operatorname{Cos} [a] \operatorname{Cos} [b x] - \operatorname{Sin} [a] \operatorname{Sin} [b x] \right] + b x \operatorname{Sin} [a] \right)}{b \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} - \left( 3 c^2 d \operatorname{Csc} [a] \left( b^2 e^{-i \operatorname{ArcTan} [\operatorname{Cot} [a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot} [a]^2}} \right. \right. \right. \\ \left. \left. \left. \operatorname{Cot} [a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) - \pi \operatorname{Log} \left[ 1 + e^{-2 i b x} \right] - 2 \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) \operatorname{Log} \left[ 1 - e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a]])} \right] + \right. \right. \right. \right. \\ \left. \left. \left. \pi \operatorname{Log} [\operatorname{Cos} [b x]] - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \operatorname{Log} [\operatorname{Sin} [b x - \operatorname{ArcTan} [\operatorname{Cot} [a]]] \right] + i \operatorname{PolyLog} \left[ 2, e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a]])} \right] \right) \right) \operatorname{Sec} [a] \right) / \\ \left( 2 b^2 \sqrt{\operatorname{Csc} [a]^2 \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} \right) + \frac{1}{4} x \left( 4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \operatorname{Tan} [a]$$

### Problem 211: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Tan} [a + b x] dx$$

Optimal (type 4, 96 leaves, 5 steps):

$$\frac{i (c + d x)^3}{3 d} - \frac{(c + d x)^2 \operatorname{Log} [1 + e^{2 i (a+b x)}]}{b} + \frac{i d (c + d x) \operatorname{PolyLog} [2, -e^{2 i (a+b x)}]}{b^2} - \frac{d^2 \operatorname{PolyLog} [3, -e^{2 i (a+b x)}]}{2 b^3}$$

Result (type 4, 363 leaves):

$$\frac{1}{12 b^3} d^2 e^{-i a} \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] \right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] \right) \\ \operatorname{Sec} [a] - \frac{c^2 \operatorname{Sec} [a] \left( \operatorname{Cos} [a] \operatorname{Log} \left[ \operatorname{Cos} [a] \operatorname{Cos} [b x] - \operatorname{Sin} [a] \operatorname{Sin} [b x] \right] + b x \operatorname{Sin} [a] \right)}{b \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} - \\ \left( c d \operatorname{Csc} [a] \left( b^2 e^{-i \operatorname{ArcTan} [\operatorname{Cot} [a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot} [a]^2}} \operatorname{Cot} [a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) - \pi \operatorname{Log} \left[ 1 + e^{-2 i b x} \right] - \right. \right. \right. \right. \\ \left. \left. \left. 2 \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) \operatorname{Log} \left[ 1 - e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a]])} \right] + \pi \operatorname{Log} [\operatorname{Cos} [b x]] - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \operatorname{Log} [\operatorname{Sin} [b x - \operatorname{ArcTan} [\operatorname{Cot} [a]]] \right] + \right. \right. \right. \\ \left. \left. \left. i \operatorname{PolyLog} \left[ 2, e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a]])} \right] \right) \right) \operatorname{Sec} [a] \right) / \left( b^2 \sqrt{\operatorname{Csc} [a]^2 \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} \right) + \frac{1}{3} x \left( 3 c^2 + 3 c d x + d^2 x^2 \right) \operatorname{Tan} [a]$$

### Problem 212: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Tan}[a + b x] dx$$

Optimal (type 4, 66 leaves, 4 steps):

$$\frac{i (c + d x)^2}{2 d} - \frac{(c + d x) \operatorname{Log}[1 + e^{2 i (a + b x)}]}{b} + \frac{i d \operatorname{PolyLog}[2, -e^{2 i (a + b x)}]}{2 b^2}$$

Result (type 4, 190 leaves):

$$\begin{aligned} & -\frac{c \operatorname{Log}[\operatorname{Cos}[a + b x]]}{b} - \\ & \left( d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \right) \right. \\ & \quad \left. \operatorname{Log}[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + \right. \\ & \quad \left. i \operatorname{PolyLog}[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}] \right) \operatorname{Sec}[a] \Big/ \left( 2 b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) + \frac{1}{2} d x^2 \operatorname{Tan}[a] \end{aligned}$$

### Problem 216: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \operatorname{Sin}[a + b x] \operatorname{Tan}[a + b x] dx$$

Optimal (type 4, 275 leaves, 14 steps):

$$\begin{aligned} & -\frac{2 i (c + d x)^3 \operatorname{ArcTan}[e^{i (a + b x)}]}{b} + \frac{6 d^3 \operatorname{Cos}[a + b x]}{b^4} - \frac{3 d (c + d x)^2 \operatorname{Cos}[a + b x]}{b^2} + \frac{3 i d (c + d x)^2 \operatorname{PolyLog}[2, -i e^{i (a + b x)}]}{b^2} - \\ & \frac{3 i d (c + d x)^2 \operatorname{PolyLog}[2, i e^{i (a + b x)}]}{b^2} - \frac{6 d^2 (c + d x) \operatorname{PolyLog}[3, -i e^{i (a + b x)}]}{b^3} + \frac{6 d^2 (c + d x) \operatorname{PolyLog}[3, i e^{i (a + b x)}]}{b^3} - \\ & \frac{6 i d^3 \operatorname{PolyLog}[4, -i e^{i (a + b x)}]}{b^4} + \frac{6 i d^3 \operatorname{PolyLog}[4, i e^{i (a + b x)}]}{b^4} + \frac{6 d^2 (c + d x) \operatorname{Sin}[a + b x]}{b^3} - \frac{(c + d x)^3 \operatorname{Sin}[a + b x]}{b} \end{aligned}$$

Result (type 4, 557 leaves):

$$\begin{aligned}
& -\frac{1}{b^4} \\
& \left( 2 i b^3 c^3 \operatorname{ArcTan}\left[e^{i(a+bx)}\right] + 3 b^2 c^2 d \operatorname{Cos}[a+bx] - 6 d^3 \operatorname{Cos}[a+bx] + 6 b^2 c d^2 x \operatorname{Cos}[a+bx] + 3 b^2 d^3 x^2 \operatorname{Cos}[a+bx] - 3 b^3 c^2 d x \operatorname{Log}\left[1-i e^{i(a+bx)}\right] - \right. \\
& \quad 3 b^3 c d^2 x^2 \operatorname{Log}\left[1-i e^{i(a+bx)}\right] - b^3 d^3 x^3 \operatorname{Log}\left[1-i e^{i(a+bx)}\right] + 3 b^3 c^2 d x \operatorname{Log}\left[1+i e^{i(a+bx)}\right] + 3 b^3 c d^2 x^2 \operatorname{Log}\left[1+i e^{i(a+bx)}\right] + \\
& \quad b^3 d^3 x^3 \operatorname{Log}\left[1+i e^{i(a+bx)}\right] - 3 i b^2 d(c+dx)^2 \operatorname{PolyLog}\left[2,-i e^{i(a+bx)}\right] + 3 i b^2 d(c+dx)^2 \operatorname{PolyLog}\left[2,i e^{i(a+bx)}\right] + \\
& \quad 6 b c d^2 \operatorname{PolyLog}\left[3,-i e^{i(a+bx)}\right] + 6 b d^3 x \operatorname{PolyLog}\left[3,-i e^{i(a+bx)}\right] - 6 b c d^2 \operatorname{PolyLog}\left[3,i e^{i(a+bx)}\right] - \\
& \quad 6 b d^3 x \operatorname{PolyLog}\left[3,i e^{i(a+bx)}\right] + 6 i d^3 \operatorname{PolyLog}\left[4,-i e^{i(a+bx)}\right] - 6 i d^3 \operatorname{PolyLog}\left[4,i e^{i(a+bx)}\right] + b^3 c^3 \operatorname{Sin}[a+bx] - \\
& \quad \left. 6 b c d^2 \operatorname{Sin}[a+bx] + 3 b^3 c^2 d x \operatorname{Sin}[a+bx] - 6 b d^3 x \operatorname{Sin}[a+bx] + 3 b^3 c d^2 x^2 \operatorname{Sin}[a+bx] + b^3 d^3 x^3 \operatorname{Sin}[a+bx] \right)
\end{aligned}$$

**Problem 218: Result more than twice size of optimal antiderivative.**

$$\int (c+dx) \operatorname{Sin}[a+bx] \operatorname{Tan}[a+bx] dx$$

Optimal (type 4, 103 leaves, 8 steps):

$$-\frac{2 i(c+dx) \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b} - \frac{d \operatorname{Cos}[a+bx]}{b^2} + \frac{i d \operatorname{PolyLog}\left[2,-i e^{i(a+bx)}\right]}{b^2} - \frac{i d \operatorname{PolyLog}\left[2,i e^{i(a+bx)}\right]}{b^2} - \frac{(c+dx) \operatorname{Sin}[a+bx]}{b}$$

Result (type 4, 258 leaves):

$$\begin{aligned}
& -\frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{b} + \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{b} + \\
& \frac{1}{b^2} d \left( \left(-a + \frac{\pi}{2} - bx\right) \left(\operatorname{Log}\left[1 - e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right] - \operatorname{Log}\left[1 + e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right]\right) - \left(-a + \frac{\pi}{2}\right) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}\left(-a + \frac{\pi}{2} - bx\right)\right]\right] + \right. \\
& \quad \left. i \left(\operatorname{PolyLog}\left[2,-e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right] - \operatorname{PolyLog}\left[2,e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right]\right) \right) - \frac{d \operatorname{Cos}[bx] (\operatorname{Cos}[a] + bx \operatorname{Sin}[a])}{b^2} - \frac{d (bx \operatorname{Cos}[a] - \operatorname{Sin}[a]) \operatorname{Sin}[bx]}{b^2} - \frac{c \operatorname{Sin}[a+bx]}{b}
\end{aligned}$$

**Problem 222: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^3 \operatorname{Sin}[a+bx]^2 \operatorname{Tan}[a+bx] dx$$

Optimal (type 4, 251 leaves, 12 steps):

$$\begin{aligned}
& -\frac{3 d^3 x}{8 b^3} + \frac{(c+dx)^3}{4 b} + \frac{i(c+dx)^4}{4 d} - \frac{(c+dx)^3 \operatorname{Log}\left[1+e^{2i(a+bx)}\right]}{b} + \frac{3 i d(c+dx)^2 \operatorname{PolyLog}\left[2,-e^{2i(a+bx)}\right]}{2 b^2} - \\
& \frac{3 d^2(c+dx) \operatorname{PolyLog}\left[3,-e^{2i(a+bx)}\right]}{2 b^3} - \frac{3 i d^3 \operatorname{PolyLog}\left[4,-e^{2i(a+bx)}\right]}{4 b^4} + \frac{3 d^3 \operatorname{Cos}[a+bx] \operatorname{Sin}[a+bx]}{8 b^4} - \\
& \frac{3 d(c+dx)^2 \operatorname{Cos}[a+bx] \operatorname{Sin}[a+bx]}{4 b^2} + \frac{3 d^2(c+dx) \operatorname{Sin}[a+bx]^2}{4 b^3} - \frac{(c+dx)^3 \operatorname{Sin}[a+bx]^2}{2 b}
\end{aligned}$$

Result (type 4, 1734 leaves):

$$\frac{1}{4 b^3} c d^2 e^{-i a} \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] \right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] \right) \operatorname{Sec} [a] - \frac{1}{4} i d^3 e^{i a} \left( -x^4 + \left( 1 + e^{-2 i a} \right) x^4 - \frac{1}{2 b^4} e^{-2 i a} \left( 1 + e^{2 i a} \right) \left( 2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] + 6 b^2 x^2 \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] + 6 i b x \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] - 3 \operatorname{PolyLog} \left[ 4, -e^{2 i (a+b x)} \right] \right) \right) \operatorname{Sec} [a] - \frac{c^3 \operatorname{Sec} [a] \left( \operatorname{Cos} [a] \operatorname{Log} \left[ \operatorname{Cos} [a] \operatorname{Cos} [b x] - \operatorname{Sin} [a] \operatorname{Sin} [b x] \right] + b x \operatorname{Sin} [a] \right)}{b \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} - \left( 3 c^2 d \operatorname{Csc} [a] \left( b^2 e^{-i \operatorname{ArcTan} [\operatorname{Cot} [a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot} [a]^2}} \right) \operatorname{Cot} [a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) \right) - \pi \operatorname{Log} \left[ 1 + e^{-2 i b x} \right] - 2 \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) \operatorname{Log} \left[ 1 - e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a]])} \right] + \pi \operatorname{Log} \left[ \operatorname{Cos} [b x] \right] - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \operatorname{Log} \left[ \operatorname{Sin} [b x - \operatorname{ArcTan} [\operatorname{Cot} [a]]] \right] + i \operatorname{PolyLog} \left[ 2, e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a]])} \right] \right) \right) \operatorname{Sec} [a] \right) / \left( 2 b^2 \sqrt{\operatorname{Csc} [a]^2 \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} \right) + \operatorname{Sec} [a] \left( \frac{\operatorname{Cos} [2 a + 2 b x]}{64 b^4} - \frac{i \operatorname{Sin} [2 a + 2 b x]}{64 b^4} \right) \left( 8 b^3 c^3 \operatorname{Cos} [a] - 12 i b^2 c^2 d \operatorname{Cos} [a] - 12 b c d^2 \operatorname{Cos} [a] + 6 i d^3 \operatorname{Cos} [a] + 24 b^3 c^2 d x \operatorname{Cos} [a] - 24 i b^2 c d^2 x \operatorname{Cos} [a] - 12 b d^3 x \operatorname{Cos} [a] + 24 b^3 c d^2 x^2 \operatorname{Cos} [a] - 12 i b^2 d^3 x^2 \operatorname{Cos} [a] + 8 b^3 d^3 x^3 \operatorname{Cos} [a] + 32 i b^4 c^3 x \operatorname{Cos} [a + 2 b x] + 48 i b^4 c^2 d x^2 \operatorname{Cos} [a + 2 b x] + 32 i b^4 c d^2 x^3 \operatorname{Cos} [a + 2 b x] + 8 i b^4 d^3 x^4 \operatorname{Cos} [a + 2 b x] - 32 i b^4 c^3 x \operatorname{Cos} [3 a + 2 b x] - 48 i b^4 c^2 d x^2 \operatorname{Cos} [3 a + 2 b x] - 32 i b^4 c d^2 x^3 \operatorname{Cos} [3 a + 2 b x] - 8 i b^4 d^3 x^4 \operatorname{Cos} [3 a + 2 b x] + 4 b^3 c^3 \operatorname{Cos} [3 a + 4 b x] + 6 i b^2 c^2 d \operatorname{Cos} [3 a + 4 b x] - 6 b c d^2 \operatorname{Cos} [3 a + 4 b x] - 3 i d^3 \operatorname{Cos} [3 a + 4 b x] + 12 b^3 c^2 d x \operatorname{Cos} [3 a + 4 b x] + 12 i b^2 c d^2 x \operatorname{Cos} [3 a + 4 b x] - 6 b d^3 x \operatorname{Cos} [3 a + 4 b x] + 12 b^3 c d^2 x^2 \operatorname{Cos} [3 a + 4 b x] + 6 i b^2 d^3 x^2 \operatorname{Cos} [3 a + 4 b x] + 4 b^3 d^3 x^3 \operatorname{Cos} [3 a + 4 b x] + 4 b^3 c^3 \operatorname{Cos} [5 a + 4 b x] + 6 i b^2 c^2 d \operatorname{Cos} [5 a + 4 b x] - 6 b c d^2 \operatorname{Cos} [5 a + 4 b x] - 3 i d^3 \operatorname{Cos} [5 a + 4 b x] + 12 b^3 c^2 d x \operatorname{Cos} [5 a + 4 b x] + 12 i b^2 c d^2 x \operatorname{Cos} [5 a + 4 b x] - 6 b d^3 x \operatorname{Cos} [5 a + 4 b x] + 12 b^3 c d^2 x^2 \operatorname{Cos} [5 a + 4 b x] + 6 i b^2 d^3 x^2 \operatorname{Cos} [5 a + 4 b x] + 4 b^3 d^3 x^3 \operatorname{Cos} [5 a + 4 b x] - 32 b^4 c^3 x \operatorname{Sin} [a + 2 b x] - 48 b^4 c^2 d x^2 \operatorname{Sin} [a + 2 b x] - 32 b^4 c d^2 x^3 \operatorname{Sin} [a + 2 b x] - 8 b^4 d^3 x^4 \operatorname{Sin} [a + 2 b x] + 32 b^4 c^3 x \operatorname{Sin} [3 a + 2 b x] + 48 b^4 c^2 d x^2 \operatorname{Sin} [3 a + 2 b x] + 32 b^4 c d^2 x^3 \operatorname{Sin} [3 a + 2 b x] + 8 b^4 d^3 x^4 \operatorname{Sin} [3 a + 2 b x] + 4 i b^3 c^3 \operatorname{Sin} [3 a + 4 b x] - 6 b^2 c^2 d \operatorname{Sin} [3 a + 4 b x] - 6 i b c d^2 \operatorname{Sin} [3 a + 4 b x] + 3 d^3 \operatorname{Sin} [3 a + 4 b x] + 12 i b^3 c^2 d x \operatorname{Sin} [3 a + 4 b x] - 12 b^2 c d^2 x \operatorname{Sin} [3 a + 4 b x] - 6 i b d^3 x \operatorname{Sin} [3 a + 4 b x] + 12 i b^3 c d^2 x^2 \operatorname{Sin} [3 a + 4 b x] - 6 b^2 d^3 x^2 \operatorname{Sin} [3 a + 4 b x] + 4 i b^3 d^3 x^3 \operatorname{Sin} [3 a + 4 b x] + 4 i b^3 c^3 \operatorname{Sin} [5 a + 4 b x] - 6 b^2 c^2 d \operatorname{Sin} [5 a + 4 b x] - 6 i b c d^2 \operatorname{Sin} [5 a + 4 b x] + 3 d^3 \operatorname{Sin} [5 a + 4 b x] + 12 i b^3 c^2 d x \operatorname{Sin} [5 a + 4 b x] - 12 b^2 c d^2 x \operatorname{Sin} [5 a + 4 b x] - 6 i b d^3 x \operatorname{Sin} [5 a + 4 b x] + 12 i b^3 c d^2 x^2 \operatorname{Sin} [5 a + 4 b x] - 6 b^2 d^3 x^2 \operatorname{Sin} [5 a + 4 b x] + 4 i b^3 d^3 x^3 \operatorname{Sin} [5 a + 4 b x] \right)$$

Problem 223: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Sin} [a + b x]^2 \operatorname{Tan} [a + b x] dx$$

Optimal (type 4, 184 leaves, 9 steps):

$$\frac{c d x}{2 b} + \frac{d^2 x^2}{4 b} + \frac{i (c + d x)^3}{3 d} - \frac{(c + d x)^2 \operatorname{Log}[1 + e^{2 i (a+b x)}]}{b} + \frac{i d (c + d x) \operatorname{PolyLog}[2, -e^{2 i (a+b x)}]}{b^2} -$$

$$\frac{d^2 \operatorname{PolyLog}[3, -e^{2 i (a+b x)}]}{2 b^3} - \frac{d (c + d x) \operatorname{Cos}[a + b x] \operatorname{Sin}[a + b x]}{2 b^2} + \frac{d^2 \operatorname{Sin}[a + b x]^2}{4 b^3} - \frac{(c + d x)^2 \operatorname{Sin}[a + b x]^2}{2 b}$$

Result (type 4, 525 leaves):

$$\frac{1}{12 b^3} d^2 e^{-i a} \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log}[1 + e^{2 i (a+b x)}] \right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog}[2, -e^{2 i (a+b x)}] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog}[3, -e^{2 i (a+b x)}] \right)$$

$$\operatorname{Sec}[a] - \frac{c^2 \operatorname{Sec}[a] \left( \operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a] \right)}{b \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} -$$

$$\left( c d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Cot}[a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) - \pi \operatorname{Log}[1 + e^{-2 i b x}] \right) - \right. \right.$$

$$2 \left( b x - \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) \operatorname{Log}[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] +$$

$$\left. i \operatorname{PolyLog}[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}] \right) \operatorname{Sec}[a] \right) / \left( b^2 \sqrt{\operatorname{Csc}[a]^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right) + \frac{1}{8 b^3}$$

$$\operatorname{Cos}[2 b x] \left( 2 b^2 c^2 \operatorname{Cos}[2 a] - d^2 \operatorname{Cos}[2 a] + 4 b^2 c d x \operatorname{Cos}[2 a] + 2 b^2 d^2 x^2 \operatorname{Cos}[2 a] - 2 b c d \operatorname{Sin}[2 a] - 2 b d^2 x \operatorname{Sin}[2 a] \right) -$$

$$\frac{1}{8 b^3} \left( 2 b c d \operatorname{Cos}[2 a] + 2 b d^2 x \operatorname{Cos}[2 a] + 2 b^2 c^2 \operatorname{Sin}[2 a] - d^2 \operatorname{Sin}[2 a] + 4 b^2 c d x \operatorname{Sin}[2 a] + 2 b^2 d^2 x^2 \operatorname{Sin}[2 a] \right) \operatorname{Sin}[2 b x] +$$

$$\frac{1}{3} x \left( 3 c^2 + 3 c d x + d^2 x^2 \right) \operatorname{Tan}[a]$$

Problem 228: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^4 \operatorname{Csc}[a + b x] \operatorname{Sec}[a + b x] dx$$

Optimal (type 4, 247 leaves, 12 steps):

$$-\frac{2 (c + d x)^4 \operatorname{ArcTanh}[e^{2 i (a+b x)}]}{b} + \frac{2 i d (c + d x)^3 \operatorname{PolyLog}[2, -e^{2 i (a+b x)}]}{b^2} - \frac{2 i d (c + d x)^3 \operatorname{PolyLog}[2, e^{2 i (a+b x)}]}{b^2} -$$

$$\frac{3 d^2 (c + d x)^2 \operatorname{PolyLog}[3, -e^{2 i (a+b x)}]}{b^3} + \frac{3 d^2 (c + d x)^2 \operatorname{PolyLog}[3, e^{2 i (a+b x)}]}{b^3} - \frac{3 i d^3 (c + d x) \operatorname{PolyLog}[4, -e^{2 i (a+b x)}]}{b^4} +$$

$$\frac{3 i d^3 (c + d x) \operatorname{PolyLog}[4, e^{2 i (a+b x)}]}{b^4} + \frac{3 d^4 \operatorname{PolyLog}[5, -e^{2 i (a+b x)}]}{2 b^5} - \frac{3 d^4 \operatorname{PolyLog}[5, e^{2 i (a+b x)}]}{2 b^5}$$

Result (type 4, 578 leaves):

$$\begin{aligned} & \frac{1}{2 b^5} \left( -4 b^4 c^4 \operatorname{ArcTanh}\left[e^{2 i(a+b x)}\right] + 8 b^4 c^3 d x \operatorname{Log}\left[1 - e^{2 i(a+b x)}\right] + 12 b^4 c^2 d^2 x^2 \operatorname{Log}\left[1 - e^{2 i(a+b x)}\right] + 8 b^4 c d^3 x^3 \operatorname{Log}\left[1 - e^{2 i(a+b x)}\right] + \right. \\ & 2 b^4 d^4 x^4 \operatorname{Log}\left[1 - e^{2 i(a+b x)}\right] - 8 b^4 c^3 d x \operatorname{Log}\left[1 + e^{2 i(a+b x)}\right] - 12 b^4 c^2 d^2 x^2 \operatorname{Log}\left[1 + e^{2 i(a+b x)}\right] - 8 b^4 c d^3 x^3 \operatorname{Log}\left[1 + e^{2 i(a+b x)}\right] - \\ & 2 b^4 d^4 x^4 \operatorname{Log}\left[1 + e^{2 i(a+b x)}\right] + 4 i b^3 d (c+d x)^3 \operatorname{PolyLog}\left[2, -e^{2 i(a+b x)}\right] - 4 i b^3 d (c+d x)^3 \operatorname{PolyLog}\left[2, e^{2 i(a+b x)}\right] - \\ & 6 b^2 c^2 d^2 \operatorname{PolyLog}\left[3, -e^{2 i(a+b x)}\right] - 12 b^2 c d^3 x \operatorname{PolyLog}\left[3, -e^{2 i(a+b x)}\right] - 6 b^2 d^4 x^2 \operatorname{PolyLog}\left[3, -e^{2 i(a+b x)}\right] + 6 b^2 c^2 d^2 \operatorname{PolyLog}\left[3, e^{2 i(a+b x)}\right] + \\ & 12 b^2 c d^3 x \operatorname{PolyLog}\left[3, e^{2 i(a+b x)}\right] + 6 b^2 d^4 x^2 \operatorname{PolyLog}\left[3, e^{2 i(a+b x)}\right] - 6 i b c d^3 \operatorname{PolyLog}\left[4, -e^{2 i(a+b x)}\right] - 6 i b d^4 x \operatorname{PolyLog}\left[4, -e^{2 i(a+b x)}\right] + \\ & \left. 6 i b c d^3 \operatorname{PolyLog}\left[4, e^{2 i(a+b x)}\right] + 6 i b d^4 x \operatorname{PolyLog}\left[4, e^{2 i(a+b x)}\right] + 3 d^4 \operatorname{PolyLog}\left[5, -e^{2 i(a+b x)}\right] - 3 d^4 \operatorname{PolyLog}\left[5, e^{2 i(a+b x)}\right] \right) \end{aligned}$$

Problem 235: Result more than twice size of optimal antiderivative.

$$\int (c+d x)^3 \operatorname{Csc}[a+b x]^2 \operatorname{Sec}[a+b x] d x$$

Optimal (type 4, 350 leaves, 23 steps):

$$\begin{aligned} & -\frac{2 i(c+d x)^3 \operatorname{ArcTan}\left[e^{i(a+b x)}\right]}{b} - \frac{6 d(c+d x)^2 \operatorname{ArcTanh}\left[e^{i(a+b x)}\right]}{b^2} - \frac{(c+d x)^3 \operatorname{Csc}[a+b x]}{b} + \\ & \frac{6 i d^2(c+d x) \operatorname{PolyLog}\left[2, -e^{i(a+b x)}\right]}{b^3} + \frac{3 i d(c+d x)^2 \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right]}{b^2} - \frac{3 i d(c+d x)^2 \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right]}{b^2} - \\ & \frac{6 i d^2(c+d x) \operatorname{PolyLog}\left[2, e^{i(a+b x)}\right]}{b^3} - \frac{6 d^3 \operatorname{PolyLog}\left[3, -e^{i(a+b x)}\right]}{b^4} - \frac{6 d^2(c+d x) \operatorname{PolyLog}\left[3, -i e^{i(a+b x)}\right]}{b^3} + \\ & \frac{6 d^2(c+d x) \operatorname{PolyLog}\left[3, i e^{i(a+b x)}\right]}{b^3} + \frac{6 d^3 \operatorname{PolyLog}\left[3, e^{i(a+b x)}\right]}{b^4} - \frac{6 i d^3 \operatorname{PolyLog}\left[4, -i e^{i(a+b x)}\right]}{b^4} + \frac{6 i d^3 \operatorname{PolyLog}\left[4, i e^{i(a+b x)}\right]}{b^4} \end{aligned}$$

Result (type 4, 760 leaves):

$$\begin{aligned} & -\frac{1}{b^4} \left( 2 i b^3 c^3 \operatorname{ArcTan}\left[e^{i(a+b x)}\right] + 6 b^2 c^2 d \operatorname{ArcTanh}\left[e^{i(a+b x)}\right] + b^3 c^3 \operatorname{Csc}[a+b x] + 3 b^3 c^2 d x \operatorname{Csc}[a+b x] + 3 b^3 c d^2 x^2 \operatorname{Csc}[a+b x] + \right. \\ & b^3 d^3 x^3 \operatorname{Csc}[a+b x] - 6 b^2 c d^2 x \operatorname{Log}\left[1 - e^{i(a+b x)}\right] - 3 b^2 d^3 x^2 \operatorname{Log}\left[1 - e^{i(a+b x)}\right] - 3 b^3 c^2 d x \operatorname{Log}\left[1 - i e^{i(a+b x)}\right] - \\ & 3 b^3 c d^2 x^2 \operatorname{Log}\left[1 - i e^{i(a+b x)}\right] - b^3 d^3 x^3 \operatorname{Log}\left[1 - i e^{i(a+b x)}\right] + 3 b^3 c^2 d x \operatorname{Log}\left[1 + i e^{i(a+b x)}\right] + 3 b^3 c d^2 x^2 \operatorname{Log}\left[1 + i e^{i(a+b x)}\right] + \\ & b^3 d^3 x^3 \operatorname{Log}\left[1 + i e^{i(a+b x)}\right] + 6 b^2 c d^2 x \operatorname{Log}\left[1 + e^{i(a+b x)}\right] + 3 b^2 d^3 x^2 \operatorname{Log}\left[1 + e^{i(a+b x)}\right] - 6 i b d^2 (c+d x) \operatorname{PolyLog}\left[2, -e^{i(a+b x)}\right] - \\ & 3 i b^2 d (c+d x)^2 \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right] + 3 i b^2 c^2 d \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right] + 6 i b^2 c d^2 x \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right] + \\ & 3 i b^2 d^3 x^2 \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right] + 6 i b c d^2 \operatorname{PolyLog}\left[2, e^{i(a+b x)}\right] + 6 i b d^3 x \operatorname{PolyLog}\left[2, e^{i(a+b x)}\right] + \\ & 6 d^3 \operatorname{PolyLog}\left[3, -e^{i(a+b x)}\right] + 6 b c d^2 \operatorname{PolyLog}\left[3, -i e^{i(a+b x)}\right] + 6 b d^3 x \operatorname{PolyLog}\left[3, -i e^{i(a+b x)}\right] - 6 b c d^2 \operatorname{PolyLog}\left[3, i e^{i(a+b x)}\right] - \\ & \left. 6 b d^3 x \operatorname{PolyLog}\left[3, i e^{i(a+b x)}\right] - 6 d^3 \operatorname{PolyLog}\left[3, e^{i(a+b x)}\right] + 6 i d^3 \operatorname{PolyLog}\left[4, -i e^{i(a+b x)}\right] - 6 i d^3 \operatorname{PolyLog}\left[4, i e^{i(a+b x)}\right] \right) \end{aligned}$$

### Problem 236: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Csc}[a + b x]^2 \operatorname{Sec}[a + b x] dx$$

Optimal (type 4, 226 leaves, 19 steps):

$$\begin{aligned} & -\frac{2 i (c + d x)^2 \operatorname{ArcTan}\left[e^{i(a+b x)}\right]}{b} - \frac{4 d (c + d x) \operatorname{ArcTanh}\left[e^{i(a+b x)}\right]}{b^2} - \frac{(c + d x)^2 \operatorname{Csc}[a + b x]}{b} + \\ & \frac{2 i d^2 \operatorname{PolyLog}\left[2, -e^{i(a+b x)}\right]}{b^3} + \frac{2 i d (c + d x) \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right]}{b^2} - \frac{2 i d (c + d x) \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right]}{b^2} - \\ & \frac{2 i d^2 \operatorname{PolyLog}\left[2, e^{i(a+b x)}\right]}{b^3} - \frac{2 d^2 \operatorname{PolyLog}\left[3, -i e^{i(a+b x)}\right]}{b^3} + \frac{2 d^2 \operatorname{PolyLog}\left[3, i e^{i(a+b x)}\right]}{b^3} \end{aligned}$$

Result (type 4, 593 leaves):

$$\begin{aligned} & -\frac{(c + d x)^2 \operatorname{Csc}[a]}{b} + \frac{1}{b^3} \\ & \left( -2 i b^2 c^2 \operatorname{ArcTan}\left[e^{i(a+b x)}\right] + 2 b^2 c d x \operatorname{Log}\left[1 - i e^{i(a+b x)}\right] + b^2 d^2 x^2 \operatorname{Log}\left[1 - i e^{i(a+b x)}\right] - 2 b^2 c d x \operatorname{Log}\left[1 + i e^{i(a+b x)}\right] - b^2 d^2 x^2 \operatorname{Log}\left[1 + i e^{i(a+b x)}\right] + \right. \\ & \left. 2 i b d (c + d x) \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right] - 2 i b d (c + d x) \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right] - 2 d^2 \operatorname{PolyLog}\left[3, -i e^{i(a+b x)}\right] + 2 d^2 \operatorname{PolyLog}\left[3, i e^{i(a+b x)}\right] \right) + \\ & \frac{4 i c d \operatorname{ArcTan}\left[\frac{i \operatorname{Cos}[a] - i \operatorname{Sin}[a] \operatorname{Tan}\left[\frac{b x}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{b^2 \sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} + \frac{\operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2} + \frac{b x}{2}\right] \left(-c^2 \operatorname{Sin}\left[\frac{b x}{2}\right] - 2 c d x \operatorname{Sin}\left[\frac{b x}{2}\right] - d^2 x^2 \operatorname{Sin}\left[\frac{b x}{2}\right]\right)}{2 b} + \\ & \frac{\operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{a}{2} + \frac{b x}{2}\right] \left(c^2 \operatorname{Sin}\left[\frac{b x}{2}\right] + 2 c d x \operatorname{Sin}\left[\frac{b x}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{b x}{2}\right]\right)}{2 b} + \frac{1}{b^3} \\ & 2 d^2 \left( -\frac{2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{ArcTanh}\left[\frac{-\operatorname{Cos}[a] + \operatorname{Sin}[a] \operatorname{Tan}\left[\frac{b x}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \\ & \left. \left( (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \left( \operatorname{Log}\left[1 - e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] - \operatorname{Log}\left[1 + e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) + \right. \right. \\ & \left. \left. i \left( \operatorname{PolyLog}\left[2, -e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] - \operatorname{PolyLog}\left[2, e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) \right) \operatorname{Sec}[a] \right) \end{aligned}$$

### Problem 237: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Csc}[a + b x]^2 \operatorname{Sec}[a + b x] dx$$

Optimal (type 4, 131 leaves, 10 steps):

$$\begin{aligned} & -\frac{2 i d x \operatorname{ArcTan}\left[e^{i(a+b x)}\right]}{b} - \frac{d \operatorname{ArcTanh}[\operatorname{Cos}[a+b x]]}{b^2} - \frac{d x \operatorname{ArcTanh}[\operatorname{Sin}[a+b x]]}{b} + \\ & \frac{(c+d x) \operatorname{ArcTanh}[\operatorname{Sin}[a+b x]]}{b} - \frac{(c+d x) \operatorname{Csc}[a+b x]}{b} + \frac{i d \operatorname{PolyLog}\left[2,-i e^{i(a+b x)}\right]}{b^2} - \frac{i d \operatorname{PolyLog}\left[2,i e^{i(a+b x)}\right]}{b^2} \end{aligned}$$

Result (type 4, 550 leaves):

$$\begin{aligned} & -\frac{c \operatorname{Cot}\left[\frac{1}{2}(a+b x)\right]}{2 b} + \frac{d\left(a \operatorname{Cos}\left[\frac{1}{2}(a+b x)\right] - (a+b x) \operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(a+b x)\right]}{2 b^2} - \frac{d \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right]}{b^2} - \\ & \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right]}{b} + \frac{d \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right]}{b^2} + \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+b x)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right]}{b} + \\ & \frac{1}{b^2} d\left(a\left(\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right] - \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right]\right) + (a+b x)\left(-\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right] + \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right]\right) - \\ & i\left(\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right] \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right)\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right] - \operatorname{Log}\left[\frac{1}{2}\left((1+i) - (1-i) \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right] + \right. \\ & \left. \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right)\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right] - \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right] \operatorname{Log}\left[\frac{1}{2}\left((1+i) + (1-i) \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right]\right) + \\ & \operatorname{PolyLog}\left[2,\left(-\frac{1}{2} - \frac{i}{2}\right)\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right] - \operatorname{PolyLog}\left[2,\left(-\frac{1}{2} + \frac{i}{2}\right)\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right] - \\ & \operatorname{PolyLog}\left[2,\left(\frac{1}{2} - \frac{i}{2}\right)\left(1 + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right] + \operatorname{PolyLog}\left[2,\left(\frac{1}{2} + \frac{i}{2}\right)\left(1 + \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)\right]\right) + \\ & \frac{d \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]\left(a \operatorname{Sin}\left[\frac{1}{2}(a+b x)\right] - (a+b x) \operatorname{Sin}\left[\frac{1}{2}(a+b x)\right]\right)}{2 b^2} - \frac{c \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]}{2 b} \end{aligned}$$

### Problem 241: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \operatorname{Csc}[a + b x]^3 \operatorname{Sec}[a + b x] dx$$

Optimal (type 4, 325 leaves, 22 steps):



$$\begin{aligned}
& - \frac{3 \, i \, d \, (c + d x)^2}{2 b^2} - \frac{(c + d x)^3}{2 b} - \frac{2 (c + d x)^3 \operatorname{ArcTanh}\left[e^{2 i (a+b x)}\right]}{b} - \frac{3 d (c + d x)^2 \operatorname{Cot}[a + b x]}{2 b^2} - \frac{(c + d x)^3 \operatorname{Cot}[a + b x]^2}{2 b} + \\
& \frac{3 d^2 (c + d x) \operatorname{Log}\left[1 - e^{2 i (a+b x)}\right]}{b^3} + \frac{3 \, i \, d (c + d x)^2 \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right]}{2 b^2} - \frac{3 \, i \, d^3 \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right]}{2 b^4} - \frac{3 \, i \, d (c + d x)^2 \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right]}{2 b^2} \\
& \frac{3 d^2 (c + d x) \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right]}{2 b^3} + \frac{3 d^2 (c + d x) \operatorname{PolyLog}\left[3, e^{2 i (a+b x)}\right]}{2 b^3} - \frac{3 \, i \, d^3 \operatorname{PolyLog}\left[4, -e^{2 i (a+b x)}\right]}{4 b^4} + \frac{3 \, i \, d^3 \operatorname{PolyLog}\left[4, e^{2 i (a+b x)}\right]}{4 b^4}
\end{aligned}$$

Result (type 4, 1285 leaves):

$$\begin{aligned}
& - \frac{(c + d x)^3 \operatorname{Csc}[a + b x]^2}{2 b} - \frac{1}{4 b^3} c d^2 e^{-i a} \operatorname{Csc}[a] \\
& \left( 2 b^2 x^2 (2 b e^{2 i a} x + 3 \, i (-1 + e^{2 i a}) \operatorname{Log}\left[1 - e^{2 i (a+b x)}\right]) + 6 b (-1 + e^{2 i a}) x \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right] + 3 \, i (-1 + e^{2 i a}) \operatorname{PolyLog}\left[3, e^{2 i (a+b x)}\right] \right) - \\
& \frac{1}{4} d^3 e^{i a} \operatorname{Csc}[a] \left( x^4 + (-1 + e^{-2 i a}) x^4 + \frac{1}{2 b^4} e^{-2 i a} (-1 + e^{2 i a}) \right. \\
& \left. (2 b^4 x^4 + 4 \, i b^3 x^3 \operatorname{Log}\left[1 - e^{2 i (a+b x)}\right] + 6 b^2 x^2 \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right] + 6 \, i b x \operatorname{PolyLog}\left[3, e^{2 i (a+b x)}\right] - 3 \operatorname{PolyLog}\left[4, e^{2 i (a+b x)}\right]) \right) + \\
& \frac{1}{4} x (4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3) \operatorname{Csc}[a] \operatorname{Sec}[a] + \frac{1}{4 b^3} c d^2 e^{-i a} \\
& \left( 2 \, i b^2 x^2 (2 b e^{2 i a} x + 3 \, i (1 + e^{2 i a}) \operatorname{Log}\left[1 + e^{2 i (a+b x)}\right]) + 6 \, i b (1 + e^{2 i a}) x \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right] - 3 (1 + e^{2 i a}) \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right] \right) \\
& \operatorname{Sec}[a] - \frac{1}{4} \, i d^3 e^{i a} \left( -x^4 + (1 + e^{-2 i a}) x^4 - \frac{1}{2 b^4} e^{-2 i a} (1 + e^{2 i a}) \right. \\
& \left. (2 b^4 x^4 + 4 \, i b^3 x^3 \operatorname{Log}\left[1 + e^{2 i (a+b x)}\right] + 6 b^2 x^2 \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right] + 6 \, i b x \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right] - 3 \operatorname{PolyLog}\left[4, -e^{2 i (a+b x)}\right]) \right) \operatorname{Sec}[a] - \\
& \frac{c^3 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x]] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{c^3 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{3 c d^2 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} - \\
& \left( 3 c^2 d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right. \\
& \left. \left. \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}\left[1 + e^{-2 i b x}\right] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}\left[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right]) \right) + \right. \\
& \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \right) \operatorname{Sec}[a] \Big/ \\
& \left( 2 b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) + \frac{3 \operatorname{Csc}[a] \operatorname{Csc}[a + b x] (c^2 d \operatorname{Sin}[b x] + 2 c d^2 x \operatorname{Sin}[b x] + d^3 x^2 \operatorname{Sin}[b x])}{2 b^2} -
\end{aligned}$$

$$\left( 3 c^2 d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\ \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] + \right. \right. \\ \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] \operatorname{Tan}[a] \right) \right) / \\ \left( 2 b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) - \left( 3 d^3 \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\ \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}]] \operatorname{Tan}[a] \right) \right) / \left( 2 b^4 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)$$

**Problem 242: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Csc}[a + b x]^3 \operatorname{Sec}[a + b x] dx$$

Optimal (type 4, 201 leaves, 17 steps):

$$-\frac{c d x}{b} - \frac{d^2 x^2}{2 b} - \frac{2 (c + d x)^2 \operatorname{ArcTanh}\left[e^{2 i (a + b x)}\right]}{b} - \frac{d (c + d x) \operatorname{Cot}[a + b x]}{b^2} - \frac{(c + d x)^2 \operatorname{Cot}[a + b x]^2}{2 b} + \frac{d^2 \operatorname{Log}[\operatorname{Sin}[a + b x]]}{b^3} + \\ \frac{i d (c + d x) \operatorname{PolyLog}\left[2, -e^{2 i (a + b x)}\right]}{b^2} - \frac{i d (c + d x) \operatorname{PolyLog}\left[2, e^{2 i (a + b x)}\right]}{b^2} - \frac{d^2 \operatorname{PolyLog}\left[3, -e^{2 i (a + b x)}\right]}{2 b^3} + \frac{d^2 \operatorname{PolyLog}\left[3, e^{2 i (a + b x)}\right]}{2 b^3}$$

Result (type 4, 785 leaves):

$$\begin{aligned}
& - \frac{(c+dx)^2 \operatorname{Csc}[a+bx]^2}{2b} - \frac{1}{12b^3} d^2 e^{-ia} \operatorname{Csc}[a] \\
& \left( 2b^2 x^2 (2b e^{2ia} x + 3i(-1 + e^{2ia}) \operatorname{Log}[1 - e^{2i(a+bx)}]) + 6b(-1 + e^{2ia}) x \operatorname{PolyLog}[2, e^{2i(a+bx)}] + 3i(-1 + e^{2ia}) \operatorname{PolyLog}[3, e^{2i(a+bx)}] \right) + \\
& \frac{1}{3} x (3c^2 + 3cdx + d^2 x^2) \operatorname{Csc}[a] \operatorname{Sec}[a] + \frac{1}{12b^3} \\
& d^2 e^{-ia} (2ib^2 x^2 (2b e^{2ia} x + 3i(1 + e^{2ia}) \operatorname{Log}[1 + e^{2i(a+bx)}]) + 6ib(1 + e^{2ia}) x \operatorname{PolyLog}[2, -e^{2i(a+bx)}] - 3(1 + e^{2ia}) \operatorname{PolyLog}[3, -e^{2i(a+bx)}]) \\
& \operatorname{Sec}[a] - \frac{c^2 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[bx] - \operatorname{Sin}[a] \operatorname{Sin}[bx]] + bx \operatorname{Sin}[a])}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{c^2 \operatorname{Csc}[a] (-bx \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[bx] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[bx]]) \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{d^2 \operatorname{Csc}[a] (-bx \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[bx] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[bx]]) \operatorname{Sin}[a]}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} - \\
& \left( cd \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Cot}[a] (ibx(-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}] - \right. \right. \\
& \left. \left. 2(bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}[1 - e^{2i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + \right. \right. \\
& \left. \left. i \operatorname{PolyLog}[2, e^{2i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}] \right) \right) \operatorname{Sec}[a] \Bigg) / \left( b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) + \\
& \frac{\operatorname{Csc}[a] \operatorname{Csc}[a+bx] (cd \operatorname{Sin}[bx] + d^2 x \operatorname{Sin}[bx])}{b^2} - \left( cd \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. (ibx(-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2ibx}] - 2(bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}[1 - e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}] + \pi \operatorname{Log}[\operatorname{Cos}[bx]] + 2 \right. \right. \\
& \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[bx + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}[2, e^{2i(bx + \operatorname{ArcTan}[\operatorname{Tan}[a])}] \right) \operatorname{Tan}[a] \right) \Bigg) / \left( b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)
\end{aligned}$$

**Problem 250: Result more than twice size of optimal antiderivative.**

$$\int (c+dx) \operatorname{Sec}[a+bx] \operatorname{Tan}[a+bx] dx$$

Optimal (type 3, 29 leaves, 2 steps):

$$-\frac{d \operatorname{ArcTanh}[\operatorname{Sin}[a+bx]]}{b^2} + \frac{(c+dx) \operatorname{Sec}[a+bx]}{b}$$

Result (type 3, 93 leaves):

$$\frac{d \operatorname{Log} \left[ \cos \left[ \frac{a}{2} + \frac{bx}{2} \right] - \sin \left[ \frac{a}{2} + \frac{bx}{2} \right] \right]}{b^2} - \frac{d \operatorname{Log} \left[ \cos \left[ \frac{a}{2} + \frac{bx}{2} \right] + \sin \left[ \frac{a}{2} + \frac{bx}{2} \right] \right]}{b^2} + \frac{c \operatorname{Sec} [a + bx]}{b} + \frac{d x \operatorname{Sec} [a + bx]}{b}$$

**Problem 254: Result more than twice size of optimal antiderivative.**

$$\int (c + dx)^3 \operatorname{Tan} [a + bx]^2 dx$$

Optimal (type 4, 128 leaves, 7 steps):

$$-\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \operatorname{Log} [1 + e^{2i(a+bx)}]}{b^2} - \frac{3id^2(c+dx) \operatorname{PolyLog} [2, -e^{2i(a+bx)}]}{b^3} + \frac{3d^3 \operatorname{PolyLog} [3, -e^{2i(a+bx)}]}{2b^4} + \frac{(c+dx)^3 \operatorname{Tan} [a + bx]}{b}$$

Result (type 4, 431 leaves):

$$-\frac{1}{4} x (4c^3 + 6c^2 dx + 4cd^2 x^2 + d^3 x^3) - \frac{1}{4b^4} d^3 e^{-ia} (2ib^2 x^2 (2be^{2ia} x + 3i(1 + e^{2ia}) \operatorname{Log} [1 + e^{2i(a+bx)}]) + 6ib(1 + e^{2ia}) x \operatorname{PolyLog} [2, -e^{2i(a+bx)}] - 3(1 + e^{2ia}) \operatorname{PolyLog} [3, -e^{2i(a+bx)}]) \operatorname{Sec} [a] + \frac{3c^2 d \operatorname{Sec} [a] (\cos [a] \operatorname{Log} [\cos [a] \cos [bx] - \sin [a] \sin [bx]] + bx \sin [a])}{b^2 (\cos [a]^2 + \sin [a]^2)} + \left( 3cd^2 \operatorname{Csc} [a] \left( b^2 e^{-i \operatorname{ArcTan} [\operatorname{Cot} [a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot} [a]^2}} \right. \right. \\ \left. \left. \operatorname{Cot} [a] (ibx (-\pi - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]]) - \pi \operatorname{Log} [1 + e^{-2ibx}] - 2(bx - \operatorname{ArcTan} [\operatorname{Cot} [a]]) \operatorname{Log} [1 - e^{2i(bx - \operatorname{ArcTan} [\operatorname{Cot} [a]])}] + \right. \right. \\ \left. \left. \pi \operatorname{Log} [\cos [bx]] - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \operatorname{Log} [\sin [bx - \operatorname{ArcTan} [\operatorname{Cot} [a]]]] + i \operatorname{PolyLog} [2, e^{2i(bx - \operatorname{ArcTan} [\operatorname{Cot} [a]])}] \right) \operatorname{Sec} [a] \right) / \\ \left( b^3 \sqrt{\operatorname{Csc} [a]^2 (\cos [a]^2 + \sin [a]^2)} \right) + \frac{\operatorname{Sec} [a] \operatorname{Sec} [a + bx] (c^3 \sin [bx] + 3c^2 dx \sin [bx] + 3cd^2 x^2 \sin [bx] + d^3 x^3 \sin [bx])}{b}$$

**Problem 255: Result more than twice size of optimal antiderivative.**

$$\int (c + dx)^2 \operatorname{Tan} [a + bx]^2 dx$$

Optimal (type 4, 96 leaves, 6 steps):

$$-\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx) \operatorname{Log} [1 + e^{2i(a+bx)}]}{b^2} - \frac{id^2 \operatorname{PolyLog} [2, -e^{2i(a+bx)}]}{b^3} + \frac{(c+dx)^2 \operatorname{Tan} [a + bx]}{b}$$

Result (type 4, 276 leaves):

$$\begin{aligned}
& -\frac{1}{3} x (3 c^2 + 3 c d x + d^2 x^2) + \frac{2 c d \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \left( d^2 \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right. \\
& \quad \left. \left. \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{2 i b x}] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]] \right) + \right. \\
& \quad \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}] \right) \operatorname{Sec}[a] \Big/ \\
& \left( b^3 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) + \frac{\operatorname{Sec}[a] \operatorname{Sec}[a + b x] (c^2 \operatorname{Sin}[b x] + 2 c d x \operatorname{Sin}[b x] + d^2 x^2 \operatorname{Sin}[b x])}{b}
\end{aligned}$$

**Problem 260: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 \operatorname{Sin}[a + b x] \operatorname{Tan}[a + b x]^2 dx$$

Optimal (type 4, 228 leaves, 13 steps):

$$\begin{aligned}
& \frac{6 i d (c + d x)^2 \operatorname{ArcTan}[e^{i (a + b x)}]}{b^2} - \frac{6 d^2 (c + d x) \operatorname{Cos}[a + b x]}{b^3} + \frac{(c + d x)^3 \operatorname{Cos}[a + b x]}{b} - \\
& \frac{6 i d^2 (c + d x) \operatorname{PolyLog}[2, -i e^{i (a + b x)}]}{b^3} + \frac{6 i d^2 (c + d x) \operatorname{PolyLog}[2, i e^{i (a + b x)}]}{b^3} + \frac{6 d^3 \operatorname{PolyLog}[3, -i e^{i (a + b x)}]}{b^4} - \\
& \frac{6 d^3 \operatorname{PolyLog}[3, i e^{i (a + b x)}]}{b^4} + \frac{(c + d x)^3 \operatorname{Sec}[a + b x]}{b} + \frac{6 d^3 \operatorname{Sin}[a + b x]}{b^4} - \frac{3 d (c + d x)^2 \operatorname{Sin}[a + b x]}{b^2}
\end{aligned}$$

Result (type 4, 532 leaves):

$$\begin{aligned}
& \frac{1}{2 b^4} \operatorname{Sec}[a + b x] (3 b^3 c^3 - 6 b c d^2 + 9 b^3 c^2 d x - 6 b d^3 x + 9 b^3 c d^2 x^2 + 3 b^3 d^3 x^3 + 12 i b^2 c^2 d \operatorname{ArcTan}[e^{i (a + b x)}] \operatorname{Cos}[a + b x] + \\
& b^3 c^3 \operatorname{Cos}[2 (a + b x)] - 6 b c d^2 \operatorname{Cos}[2 (a + b x)] + 3 b^3 c^2 d x \operatorname{Cos}[2 (a + b x)] - 6 b d^3 x \operatorname{Cos}[2 (a + b x)] + 3 b^3 c d^2 x^2 \operatorname{Cos}[2 (a + b x)] + \\
& b^3 d^3 x^3 \operatorname{Cos}[2 (a + b x)] - 12 b^2 c d^2 x \operatorname{Cos}[a + b x] \operatorname{Log}[1 - i e^{i (a + b x)}] - 6 b^2 d^3 x^2 \operatorname{Cos}[a + b x] \operatorname{Log}[1 - i e^{i (a + b x)}] + \\
& 12 b^2 c d^2 x \operatorname{Cos}[a + b x] \operatorname{Log}[1 + i e^{i (a + b x)}] + 6 b^2 d^3 x^2 \operatorname{Cos}[a + b x] \operatorname{Log}[1 + i e^{i (a + b x)}] - 12 i b d^2 (c + d x) \operatorname{Cos}[a + b x] \operatorname{PolyLog}[2, -i e^{i (a + b x)}] + \\
& 12 i b d^2 (c + d x) \operatorname{Cos}[a + b x] \operatorname{PolyLog}[2, i e^{i (a + b x)}] + 12 d^3 \operatorname{Cos}[a + b x] \operatorname{PolyLog}[3, -i e^{i (a + b x)}] - 12 d^3 \operatorname{Cos}[a + b x] \operatorname{PolyLog}[3, i e^{i (a + b x)}] - \\
& 3 b^2 c^2 d \operatorname{Sin}[2 (a + b x)] + 6 d^3 \operatorname{Sin}[2 (a + b x)] - 6 b^2 c d^2 x \operatorname{Sin}[2 (a + b x)] - 3 b^2 d^3 x^2 \operatorname{Sin}[2 (a + b x)])
\end{aligned}$$

**Problem 261: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Sin}[a + b x] \operatorname{Tan}[a + b x]^2 dx$$

Optimal (type 4, 145 leaves, 10 steps):

$$\frac{4 i d (c+d x) \operatorname{ArcTan}\left[e^{i(a+b x)}\right]}{b^2} - \frac{2 d^2 \operatorname{Cos}[a+b x]}{b^3} + \frac{(c+d x)^2 \operatorname{Cos}[a+b x]}{b} - \frac{2 i d^2 \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right]}{b^3} + \frac{2 i d^2 \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right]}{b^3} + \frac{(c+d x)^2 \operatorname{Sec}[a+b x]}{b} - \frac{2 d (c+d x) \operatorname{Sin}[a+b x]}{b^2}$$

Result (type 4, 362 leaves):

$$\frac{1}{b^3} \left( -4 b c d \operatorname{ArcTanh}\left[\operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Tan}\left[\frac{b x}{2}\right]\right] - 4 d^2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{ArcTanh}\left[\operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Tan}\left[\frac{b x}{2}\right]\right] + \frac{1}{\sqrt{\operatorname{Csc}[a]^2}} 2 d^2 \operatorname{Csc}[a] \left( (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \left( \operatorname{Log}\left[1 - e^{i(b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] - \operatorname{Log}\left[1 + e^{i(b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right]\right) + i \operatorname{PolyLog}\left[2, -e^{i(b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] - i \operatorname{PolyLog}\left[2, e^{i(b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) + b^2 (c+d x)^2 \operatorname{Sec}[a] + \operatorname{Cos}[b x] \left( \left( -2 d^2 + b^2 (c+d x)^2 \right) \operatorname{Cos}[a] - 2 b d (c+d x) \operatorname{Sin}[a] \right) - \left( 2 b d (c+d x) \operatorname{Cos}[a] + \left( -2 d^2 + b^2 (c+d x)^2 \right) \operatorname{Sin}[a] \right) \operatorname{Sin}[b x] + \frac{b^2 (c+d x)^2 \operatorname{Sin}\left[\frac{b x}{2}\right]}{\left( \operatorname{Cos}\left[\frac{a}{2}\right] - \operatorname{Sin}\left[\frac{a}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{1}{2}(a+b x)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+b x)\right] \right)} - \frac{b^2 (c+d x)^2 \operatorname{Sin}\left[\frac{b x}{2}\right]}{\left( \operatorname{Cos}\left[\frac{a}{2}\right] + \operatorname{Sin}\left[\frac{a}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{1}{2}(a+b x)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+b x)\right] \right)} \right)$$

**Problem 266: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^4 \operatorname{Csc}[a+b x] \operatorname{Sec}[a+b x]^2 dx$$

Optimal (type 4, 469 leaves, 27 steps):

$$\frac{8 i d (c+d x)^3 \operatorname{ArcTan}\left[e^{i(a+b x)}\right]}{b^2} - \frac{2 (c+d x)^4 \operatorname{ArcTanh}\left[e^{i(a+b x)}\right]}{b} + \frac{4 i d (c+d x)^3 \operatorname{PolyLog}\left[2, -e^{i(a+b x)}\right]}{b^2} - \frac{12 i d^2 (c+d x)^2 \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right]}{b^3} + \frac{12 i d^2 (c+d x)^2 \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right]}{b^3} - \frac{4 i d (c+d x)^3 \operatorname{PolyLog}\left[2, e^{i(a+b x)}\right]}{b^2} - \frac{12 d^2 (c+d x)^2 \operatorname{PolyLog}\left[3, -e^{i(a+b x)}\right]}{b^3} + \frac{24 d^3 (c+d x) \operatorname{PolyLog}\left[3, -i e^{i(a+b x)}\right]}{b^4} - \frac{24 d^3 (c+d x) \operatorname{PolyLog}\left[3, i e^{i(a+b x)}\right]}{b^4} + \frac{12 d^2 (c+d x)^2 \operatorname{PolyLog}\left[3, e^{i(a+b x)}\right]}{b^3} - \frac{24 i d^3 (c+d x) \operatorname{PolyLog}\left[4, -e^{i(a+b x)}\right]}{b^4} + \frac{24 i d^4 \operatorname{PolyLog}\left[4, -i e^{i(a+b x)}\right]}{b^5} - \frac{24 i d^4 \operatorname{PolyLog}\left[4, i e^{i(a+b x)}\right]}{b^5} + \frac{24 i d^3 (c+d x) \operatorname{PolyLog}\left[4, e^{i(a+b x)}\right]}{b^4} + \frac{24 d^4 \operatorname{PolyLog}\left[5, -e^{i(a+b x)}\right]}{b^5} - \frac{24 d^4 \operatorname{PolyLog}\left[5, e^{i(a+b x)}\right]}{b^5} + \frac{(c+d x)^4 \operatorname{Sec}[a+b x]}{b}$$

Result (type 4, 998 leaves):

$$\frac{1}{b^5} \left( -2 b^4 c^4 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right] + 4 b^4 c^3 d x \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 6 b^4 c^2 d^2 x^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 4 b^4 c d^3 x^3 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + \right. \\
b^4 d^4 x^4 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - 4 b^4 c^3 d x \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 6 b^4 c^2 d^2 x^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 4 b^4 c d^3 x^3 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - \\
b^4 d^4 x^4 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + 4 i b^3 d (c + d x)^3 \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] - 4 i b^3 d (c + d x)^3 \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] - \\
12 b^2 c^2 d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] - 24 b^2 c d^3 x \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] - 12 b^2 d^4 x^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] + 12 b^2 c^2 d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] + \\
24 b^2 c d^3 x \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] + 12 b^2 d^4 x^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] - 24 i b c d^3 \operatorname{PolyLog}\left[4, -e^{i(a+bx)}\right] - 24 i b d^4 x \operatorname{PolyLog}\left[4, -e^{i(a+bx)}\right] - \\
4 d \left( -2 i b^3 c^3 \operatorname{ArcTan}\left[e^{i(a+bx)}\right] + 3 b^3 c^2 d x \operatorname{Log}\left[1 - i e^{i(a+bx)}\right] + 3 b^3 c d^2 x^2 \operatorname{Log}\left[1 - i e^{i(a+bx)}\right] + b^3 d^3 x^3 \operatorname{Log}\left[1 - i e^{i(a+bx)}\right] - \right. \\
3 b^3 c^2 d x \operatorname{Log}\left[1 + i e^{i(a+bx)}\right] - 3 b^3 c d^2 x^2 \operatorname{Log}\left[1 + i e^{i(a+bx)}\right] - b^3 d^3 x^3 \operatorname{Log}\left[1 + i e^{i(a+bx)}\right] + 3 i b^2 d (c + d x)^2 \operatorname{PolyLog}\left[2, -i e^{i(a+bx)}\right] - \\
3 i b^2 d (c + d x)^2 \operatorname{PolyLog}\left[2, i e^{i(a+bx)}\right] - 6 b c d^2 \operatorname{PolyLog}\left[3, -i e^{i(a+bx)}\right] - 6 b d^3 x \operatorname{PolyLog}\left[3, -i e^{i(a+bx)}\right] + \\
6 b c d^2 \operatorname{PolyLog}\left[3, i e^{i(a+bx)}\right] + 6 b d^3 x \operatorname{PolyLog}\left[3, i e^{i(a+bx)}\right] - 6 i d^3 \operatorname{PolyLog}\left[4, -i e^{i(a+bx)}\right] + 6 i d^3 \operatorname{PolyLog}\left[4, i e^{i(a+bx)}\right] \left. \right) + \\
24 i b c d^3 \operatorname{PolyLog}\left[4, e^{i(a+bx)}\right] + 24 i b d^4 x \operatorname{PolyLog}\left[4, e^{i(a+bx)}\right] + 24 d^4 \operatorname{PolyLog}\left[5, -e^{i(a+bx)}\right] - \\
24 d^4 \operatorname{PolyLog}\left[5, e^{i(a+bx)}\right] + b^4 (c + d x)^4 \operatorname{Sec}[a + b x] \Big)$$

**Problem 268: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Csc}[a + b x] \operatorname{Sec}[a + b x]^2 dx$$

Optimal (type 4, 219 leaves, 19 steps):

$$\frac{4 i d (c + d x) \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b^2} - \frac{2 (c + d x)^2 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} + \\
\frac{2 i d (c + d x) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{b^2} - \frac{2 i d^2 \operatorname{PolyLog}\left[2, -i e^{i(a+bx)}\right]}{b^3} + \frac{2 i d^2 \operatorname{PolyLog}\left[2, i e^{i(a+bx)}\right]}{b^3} - \\
\frac{2 i d (c + d x) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{b^2} - \frac{2 d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right]}{b^3} + \frac{2 d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right]}{b^3} + \frac{(c + d x)^2 \operatorname{Sec}[a + b x]}{b}$$

Result (type 4, 449 leaves):

$$\frac{1}{b^3} \left( -2 b^2 c^2 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right] + 2 b^2 c d x \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + b^2 d^2 x^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - 2 b^2 c d x \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - b^2 d^2 x^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + \right. \\ \left. 2 i b d (c + d x) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] - 2 i b d (c + d x) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] - 2 d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] + 2 d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] \right) + \\ \frac{(c + d x)^2 \operatorname{Sec}[a + b x]}{b} - \frac{4 i c d \operatorname{ArcTan}\left[\frac{-i \operatorname{Sin}[a] - i \operatorname{Cos}[a] \operatorname{Tan}\left[\frac{bx}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{b^2 \sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} - \frac{1}{b^3} \\ 2 d^2 \left( -\frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Csc}[a] \left( (bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \left( \operatorname{Log}\left[1 - e^{i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] - \operatorname{Log}\left[1 + e^{i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \right) + \right. \\ \left. i \left( \operatorname{PolyLog}\left[2, -e^{i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] - \operatorname{PolyLog}\left[2, e^{i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \right) + \frac{2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{ArcTanh}\left[\frac{\operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Tan}\left[\frac{bx}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} \right)$$

**Problem 280: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^2 \operatorname{Csc}[a + b x]^3 \operatorname{Sec}[a + b x]^2 dx$$

Optimal (type 4, 305 leaves, 36 steps):

$$\frac{4 i d^2 x \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b^2} - \frac{3 (c + d x)^2 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{d^2 \operatorname{ArcTanh}[\operatorname{Cos}[a + b x]]}{b^3} - \\ \frac{2 c d \operatorname{ArcTanh}[\operatorname{Sin}[a + b x]]}{b^2} - \frac{c d \operatorname{Csc}[a + b x]}{b^2} - \frac{d^2 x \operatorname{Csc}[a + b x]}{b^2} + \frac{3 i d (c + d x) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{b^2} - \\ \frac{2 i d^2 \operatorname{PolyLog}\left[2, -i e^{i(a+bx)}\right]}{b^3} + \frac{2 i d^2 \operatorname{PolyLog}\left[2, i e^{i(a+bx)}\right]}{b^3} - \frac{3 i d (c + d x) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{b^2} - \\ \frac{3 d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right]}{b^3} + \frac{3 d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right]}{b^3} + \frac{3 (c + d x)^2 \operatorname{Sec}[a + b x]}{2 b} - \frac{(c + d x)^2 \operatorname{Csc}[a + b x]^2 \operatorname{Sec}[a + b x]}{2 b}$$

Result (type 4, 889 leaves):



$$\begin{aligned}
& \frac{(-c^2 - 2cdx - d^2x^2) \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} + \\
& \frac{1}{2b^3} \left( 3b^2c^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 2d^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 6b^2cdx \operatorname{Log}\left[1 - e^{i(a+bx)}\right] + 3b^2d^2x^2 \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - 3b^2c^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - \right. \\
& \quad \left. 2d^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 6b^2cdx \operatorname{Log}\left[1 + e^{i(a+bx)}\right] - 3b^2d^2x^2 \operatorname{Log}\left[1 + e^{i(a+bx)}\right] + 6ibd(c+dx) \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] - \right. \\
& \quad \left. 6ibd(c+dx) \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] - 6d^2 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right] + 6d^2 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right] \right) + \\
& \frac{(c^2 + 2cdx + d^2x^2) \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} + \frac{(c+dx) \operatorname{Csc}[a] \operatorname{Sec}[a] (-d \operatorname{Cos}[a] + bc \operatorname{Sin}[a] + bdx \operatorname{Sin}[a])}{b^2} - \\
& \frac{4icd \operatorname{ArcTan}\left[\frac{-i \operatorname{Sin}[a] - i \operatorname{Cos}[a] \operatorname{Tan}\left[\frac{bx}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{b^2 \sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} - \frac{1}{b^3} \\
& 2d^2 \left( -\frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Csc}[a] \left( (bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \left( \operatorname{Log}\left[1 - e^{i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) - \operatorname{Log}\left[1 + e^{i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \right) + \right. \\
& \quad \left. i \left( \operatorname{PolyLog}\left[2, -e^{i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) - \operatorname{PolyLog}\left[2, e^{i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \right) + \frac{2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{ArcTanh}\left[\frac{\operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Tan}\left[\frac{bx}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} \right) + \\
& \frac{\operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right] (-cd \operatorname{Sin}\left[\frac{bx}{2}\right] - d^2x \operatorname{Sin}\left[\frac{bx}{2}\right])}{2b^2} + \frac{\operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right] (cd \operatorname{Sin}\left[\frac{bx}{2}\right] + d^2x \operatorname{Sin}\left[\frac{bx}{2}\right])}{2b^2} + \\
& \frac{c^2 \operatorname{Sin}\left[\frac{bx}{2}\right] + 2cdx \operatorname{Sin}\left[\frac{bx}{2}\right] + d^2x^2 \operatorname{Sin}\left[\frac{bx}{2}\right]}{b \left( \operatorname{Cos}\left[\frac{a}{2}\right] - \operatorname{Sin}\left[\frac{a}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right] \right)} + \\
& \frac{-c^2 \operatorname{Sin}\left[\frac{bx}{2}\right] - 2cdx \operatorname{Sin}\left[\frac{bx}{2}\right] - d^2x^2 \operatorname{Sin}\left[\frac{bx}{2}\right]}{b \left( \operatorname{Cos}\left[\frac{a}{2}\right] + \operatorname{Sin}\left[\frac{a}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right] \right)}
\end{aligned}$$

**Problem 281: Result more than twice size of optimal antiderivative.**

$$\int (c+dx) \operatorname{Csc}[a+bx]^3 \operatorname{Sec}[a+bx]^2 dx$$

Optimal (type 4, 154 leaves, 13 steps):

$$\begin{aligned}
& - \frac{3 d x \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{3 c \operatorname{ArcTanh}[\operatorname{Cos}[a+bx]]}{2 b} - \frac{d \operatorname{ArcTanh}[\operatorname{Sin}[a+bx]]}{b^2} - \frac{d \operatorname{Csc}[a+bx]}{2 b^2} + \\
& \frac{3 i d \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{2 b^2} - \frac{3 i d \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{2 b^2} + \frac{3(c+dx) \operatorname{Sec}[a+bx]}{2 b} - \frac{(c+dx) \operatorname{Csc}[a+bx]^2 \operatorname{Sec}[a+bx]}{2 b}
\end{aligned}$$

Result (type 4, 520 leaves):

$$\begin{aligned}
& \frac{d x}{b} - \frac{d \operatorname{Cot}\left[\frac{1}{2}(a+bx)\right]}{4 b^2} - \frac{c \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{8 b} - \frac{d x \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]^2}{8 b} - \frac{3 c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right]}{2 b} + \\
& \frac{d \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{b^2} + \frac{3 c \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2 b} - \frac{d \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{b^2} - \\
& \frac{3 a d \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]}{2 b^2} + \frac{1}{2 b^2} 3 d \left( (a+bx) \left( \operatorname{Log}\left[1 - e^{i(a+bx)}\right] - \operatorname{Log}\left[1 + e^{i(a+bx)}\right] \right) + i \left( \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] - \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right] \right) \right) + \\
& \frac{c \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{8 b} + \frac{d x \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{8 b} + \frac{c \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]}{b \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)} - \frac{c \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]}{b \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)} + \\
& \frac{d \left( a \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] - (a+bx) \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)}{b^2 \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)} + \frac{d \left( -a \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] + (a+bx) \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)}{b^2 \left( \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] \right)} - \frac{d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{4 b^2}
\end{aligned}$$

Problem 286: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{Csc}[a+bx]^3 \operatorname{Sec}[a+bx]^2 dx$$

Optimal (type 4, 235 leaves, 29 steps):

$$\begin{aligned}
& \frac{4 i x \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b^2} - \frac{3 x^2 \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{\operatorname{ArcTanh}[\operatorname{Cos}[a+bx]]}{b^3} - \frac{x \operatorname{Csc}[a+bx]}{b^2} + \\
& \frac{3 i x \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{b^2} - \frac{2 i \operatorname{PolyLog}\left[2, -i e^{i(a+bx)}\right]}{b^3} + \frac{2 i \operatorname{PolyLog}\left[2, i e^{i(a+bx)}\right]}{b^3} - \frac{3 i x \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{b^2} - \\
& \frac{3 \operatorname{PolyLog}\left[3, -e^{i(a+bx)}\right]}{b^3} + \frac{3 \operatorname{PolyLog}\left[3, e^{i(a+bx)}\right]}{b^3} + \frac{3 x^2 \operatorname{Sec}[a+bx]}{2 b} - \frac{x^2 \operatorname{Csc}[a+bx]^2 \operatorname{Sec}[a+bx]}{2 b}
\end{aligned}$$

Result (type 4, 557 leaves):

$$\begin{aligned}
& - \frac{x^2 \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} - \frac{1}{b^3} 2 \left( \left(-a + \frac{\pi}{2} - bx\right) \left(\operatorname{Log}\left[1 - e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right] - \operatorname{Log}\left[1 + e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right]\right) - \right. \\
& \quad \left. \left(-a + \frac{\pi}{2}\right) \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}\left(-a + \frac{\pi}{2} - bx\right)\right]\right] + i \left(\operatorname{PolyLog}\left[2, -e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right] - \operatorname{PolyLog}\left[2, e^{i\left(-a + \frac{\pi}{2} - bx\right)}\right]\right) \right) - \frac{1}{b^3} \\
& \quad \left(2 \operatorname{ArcTanh}\left[\operatorname{Cos}\left[a + bx\right] + i \operatorname{Sin}\left[a + bx\right]\right] + 3 b^2 x^2 \operatorname{ArcTanh}\left[\operatorname{Cos}\left[a + bx\right] + i \operatorname{Sin}\left[a + bx\right]\right] - 3 i b x \operatorname{PolyLog}\left[2, -\operatorname{Cos}\left[a + bx\right] - i \operatorname{Sin}\left[a + bx\right]\right] + \right. \\
& \quad \left. 3 i b x \operatorname{PolyLog}\left[2, \operatorname{Cos}\left[a + bx\right] + i \operatorname{Sin}\left[a + bx\right]\right] + 3 \operatorname{PolyLog}\left[3, -\operatorname{Cos}\left[a + bx\right] - i \operatorname{Sin}\left[a + bx\right]\right] - 3 \operatorname{PolyLog}\left[3, \operatorname{Cos}\left[a + bx\right] + i \operatorname{Sin}\left[a + bx\right]\right] \right) + \\
& \quad \frac{x^2 \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right]^2}{8b} + \frac{x \operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2}\right] \left(-\operatorname{Cos}\left[\frac{a}{2}\right] + bx \operatorname{Sin}\left[\frac{a}{2}\right]\right)}{b^2} + \frac{x \operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right] \operatorname{Sin}\left[\frac{bx}{2}\right]}{2b^2} - \frac{x \operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right] \operatorname{Sin}\left[\frac{bx}{2}\right]}{2b^2} + \\
& \quad \frac{x^2 \operatorname{Sin}\left[\frac{bx}{2}\right]}{b \left(\operatorname{Cos}\left[\frac{a}{2}\right] - \operatorname{Sin}\left[\frac{a}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)} - \frac{x^2 \operatorname{Sin}\left[\frac{bx}{2}\right]}{b \left(\operatorname{Cos}\left[\frac{a}{2}\right] + \operatorname{Sin}\left[\frac{a}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)}
\end{aligned}$$

**Problem 287: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Csc}\left[a + bx\right]^3 \operatorname{Sec}\left[a + bx\right]^2 dx$$

Optimal (type 4, 126 leaves, 13 steps):

$$\begin{aligned}
& - \frac{3 x \operatorname{ArcTanh}\left[e^{i(a+bx)}\right]}{b} - \frac{\operatorname{ArcTanh}\left[\operatorname{Sin}\left[a + bx\right]\right]}{b^2} - \frac{\operatorname{Csc}\left[a + bx\right]}{2 b^2} + \\
& \frac{3 i \operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right]}{2 b^2} - \frac{3 i \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]}{2 b^2} + \frac{3 x \operatorname{Sec}\left[a + bx\right]}{2 b} - \frac{x \operatorname{Csc}\left[a + bx\right]^2 \operatorname{Sec}\left[a + bx\right]}{2 b}
\end{aligned}$$

Result (type 4, 282 leaves):

$$\begin{aligned}
& \frac{1}{8 b^2} \\
& \left( 8 b x - 2 \operatorname{Cot}\left[\frac{1}{2}(a + bx)\right] - b x \operatorname{Csc}\left[\frac{1}{2}(a + bx)\right]^2 + 12(a + bx) \left(\operatorname{Log}\left[1 - e^{i(a+bx)}\right] - \operatorname{Log}\left[1 + e^{i(a+bx)}\right]\right) + 8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right] - \right. \\
& \quad 8 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right] - 12 a \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] + 12 i \left(\operatorname{PolyLog}\left[2, -e^{i(a+bx)}\right] - \operatorname{PolyLog}\left[2, e^{i(a+bx)}\right]\right) + \\
& \quad \left. b x \operatorname{Sec}\left[\frac{1}{2}(a + bx)\right]^2 + \frac{8 b x \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]} - \frac{8 b x \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]}{\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]} - 2 \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right] \right)
\end{aligned}$$

**Problem 291: Result more than twice size of optimal antiderivative.**

$$\int (c + dx)^4 \operatorname{Sec}\left[a + bx\right]^2 \operatorname{Tan}\left[a + bx\right] dx$$

Optimal (type 4, 139 leaves, 7 steps):

$$\frac{2 i d (c+d x)^3}{b^2} - \frac{6 d^2 (c+d x)^2 \operatorname{Log}\left[1+e^{2 i(a+b x)}\right]}{b^3} + \frac{6 i d^3 (c+d x) \operatorname{PolyLog}\left[2,-e^{2 i(a+b x)}\right]}{b^4} - \frac{3 d^4 \operatorname{PolyLog}\left[3,-e^{2 i(a+b x)}\right]}{b^5} + \frac{(c+d x)^4 \operatorname{Sec}[a+b x]^2}{2 b} - \frac{2 d (c+d x)^3 \operatorname{Tan}[a+b x]}{b^2}$$

Result (type 4, 425 leaves):

$$\frac{1}{2 b^5} d^4 e^{-i a} \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log}\left[1+e^{2 i(a+b x)}\right] \right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog}\left[2,-e^{2 i(a+b x)}\right] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog}\left[3,-e^{2 i(a+b x)}\right] \right) \operatorname{Sec}[a] + \frac{(c+d x)^4 \operatorname{Sec}[a+b x]^2}{2 b} - \frac{6 c^2 d^2 \operatorname{Sec}[a] \left( \operatorname{Cos}[a] \operatorname{Log}\left[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]\right] + b x \operatorname{Sin}[a] \right)}{b^3 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} - \left( 6 c d^3 \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1+\operatorname{Cot}[a]^2}} \right) + \operatorname{Cot}[a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) - \pi \operatorname{Log}\left[1+e^{-2 i b x}\right] - 2 \left( b x - \operatorname{ArcTan}[\operatorname{Cot}[a]] \right) \operatorname{Log}\left[1-e^{2 i(b x-\operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) + \pi \operatorname{Log}\left[\operatorname{Cos}[b x]\right] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}\left[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]\right] + i \operatorname{PolyLog}\left[2, e^{2 i(b x-\operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \operatorname{Sec}[a] \right) / \left( b^4 \sqrt{\operatorname{Csc}[a]^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right) - \frac{2 \operatorname{Sec}[a] \operatorname{Sec}[a+b x] \left( c^3 d \operatorname{Sin}[b x] + 3 c^2 d^2 x \operatorname{Sin}[b x] + 3 c d^3 x^2 \operatorname{Sin}[b x] + d^4 x^3 \operatorname{Sin}[b x] \right)}{b^2}$$

**Problem 292: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^3 \operatorname{Sec}[a+b x]^2 \operatorname{Tan}[a+b x] dx$$

Optimal (type 4, 115 leaves, 6 steps):

$$\frac{3 i d (c+d x)^2}{2 b^2} - \frac{3 d^2 (c+d x) \operatorname{Log}\left[1+e^{2 i(a+b x)}\right]}{b^3} + \frac{3 i d^3 \operatorname{PolyLog}\left[2,-e^{2 i(a+b x)}\right]}{2 b^4} + \frac{(c+d x)^3 \operatorname{Sec}[a+b x]^2}{2 b} - \frac{3 d (c+d x)^2 \operatorname{Tan}[a+b x]}{2 b^2}$$

Result (type 4, 286 leaves):

$$\frac{(c+dx)^3 \operatorname{Sec}[a+bx]^2}{2b} - \frac{3cd^2 \operatorname{Sec}[a] (\cos[a] \log[\cos[a] \cos[bx]] - \sin[a] \sin[bx]) + bx \sin[a]}{b^3 (\cos[a]^2 + \sin[a]^2)} -$$

$$\left( 3d^3 \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right.$$

$$\left. \left. \operatorname{Cot}[a] (i bx (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \log[1 + e^{-2i bx}]) - 2 (bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \log[1 - e^{2i (bx - \operatorname{ArcTan}[\operatorname{Cot}[a]])}] + \right. \right.$$

$$\left. \left. \pi \log[\cos[bx]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \log[\sin[bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}[2, e^{2i (bx - \operatorname{ArcTan}[\operatorname{Cot}[a]])}] \right) \operatorname{Sec}[a] \right) /$$

$$\left( 2b^4 \sqrt{\operatorname{Csc}[a]^2 (\cos[a]^2 + \sin[a]^2)} \right) - \frac{3 \operatorname{Sec}[a] \operatorname{Sec}[a+bx] (c^2 d \sin[bx] + 2cd^2 x \sin[bx] + d^3 x^2 \sin[bx])}{2b^2}$$

**Problem 299: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^2 \operatorname{Sec}[a+bx] \operatorname{Tan}[a+bx]^2 dx$$

Optimal (type 4, 193 leaves, 17 steps):

$$\frac{i(c+dx)^2 \operatorname{ArcTan}[e^{i(a+bx)}]}{b} + \frac{d^2 \operatorname{ArcTanh}[\sin[a+bx]]}{b^3} - \frac{id(c+dx) \operatorname{PolyLog}[2, -ie^{i(a+bx)}]}{b^2} + \frac{id(c+dx) \operatorname{PolyLog}[2, ie^{i(a+bx)}]}{b^2} +$$

$$\frac{d^2 \operatorname{PolyLog}[3, -ie^{i(a+bx)}]}{b^3} - \frac{d^2 \operatorname{PolyLog}[3, ie^{i(a+bx)}]}{b^3} - \frac{d(c+dx) \operatorname{Sec}[a+bx]}{b^2} + \frac{(c+dx)^2 \operatorname{Sec}[a+bx] \operatorname{Tan}[a+bx]}{2b}$$

Result (type 4, 526 leaves):

$$\frac{1}{b^2} \left( i b c^2 \operatorname{ArcTan}[e^{i(a+bx)}] - \frac{2id^2 \operatorname{ArcTan}[e^{i(a+bx)}]}{b} - bcdx \log[1 - ie^{i(a+bx)}] - \right.$$

$$\left. \frac{1}{2} b d^2 x^2 \log[1 - ie^{i(a+bx)}] + bcdx \log[1 + ie^{i(a+bx)}] + \frac{1}{2} b d^2 x^2 \log[1 + ie^{i(a+bx)}] - id(c+dx) \operatorname{PolyLog}[2, -ie^{i(a+bx)}] + \right.$$

$$\left. id(c+dx) \operatorname{PolyLog}[2, ie^{i(a+bx)}] + \frac{d^2 \operatorname{PolyLog}[3, -ie^{i(a+bx)}]}{b} - \frac{d^2 \operatorname{PolyLog}[3, ie^{i(a+bx)}]}{b} \right) -$$

$$\frac{d(c+dx) \operatorname{Sec}[a]}{b^2} + \frac{c^2 + 2cdx + d^2 x^2}{4b \left( \cos\left[\frac{a}{2} + \frac{bx}{2}\right] - \sin\left[\frac{a}{2} + \frac{bx}{2}\right] \right)^2} + \frac{-cd \sin\left[\frac{bx}{2}\right] - d^2 x \sin\left[\frac{bx}{2}\right]}{b^2 \left( \cos\left[\frac{a}{2}\right] - \sin\left[\frac{a}{2}\right] \right) \left( \cos\left[\frac{a}{2} + \frac{bx}{2}\right] - \sin\left[\frac{a}{2} + \frac{bx}{2}\right] \right)} +$$

$$\frac{-c^2 - 2cdx - d^2 x^2}{4b \left( \cos\left[\frac{a}{2} + \frac{bx}{2}\right] + \sin\left[\frac{a}{2} + \frac{bx}{2}\right] \right)^2} + \frac{cd \sin\left[\frac{bx}{2}\right] + d^2 x \sin\left[\frac{bx}{2}\right]}{b^2 \left( \cos\left[\frac{a}{2}\right] + \sin\left[\frac{a}{2}\right] \right) \left( \cos\left[\frac{a}{2} + \frac{bx}{2}\right] + \sin\left[\frac{a}{2} + \frac{bx}{2}\right] \right)}$$

### Problem 300: Result more than twice size of optimal antiderivative.

$$\int (c + dx) \operatorname{Sec}[a + bx] \operatorname{Tan}[a + bx]^2 dx$$

Optimal (type 4, 117 leaves, 12 steps):

$$\frac{i(c + dx) \operatorname{ArcTan}\left[e^{i(a+bx)}\right]}{b} - \frac{id \operatorname{PolyLog}\left[2, -i e^{i(a+bx)}\right]}{2b^2} + \frac{id \operatorname{PolyLog}\left[2, i e^{i(a+bx)}\right]}{2b^2} - \frac{d \operatorname{Sec}[a + bx]}{2b^2} + \frac{(c + dx) \operatorname{Sec}[a + bx] \operatorname{Tan}[a + bx]}{2b}$$

Result (type 4, 607 leaves):

$$\begin{aligned} & \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right]}{2b} - \frac{c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right]}{2b} + \\ & \frac{1}{2b^2} d \left( (a + bx) \left( \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] - \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] \right) + a \left( -\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] + \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] \right) + \right. \\ & \quad i \left( \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right)\right] - \operatorname{Log}\left[\frac{1}{2} \left((1 + i) - (1 - i) \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] + \right. \\ & \quad \left. \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] - \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right] \operatorname{Log}\left[\frac{1}{2} \left((1 + i) + (1 - i) \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right)\right] \right) + \\ & \quad \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right)\right] - \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} + \frac{i}{2}\right) \left(-1 + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right)\right] - \\ & \quad \left. \operatorname{PolyLog}\left[2, \left(\frac{1}{2} - \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right)\right] + \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(a + bx)\right]\right)\right] \right) \right) + \\ & \frac{c}{4b \left(\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right)^2} + \frac{dx}{4b \left(\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right)^2} - \\ & \frac{d \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]}{2b^2 \left(\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right)} - \\ & \frac{c}{4b \left(\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right)^2} - \\ & \frac{dx}{4b \left(\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right)^2} + \\ & \frac{d \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]}{2b^2 \left(\operatorname{Cos}\left[\frac{1}{2}(a + bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx)\right]\right)} \end{aligned}$$

### Problem 304: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^3 \operatorname{Tan}[a + b x]^3 dx$$

Optimal (type 4, 259 leaves, 13 steps):

$$\begin{aligned} & \frac{3 i d (c + d x)^2}{2 b^2} + \frac{(c + d x)^3}{2 b} - \frac{i (c + d x)^4}{4 d} - \frac{3 d^2 (c + d x) \operatorname{Log}[1 + e^{2 i (a + b x)}]}{b^3} + \\ & \frac{(c + d x)^3 \operatorname{Log}[1 + e^{2 i (a + b x)}]}{b} + \frac{3 i d^3 \operatorname{PolyLog}[2, -e^{2 i (a + b x)}]}{2 b^4} - \frac{3 i d (c + d x)^2 \operatorname{PolyLog}[2, -e^{2 i (a + b x)}]}{2 b^2} + \\ & \frac{3 d^2 (c + d x) \operatorname{PolyLog}[3, -e^{2 i (a + b x)}]}{2 b^3} + \frac{3 i d^3 \operatorname{PolyLog}[4, -e^{2 i (a + b x)}]}{4 b^4} - \frac{3 d (c + d x)^2 \operatorname{Tan}[a + b x]}{2 b^2} + \frac{(c + d x)^3 \operatorname{Tan}[a + b x]^2}{2 b} \end{aligned}$$

Result (type 4, 817 leaves):

$$\begin{aligned}
& -\frac{1}{4b^3} c d^2 e^{-i a} \\
& \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] \right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] \right) \\
& \operatorname{Sec} [a] + \frac{1}{4} i d^3 e^{i a} \left( -x^4 + \left( 1 + e^{-2 i a} \right) x^4 - \frac{1}{2 b^4} e^{-2 i a} \left( 1 + e^{2 i a} \right) \right. \\
& \left. \left( 2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] + 6 b^2 x^2 \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] + 6 i b x \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] - 3 \operatorname{PolyLog} \left[ 4, -e^{2 i (a+b x)} \right] \right) \right) \operatorname{Sec} [a] + \\
& \frac{(c+d x)^3 \operatorname{Sec} [a+b x]^2}{2 b} + \frac{c^3 \operatorname{Sec} [a] \left( \operatorname{Cos} [a] \operatorname{Log} [\operatorname{Cos} [a] \operatorname{Cos} [b x]] - \operatorname{Sin} [a] \operatorname{Sin} [b x] \right) + b x \operatorname{Sin} [a]}{b \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} - \\
& \frac{3 c d^2 \operatorname{Sec} [a] \left( \operatorname{Cos} [a] \operatorname{Log} [\operatorname{Cos} [a] \operatorname{Cos} [b x]] - \operatorname{Sin} [a] \operatorname{Sin} [b x] \right) + b x \operatorname{Sin} [a]}{b^3 \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} + \\
& \left( 3 c^2 d \operatorname{Csc} [a] \left( b^2 e^{-i \operatorname{ArcTan} [\operatorname{Cot} [a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot} [a]^2}} \right. \right. \\
& \left. \left. \operatorname{Cot} [a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) - \pi \operatorname{Log} \left[ 1 + e^{-2 i b x} \right] - 2 \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) \operatorname{Log} \left[ 1 - e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a])} \right] \right) + \right. \right. \\
& \left. \left. \pi \operatorname{Log} [\operatorname{Cos} [b x]] - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \operatorname{Log} [\operatorname{Sin} [b x - \operatorname{ArcTan} [\operatorname{Cot} [a]]]] + i \operatorname{PolyLog} \left[ 2, e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a])} \right] \right) \right) \operatorname{Sec} [a] \right) / \\
& \left( 2 b^2 \sqrt{\operatorname{Csc} [a]^2 \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} \right) - \left( 3 d^3 \operatorname{Csc} [a] \left( b^2 e^{-i \operatorname{ArcTan} [\operatorname{Cot} [a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot} [a]^2}} \right. \right. \\
& \left. \left. \operatorname{Cot} [a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) - \pi \operatorname{Log} \left[ 1 + e^{-2 i b x} \right] - 2 \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) \operatorname{Log} \left[ 1 - e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a])} \right] \right) + \right. \right. \\
& \left. \left. \pi \operatorname{Log} [\operatorname{Cos} [b x]] - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \operatorname{Log} [\operatorname{Sin} [b x - \operatorname{ArcTan} [\operatorname{Cot} [a]]]] + i \operatorname{PolyLog} \left[ 2, e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a])} \right] \right) \right) \operatorname{Sec} [a] \right) / \\
& \left( 2 b^4 \sqrt{\operatorname{Csc} [a]^2 \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} \right) - \frac{3 \operatorname{Sec} [a] \operatorname{Sec} [a+b x] \left( c^2 d \operatorname{Sin} [b x] + 2 c d^2 x \operatorname{Sin} [b x] + d^3 x^2 \operatorname{Sin} [b x] \right)}{2 b^2} - \\
& \frac{1}{4} x \left( 4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3 \right) \operatorname{Tan} [a]
\end{aligned}$$

**Problem 305: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^2 \operatorname{Tan} [a+b x]^3 dx$$

Optimal (type 4, 169 leaves, 9 steps):



$$\frac{c d x}{b} + \frac{d^2 x^2}{2 b} - \frac{i (c + d x)^3}{3 d} + \frac{(c + d x)^2 \operatorname{Log}[1 + e^{2 i (a+b x)}]}{b} - \frac{d^2 \operatorname{Log}[\operatorname{Cos}[a + b x]]}{b^3} -$$

$$\frac{i d (c + d x) \operatorname{PolyLog}[2, -e^{2 i (a+b x)}]}{b^2} + \frac{d^2 \operatorname{PolyLog}[3, -e^{2 i (a+b x)}]}{2 b^3} - \frac{d (c + d x) \operatorname{Tan}[a + b x]}{b^2} + \frac{(c + d x)^2 \operatorname{Tan}[a + b x]^2}{2 b}$$

Result (type 4, 461 leaves):

$$-\frac{1}{12 b^3}$$

$$d^2 e^{-i a} (2 i b^2 x^2 (2 b e^{2 i a} x + 3 i (1 + e^{2 i a}) \operatorname{Log}[1 + e^{2 i (a+b x)}]) + 6 i b (1 + e^{2 i a}) x \operatorname{PolyLog}[2, -e^{2 i (a+b x)}] - 3 (1 + e^{2 i a}) \operatorname{PolyLog}[3, -e^{2 i (a+b x)}])$$

$$\operatorname{Sec}[a] + \frac{(c + d x)^2 \operatorname{Sec}[a + b x]^2}{2 b} + \frac{c^2 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x]] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} -$$

$$\frac{d^2 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x]] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \left( c d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right.$$

$$\left. \left. \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]) + \right. \right.$$

$$\left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}] \right) \operatorname{Sec}[a] \right) /$$

$$\left( b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) + \frac{\operatorname{Sec}[a] \operatorname{Sec}[a + b x] (-c d \operatorname{Sin}[b x] - d^2 x \operatorname{Sin}[b x])}{b^2} - \frac{1}{3} x (3 c^2 + 3 c d x + d^2 x^2) \operatorname{Tan}[a]$$

Problem 306: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Tan}[a + b x]^3 dx$$

Optimal (type 4, 108 leaves, 7 steps):

$$\frac{d x}{2 b} - \frac{i (c + d x)^2}{2 d} + \frac{(c + d x) \operatorname{Log}[1 + e^{2 i (a+b x)}]}{b} - \frac{i d \operatorname{PolyLog}[2, -e^{2 i (a+b x)}]}{2 b^2} - \frac{d \operatorname{Tan}[a + b x]}{2 b^2} + \frac{(c + d x) \operatorname{Tan}[a + b x]^2}{2 b}$$

Result (type 4, 242 leaves):

$$\frac{c \operatorname{Log}[\operatorname{Cos}[a + b x]]}{b} + \frac{c \operatorname{Sec}[a + b x]^2}{2 b} + \frac{d x \operatorname{Sec}[a + b x]^2}{2 b} +$$

$$\left( d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]] \right) \right)$$

$$\operatorname{Sec}[a] \Big/ \left( 2 b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) - \frac{d \operatorname{Sec}[a] \operatorname{Sec}[a + b x] \operatorname{Sin}[b x]}{2 b^2} - \frac{1}{2} d x^2 \operatorname{Tan}[a]$$

**Problem 310: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^4 \operatorname{Csc}[a + b x] \operatorname{Sec}[a + b x]^3 dx$$

Optimal (type 4, 399 leaves, 25 steps):

$$\frac{2 i d (c + d x)^3}{b^2} + \frac{(c + d x)^4}{2 b} - \frac{2 (c + d x)^4 \operatorname{ArcTanh}[e^{2 i (a + b x)}]}{b} - \frac{6 d^2 (c + d x)^2 \operatorname{Log}[1 + e^{2 i (a + b x)}]}{b^3} +$$

$$\frac{6 i d^3 (c + d x) \operatorname{PolyLog}[2, -e^{2 i (a + b x)}]}{b^4} + \frac{2 i d (c + d x)^3 \operatorname{PolyLog}[2, -e^{2 i (a + b x)}]}{b^2} -$$

$$\frac{2 i d (c + d x)^3 \operatorname{PolyLog}[2, e^{2 i (a + b x)}]}{b^2} - \frac{3 d^4 \operatorname{PolyLog}[3, -e^{2 i (a + b x)}]}{b^5} - \frac{3 d^2 (c + d x)^2 \operatorname{PolyLog}[3, -e^{2 i (a + b x)}]}{b^3} +$$

$$\frac{3 d^2 (c + d x)^2 \operatorname{PolyLog}[3, e^{2 i (a + b x)}]}{b^3} - \frac{3 i d^3 (c + d x) \operatorname{PolyLog}[4, -e^{2 i (a + b x)}]}{b^4} + \frac{3 i d^3 (c + d x) \operatorname{PolyLog}[4, e^{2 i (a + b x)}]}{b^4} +$$

$$\frac{3 d^4 \operatorname{PolyLog}[5, -e^{2 i (a + b x)}]}{2 b^5} - \frac{3 d^4 \operatorname{PolyLog}[5, e^{2 i (a + b x)}]}{2 b^5} - \frac{2 d (c + d x)^3 \operatorname{Tan}[a + b x]}{b^2} + \frac{(c + d x)^4 \operatorname{Tan}[a + b x]^2}{2 b}$$

Result (type 4, 1790 leaves):

$$-\frac{1}{2 b^3} c^2 d^2 e^{-i a} \operatorname{Csc}[a]$$

$$(2 b^2 x^2 (2 b e^{2 i a} x + 3 i (-1 + e^{2 i a}) \operatorname{Log}[1 - e^{2 i (a + b x)}]) + 6 b (-1 + e^{2 i a}) x \operatorname{PolyLog}[2, e^{2 i (a + b x)}] + 3 i (-1 + e^{2 i a}) \operatorname{PolyLog}[3, e^{2 i (a + b x)}]) -$$

$$c d^3 e^{i a} \operatorname{Csc}[a] \left( x^4 + (-1 + e^{-2 i a}) x^4 + \frac{1}{2 b^4} e^{-2 i a} (-1 + e^{2 i a}) \right.$$

$$\left. (2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log}[1 - e^{2 i (a + b x)}] + 6 b^2 x^2 \operatorname{PolyLog}[2, e^{2 i (a + b x)}] + 6 i b x \operatorname{PolyLog}[3, e^{2 i (a + b x)}] - 3 \operatorname{PolyLog}[4, e^{2 i (a + b x)}]) \right) -$$

$$\begin{aligned}
& \frac{1}{5} d^4 e^{i a} \operatorname{Csc}[a] \left( x^5 + (-1 + e^{-2 i a}) x^5 + \frac{1}{4 b^5} e^{-2 i a} (-1 + e^{2 i a}) (4 b^5 x^5 + 10 i b^4 x^4 \operatorname{Log}[1 - e^{2 i (a+b x)}] + 20 b^3 x^3 \operatorname{PolyLog}[2, e^{2 i (a+b x)}] + \right. \\
& \quad \left. 30 i b^2 x^2 \operatorname{PolyLog}[3, e^{2 i (a+b x)}] - 30 b x \operatorname{PolyLog}[4, e^{2 i (a+b x)}] - 15 i \operatorname{PolyLog}[5, e^{2 i (a+b x)}] \right) + \\
& \frac{1}{5} x (5 c^4 + 10 c^3 d x + 10 c^2 d^2 x^2 + 5 c d^3 x^3 + d^4 x^4) \operatorname{Csc}[a] \operatorname{Sec}[a] + \frac{1}{2 b^3} c^2 d^2 e^{-i a} \\
& \quad \left( 2 i b^2 x^2 (2 b e^{2 i a} x + 3 i (1 + e^{2 i a}) \operatorname{Log}[1 + e^{2 i (a+b x)}]) + 6 i b (1 + e^{2 i a}) x \operatorname{PolyLog}[2, -e^{2 i (a+b x)}] - 3 (1 + e^{2 i a}) \operatorname{PolyLog}[3, -e^{2 i (a+b x)}] \right) \\
& \operatorname{Sec}[a] + \frac{1}{2 b^5} \\
& d^4 e^{-i a} (2 i b^2 x^2 (2 b e^{2 i a} x + 3 i (1 + e^{2 i a}) \operatorname{Log}[1 + e^{2 i (a+b x)}]) + 6 i b (1 + e^{2 i a}) x \operatorname{PolyLog}[2, -e^{2 i (a+b x)}] - 3 (1 + e^{2 i a}) \operatorname{PolyLog}[3, -e^{2 i (a+b x)}]) \\
& \operatorname{Sec}[a] - i c d^3 e^{i a} \left( -x^4 + (1 + e^{-2 i a}) x^4 - \frac{1}{2 b^4} e^{-2 i a} (1 + e^{2 i a}) \right. \\
& \quad \left. (2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log}[1 + e^{2 i (a+b x)}] + 6 b^2 x^2 \operatorname{PolyLog}[2, -e^{2 i (a+b x)}] + 6 i b x \operatorname{PolyLog}[3, -e^{2 i (a+b x)}] - 3 \operatorname{PolyLog}[4, -e^{2 i (a+b x)}]) \right) \operatorname{Sec}[a] - \\
& \frac{1}{5} i d^4 e^{i a} \left( -x^5 + (1 + e^{-2 i a}) x^5 - \frac{1}{4 b^5} e^{-2 i a} (1 + e^{2 i a}) (4 b^5 x^5 + 10 i b^4 x^4 \operatorname{Log}[1 + e^{2 i (a+b x)}] + 20 b^3 x^3 \operatorname{PolyLog}[2, -e^{2 i (a+b x)}] + \right. \\
& \quad \left. 30 i b^2 x^2 \operatorname{PolyLog}[3, -e^{2 i (a+b x)}] - 30 b x \operatorname{PolyLog}[4, -e^{2 i (a+b x)}] - 15 i \operatorname{PolyLog}[5, -e^{2 i (a+b x)}] \right) \operatorname{Sec}[a] + \\
& \frac{(c + d x)^4 \operatorname{Sec}[a + b x]^2}{2 b} - \frac{c^4 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} - \\
& \frac{6 c^2 d^2 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{c^4 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} - \\
& \left( 2 c^3 d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right. \\
& \quad \left. \left. \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]) + \right. \right. \\
& \quad \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]) \right) \operatorname{Sec}[a] \right) / \\
& \left( b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) - \left( 6 c d^3 \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right. \\
& \quad \left. \left. \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]) + \right. \right. \\
& \quad \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}]) \right) \operatorname{Sec}[a] \right) /
\end{aligned}$$

$$\left( b^4 \sqrt{\text{Csc}[a]^2 (\text{Cos}[a]^2 + \text{Sin}[a]^2)} \right) - \frac{2 \text{Sec}[a] \text{Sec}[a + b x] (c^3 d \text{Sin}[b x] + 3 c^2 d^2 x \text{Sin}[b x] + 3 c d^3 x^2 \text{Sin}[b x] + d^4 x^3 \text{Sin}[b x])}{b^2} -$$

$$\left( 2 c^3 d \text{Csc}[a] \text{Sec}[a] \left( b^2 e^{i \text{ArcTan}[\text{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \text{Tan}[a]^2}} \right. \right.$$

$$\left. \left( i b x (-\pi + 2 \text{ArcTan}[\text{Tan}[a]]) - \pi \text{Log}[1 + e^{-2 i b x}] - 2 (b x + \text{ArcTan}[\text{Tan}[a]]) \text{Log}[1 - e^{2 i (b x + \text{ArcTan}[\text{Tan}[a])}] + \pi \text{Log}[\text{Cos}[b x]] + 2 \right. \right.$$

$$\left. \left. \text{ArcTan}[\text{Tan}[a]] \text{Log}[\text{Sin}[b x + \text{ArcTan}[\text{Tan}[a]]] + i \text{PolyLog}[2, e^{2 i (b x + \text{ArcTan}[\text{Tan}[a])}] \right] \text{Tan}[a] \right) \right) / \left( b^2 \sqrt{\text{Sec}[a]^2 (\text{Cos}[a]^2 + \text{Sin}[a]^2)} \right)$$

**Problem 311: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 \text{Csc}[a + b x] \text{Sec}[a + b x]^3 dx$$

Optimal (type 4, 325 leaves, 22 steps):

$$\frac{3 i d (c + d x)^2}{2 b^2} + \frac{(c + d x)^3}{2 b} - \frac{2 (c + d x)^3 \text{ArcTanh}[e^{2 i (a + b x)}]}{b} - \frac{3 d^2 (c + d x) \text{Log}[1 + e^{2 i (a + b x)}]}{b^3} +$$

$$\frac{3 i d^3 \text{PolyLog}[2, -e^{2 i (a + b x)}]}{2 b^4} + \frac{3 i d (c + d x)^2 \text{PolyLog}[2, -e^{2 i (a + b x)}]}{2 b^2} - \frac{3 i d (c + d x)^2 \text{PolyLog}[2, e^{2 i (a + b x)}]}{2 b^2} -$$

$$\frac{3 d^2 (c + d x) \text{PolyLog}[3, -e^{2 i (a + b x)}]}{2 b^3} + \frac{3 d^2 (c + d x) \text{PolyLog}[3, e^{2 i (a + b x)}]}{2 b^3} - \frac{3 i d^3 \text{PolyLog}[4, -e^{2 i (a + b x)}]}{4 b^4} +$$

$$\frac{3 i d^3 \text{PolyLog}[4, e^{2 i (a + b x)}]}{4 b^4} - \frac{3 d (c + d x)^2 \text{Tan}[a + b x]}{2 b^2} + \frac{(c + d x)^3 \text{Tan}[a + b x]^2}{2 b}$$

Result (type 4, 1294 leaves):

$$-\frac{1}{4 b^3} c d^2 e^{-i a} \text{Csc}[a]$$

$$\left( 2 b^2 x^2 (2 b e^{2 i a} x + 3 i (-1 + e^{2 i a}) \text{Log}[1 - e^{2 i (a + b x)}]) + 6 b (-1 + e^{2 i a}) x \text{PolyLog}[2, e^{2 i (a + b x)}] + 3 i (-1 + e^{2 i a}) \text{PolyLog}[3, e^{2 i (a + b x)}] \right) -$$

$$\frac{1}{4} d^3 e^{i a} \text{Csc}[a] \left( x^4 + (-1 + e^{-2 i a}) x^4 + \frac{1}{2 b^4} e^{-2 i a} (-1 + e^{2 i a}) \right.$$

$$\left. (2 b^4 x^4 + 4 i b^3 x^3 \text{Log}[1 - e^{2 i (a + b x)}] + 6 b^2 x^2 \text{PolyLog}[2, e^{2 i (a + b x)}] + 6 i b x \text{PolyLog}[3, e^{2 i (a + b x)}] - 3 \text{PolyLog}[4, e^{2 i (a + b x)}]) \right) +$$

$$\frac{1}{4} x (4 c^3 + 6 c^2 d x + 4 c d^2 x^2 + d^3 x^3) \text{Csc}[a] \text{Sec}[a] + \frac{1}{4 b^3} c d^2 e^{-i a}$$

$$(2 i b^2 x^2 (2 b e^{2 i a} x + 3 i (1 + e^{2 i a}) \text{Log}[1 + e^{2 i (a + b x)}]) + 6 i b (1 + e^{2 i a}) x \text{PolyLog}[2, -e^{2 i (a + b x)}] - 3 (1 + e^{2 i a}) \text{PolyLog}[3, -e^{2 i (a + b x)}])$$

$$\text{Sec}[a] - \frac{1}{4} i d^3 e^{i a} \left( -x^4 + (1 + e^{-2 i a}) x^4 - \frac{1}{2 b^4} e^{-2 i a} (1 + e^{2 i a}) \right.$$

$$\begin{aligned}
& \left( 2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log}\left[1 + e^{2 i (a+b x)}\right] + 6 b^2 x^2 \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right] + 6 i b x \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right] - 3 \operatorname{PolyLog}\left[4, -e^{2 i (a+b x)}\right] \right) \operatorname{Sec}[a] + \\
& \frac{(c+d x)^3 \operatorname{Sec}[a+b x]^2}{2 b} - \frac{c^3 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} - \\
& \frac{3 c d^2 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x] - \operatorname{Sin}[a] \operatorname{Sin}[b x]] + b x \operatorname{Sin}[a])}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\
& \frac{c^3 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} - \\
& \left( 3 c^2 d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right. \\
& \left. \left. \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}\left[1 + e^{-2 i b x}\right] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}\left[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) + \right. \\
& \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \right) \operatorname{Sec}[a] \Big/ \\
& \left( 2 b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) - \left( 3 d^3 \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \right. \right. \\
& \left. \left. \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}\left[1 + e^{-2 i b x}\right] - 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}\left[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) + \right. \\
& \left. \left. \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a])}\right] \right) \right) \operatorname{Sec}[a] \Big/ \\
& \left( 2 b^4 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) - \frac{3 \operatorname{Sec}[a] \operatorname{Sec}[a+b x] (c^2 d \operatorname{Sin}[b x] + 2 c d^2 x \operatorname{Sin}[b x] + d^3 x^2 \operatorname{Sin}[b x])}{2 b^2} - \\
& \left( 3 c^2 d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\
& \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}\left[1 + e^{-2 i b x}\right] - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}\left[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \operatorname{ArcTan}[\right. \\
& \left. \operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]] + i \operatorname{PolyLog}\left[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) \operatorname{Tan}[a] \right) \Big/ \left( 2 b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right)
\end{aligned}$$

**Problem 312: Result more than twice size of optimal antiderivative.**

$$\int (c+d x)^2 \operatorname{Csc}[a+b x] \operatorname{Sec}[a+b x]^3 dx$$

Optimal (type 4, 201 leaves, 17 steps):

$$\frac{c d x}{b} + \frac{d^2 x^2}{2 b} - \frac{2 (c + d x)^2 \operatorname{ArcTanh}\left[e^{2 i (a+b x)}\right]}{b} - \frac{d^2 \operatorname{Log}[\operatorname{Cos}[a + b x]]}{b^3} + \frac{i d (c + d x) \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right]}{b^2} - \frac{i d (c + d x) \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right]}{b^2} - \frac{d^2 \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right]}{2 b^3} + \frac{d^2 \operatorname{PolyLog}\left[3, e^{2 i (a+b x)}\right]}{2 b^3} - \frac{d (c + d x) \operatorname{Tan}[a + b x]}{b^2} + \frac{(c + d x)^2 \operatorname{Tan}[a + b x]^2}{2 b}$$

Result (type 4, 788 leaves):

$$\begin{aligned} & -\frac{1}{12 b^3} d^2 e^{-i a} \operatorname{Csc}[a] \\ & \left(2 b^2 x^2 \left(2 b e^{2 i a} x + 3 i (-1 + e^{2 i a}) \operatorname{Log}\left[1 - e^{2 i (a+b x)}\right]\right) + 6 b (-1 + e^{2 i a}) x \operatorname{PolyLog}\left[2, e^{2 i (a+b x)}\right] + 3 i (-1 + e^{2 i a}) \operatorname{PolyLog}\left[3, e^{2 i (a+b x)}\right]\right) + \\ & \frac{1}{3} x \left(3 c^2 + 3 c d x + d^2 x^2\right) \operatorname{Csc}[a] \operatorname{Sec}[a] + \frac{1}{12 b^3} \\ & d^2 e^{-i a} \left(2 i b^2 x^2 \left(2 b e^{2 i a} x + 3 i (1 + e^{2 i a}) \operatorname{Log}\left[1 + e^{2 i (a+b x)}\right]\right) + 6 i b (1 + e^{2 i a}) x \operatorname{PolyLog}\left[2, -e^{2 i (a+b x)}\right] - 3 (1 + e^{2 i a}) \operatorname{PolyLog}\left[3, -e^{2 i (a+b x)}\right]\right) \\ & \operatorname{Sec}[a] + \frac{(c + d x)^2 \operatorname{Sec}[a + b x]^2}{2 b} - \frac{c^2 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x]] - \operatorname{Sin}[a] \operatorname{Sin}[b x]) + b x \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} - \\ & \frac{d^2 \operatorname{Sec}[a] (\operatorname{Cos}[a] \operatorname{Log}[\operatorname{Cos}[a] \operatorname{Cos}[b x]] - \operatorname{Sin}[a] \operatorname{Sin}[b x]) + b x \operatorname{Sin}[a]}{b^3 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} + \\ & \frac{c^2 \operatorname{Csc}[a] (-b x \operatorname{Cos}[a] + \operatorname{Log}[\operatorname{Cos}[b x] \operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Sin}[b x]]) \operatorname{Sin}[a]}{b (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} - \\ & \left( c d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}]) - \right. \right. \\ & \left. \left. 2 (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}\left[1 - e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]]\right] + \right. \\ & \left. i \operatorname{PolyLog}\left[2, e^{2 i (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}\right] \right) \operatorname{Sec}[a] \right) / \left( b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) + \\ & \frac{\operatorname{Sec}[a] \operatorname{Sec}[a + b x] (-c d \operatorname{Sin}[b x] - d^2 x \operatorname{Sin}[b x])}{b^2} - \left( c d \operatorname{Csc}[a] \operatorname{Sec}[a] \left( b^2 e^{i \operatorname{ArcTan}[\operatorname{Tan}[a]]} x^2 + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \right. \\ & \left. \left. (i b x (-\pi + 2 \operatorname{ArcTan}[\operatorname{Tan}[a]]) - \pi \operatorname{Log}[1 + e^{-2 i b x}]) - 2 (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \operatorname{Log}\left[1 - e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])}\right] + \pi \operatorname{Log}[\operatorname{Cos}[b x]] + 2 \right. \right. \\ & \left. \left. \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{Log}[\operatorname{Sin}[b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]]\right] + i \operatorname{PolyLog}\left[2, e^{2 i (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]])}\right] \right) \operatorname{Tan}[a] \right) / \left( b^2 \sqrt{\operatorname{Sec}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) \end{aligned}$$

### Problem 318: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^2 \operatorname{Csc}[a + bx]^2 \operatorname{Sec}[a + bx]^3 dx$$

Optimal (type 4, 341 leaves, 31 steps):

$$\begin{aligned} & -\frac{3i(c+dx)^2 \operatorname{ArcTan}[e^{i(a+bx)}]}{b} + \frac{2d^2 x \operatorname{ArcTanh}[e^{i(a+bx)}]}{b^2} - \frac{6d(c+dx) \operatorname{ArcTanh}[e^{i(a+bx)}]}{b^2} - \frac{d^2 x \operatorname{ArcTanh}[\operatorname{Cos}[a+bx]]}{b^2} + \\ & \frac{d(c+dx) \operatorname{ArcTanh}[\operatorname{Cos}[a+bx]]}{b^2} + \frac{d^2 \operatorname{ArcTanh}[\operatorname{Sin}[a+bx]]}{b^3} - \frac{3(c+dx)^2 \operatorname{Csc}[a+bx]}{2b} + \frac{2id^2 \operatorname{PolyLog}[2, -e^{i(a+bx)}]}{b^3} + \\ & \frac{3id(c+dx) \operatorname{PolyLog}[2, -ie^{i(a+bx)}]}{b^2} - \frac{3id(c+dx) \operatorname{PolyLog}[2, ie^{i(a+bx)}]}{b^2} - \frac{2id^2 \operatorname{PolyLog}[2, e^{i(a+bx)}]}{b^3} - \\ & \frac{3d^2 \operatorname{PolyLog}[3, -ie^{i(a+bx)}]}{b^3} + \frac{3d^2 \operatorname{PolyLog}[3, ie^{i(a+bx)}]}{b^3} - \frac{d(c+dx) \operatorname{Sec}[a+bx]}{b^2} + \frac{(c+dx)^2 \operatorname{Csc}[a+bx] \operatorname{Sec}[a+bx]^2}{2b} \end{aligned}$$

Result (type 4, 889 leaves):

$$\begin{aligned}
& -\frac{1}{2b^3} \left( 6i b^2 c^2 \operatorname{ArcTan}\left[e^{i(a+bx)}\right] + 4i d^2 \operatorname{ArcTan}\left[e^{i(a+bx)}\right] - 6b^2 c d x \operatorname{Log}\left[1 - i e^{i(a+bx)}\right] - 3b^2 d^2 x^2 \operatorname{Log}\left[1 - i e^{i(a+bx)}\right] + \right. \\
& \quad \left. 6b^2 c d x \operatorname{Log}\left[1 + i e^{i(a+bx)}\right] + 3b^2 d^2 x^2 \operatorname{Log}\left[1 + i e^{i(a+bx)}\right] - 6i b d (c + d x) \operatorname{PolyLog}\left[2, -i e^{i(a+bx)}\right] + \right. \\
& \quad \left. 6i b d (c + d x) \operatorname{PolyLog}\left[2, i e^{i(a+bx)}\right] + 6d^2 \operatorname{PolyLog}\left[3, -i e^{i(a+bx)}\right] - 6d^2 \operatorname{PolyLog}\left[3, i e^{i(a+bx)}\right] \right) - \\
& \quad \frac{(c + d x) \operatorname{Csc}[a] \operatorname{Sec}[a] (b c \operatorname{Cos}[a] + b d x \operatorname{Cos}[a] + d \operatorname{Sin}[a])}{b^2} + \frac{4i c d \operatorname{ArcTan}\left[\frac{i \operatorname{Cos}[a] - i \operatorname{Sin}[a] \operatorname{Tan}\left[\frac{bx}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{b^2 \sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} + \\
& \quad \frac{\operatorname{Sec}\left[\frac{a}{2}\right] \operatorname{Sec}\left[\frac{a}{2} + \frac{bx}{2}\right] \left(-c^2 \operatorname{Sin}\left[\frac{bx}{2}\right] - 2c d x \operatorname{Sin}\left[\frac{bx}{2}\right] - d^2 x^2 \operatorname{Sin}\left[\frac{bx}{2}\right]\right)}{2b} + \\
& \quad \frac{\operatorname{Csc}\left[\frac{a}{2}\right] \operatorname{Csc}\left[\frac{a}{2} + \frac{bx}{2}\right] \left(c^2 \operatorname{Sin}\left[\frac{bx}{2}\right] + 2c d x \operatorname{Sin}\left[\frac{bx}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{bx}{2}\right]\right)}{2b} + \\
& \quad \frac{c^2 + 2c d x + d^2 x^2}{4b \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)^2} + \frac{-c d \operatorname{Sin}\left[\frac{bx}{2}\right] - d^2 x \operatorname{Sin}\left[\frac{bx}{2}\right]}{b^2 \left(\operatorname{Cos}\left[\frac{a}{2}\right] - \operatorname{Sin}\left[\frac{a}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)} + \\
& \quad \frac{-c^2 - 2c d x - d^2 x^2}{4b \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)^2} + \frac{c d \operatorname{Sin}\left[\frac{bx}{2}\right] + d^2 x \operatorname{Sin}\left[\frac{bx}{2}\right]}{b^2 \left(\operatorname{Cos}\left[\frac{a}{2}\right] + \operatorname{Sin}\left[\frac{a}{2}\right]\right) \left(\operatorname{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right)} + \\
& \quad \frac{1}{b^3} 2 d^2 \left( -\frac{2 \operatorname{ArcTan}[\operatorname{Tan}[a]] \operatorname{ArcTanh}\left[\frac{-\operatorname{Cos}[a] + \operatorname{Sin}[a] \operatorname{Tan}\left[\frac{bx}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} + \frac{1}{\sqrt{1 + \operatorname{Tan}[a]^2}} \right. \\
& \quad \left. \left( (b x + \operatorname{ArcTan}[\operatorname{Tan}[a]]) \left( \operatorname{Log}\left[1 - e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] - \operatorname{Log}\left[1 + e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) \right) + \right. \\
& \quad \left. \left. i \left( \operatorname{PolyLog}\left[2, -e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] - \operatorname{PolyLog}\left[2, e^{i(b x + \operatorname{ArcTan}[\operatorname{Tan}[a])}\right] \right) \right) \operatorname{Sec}[a] \right)
\end{aligned}$$

**Problem 319: Result more than twice size of optimal antiderivative.**

$$\int (c + d x) \operatorname{Csc}[a + b x]^2 \operatorname{Sec}[a + b x]^3 dx$$

Optimal (type 4, 162 leaves, 13 steps):



$$\begin{aligned}
& - \frac{3 i d x \operatorname{ArcTan}\left[e^{i(a+b x)}\right]}{b} - \frac{d \operatorname{ArcTanh}[\operatorname{Cos}[a+b x]]}{b^2} + \frac{3 c \operatorname{ArcTanh}[\operatorname{Sin}[a+b x]]}{2 b} - \frac{3(c+d x) \operatorname{Csc}[a+b x]}{2 b} + \\
& \frac{3 i d \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right]}{2 b^2} - \frac{3 i d \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right]}{2 b^2} - \frac{d \operatorname{Sec}[a+b x]}{2 b^2} + \frac{(c+d x) \operatorname{Csc}[a+b x] \operatorname{Sec}[a+b x]^2}{2 b}
\end{aligned}$$

Result (type 4, 772 leaves):

$$\begin{aligned}
& - \frac{c \operatorname{Cot}\left[\frac{1}{2}(a+bx)\right]}{2b} + \frac{d\left(a \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - (a+bx) \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right) \operatorname{Csc}\left[\frac{1}{2}(a+bx)\right]}{2b^2} - \frac{d \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right]}{b^2} - \\
& \frac{3c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \frac{d \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{b^2} + \frac{3c \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right]}{2b} + \\
& \frac{1}{2b^2} 3d\left(a\left(\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] - \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\right) + (a+bx)\left(-\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] + \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\right) - \\
& i\left(\operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right)\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)\right] - \operatorname{Log}\left[\frac{1}{2}\left((1+i) - (1-i)\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] + \right. \\
& \left. \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right)\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)\right] \operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] - \operatorname{Log}\left[1 - \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Log}\left[\frac{1}{2}\left((1+i) + (1-i)\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)\right]\right) + \\
& \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} - \frac{i}{2}\right)\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)\right] - \operatorname{PolyLog}\left[2, \left(-\frac{1}{2} + \frac{i}{2}\right)\left(-1 + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)\right] - \\
& \operatorname{PolyLog}\left[2, \left(\frac{1}{2} - \frac{i}{2}\right)\left(1 + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)\right] + \operatorname{PolyLog}\left[2, \left(\frac{1}{2} + \frac{i}{2}\right)\left(1 + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)\right] \Big) + \\
& \frac{c}{4b\left(\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^2} + \frac{dx}{4b\left(\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^2} - \\
& \frac{d \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]}{2b^2\left(\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] - \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
& \frac{c}{4b\left(\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^2} - \\
& \frac{dx}{4b\left(\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)^2} + \\
& \frac{d \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]}{2b^2\left(\operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] + \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
& \frac{d \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]\left(a \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right] - (a+bx) \operatorname{Sin}\left[\frac{1}{2}(a+bx)\right]\right)}{2b^2} - \\
& \frac{c \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{2b}
\end{aligned}$$

**Problem 324:** Result more than twice size of optimal antiderivative.

$$\int (c+dx)^2 \operatorname{Csc}[a+bx]^3 \operatorname{Sec}[a+bx]^3 dx$$

Optimal (type 4, 190 leaves, 10 steps):

$$\begin{aligned} & -\frac{4(c+dx)^2 \operatorname{ArcTanh}\left[e^{2i(a+bx)}\right]}{b} - \frac{d^2 \operatorname{ArcTanh}\left[\cos[2a+2bx]\right]}{b^3} - \frac{2d(c+dx) \operatorname{Csc}[2a+2bx]}{b^2} - \frac{2(c+dx)^2 \operatorname{Cot}[2a+2bx] \operatorname{Csc}[2a+2bx]}{b} + \\ & \frac{2id(c+dx) \operatorname{PolyLog}\left[2, -e^{2i(a+bx)}\right]}{b^2} - \frac{2id(c+dx) \operatorname{PolyLog}\left[2, e^{2i(a+bx)}\right]}{b^2} - \frac{d^2 \operatorname{PolyLog}\left[3, -e^{2i(a+bx)}\right]}{b^3} + \frac{d^2 \operatorname{PolyLog}\left[3, e^{2i(a+bx)}\right]}{b^3} \end{aligned}$$

Result (type 4, 429 leaves):

$$\begin{aligned} & 8 \left( -\frac{d(c+dx) \operatorname{Csc}[2a]}{4b^2} + \frac{(-c^2 - 2cdx - d^2x^2) \operatorname{Csc}[a+bx]^2}{16b} + \right. \\ & \frac{1}{8b^3} \left( 2b^2c^2 \operatorname{Log}\left[1 - e^{2i(a+bx)}\right] + d^2 \operatorname{Log}\left[1 - e^{2i(a+bx)}\right] + 4b^2cdx \operatorname{Log}\left[1 - e^{2i(a+bx)}\right] + 2b^2d^2x^2 \operatorname{Log}\left[1 - e^{2i(a+bx)}\right] - \right. \\ & \left. 2b^2c^2 \operatorname{Log}\left[1 + e^{2i(a+bx)}\right] - d^2 \operatorname{Log}\left[1 + e^{2i(a+bx)}\right] - 4b^2cdx \operatorname{Log}\left[1 + e^{2i(a+bx)}\right] - 2b^2d^2x^2 \operatorname{Log}\left[1 + e^{2i(a+bx)}\right] + \right. \\ & \left. 2idb d(c+dx) \operatorname{PolyLog}\left[2, -e^{2i(a+bx)}\right] - 2idb d(c+dx) \operatorname{PolyLog}\left[2, e^{2i(a+bx)}\right] - d^2 \operatorname{PolyLog}\left[3, -e^{2i(a+bx)}\right] + d^2 \operatorname{PolyLog}\left[3, e^{2i(a+bx)}\right] \right) + \\ & \left. \frac{(c^2 + 2cdx + d^2x^2) \operatorname{Sec}[a+bx]^2}{16b} + \frac{\operatorname{Sec}[a] \operatorname{Sec}[a+bx] (-cd \operatorname{Sin}[bx] - d^2x \operatorname{Sin}[bx])}{8b^2} + \frac{\operatorname{Csc}[a] \operatorname{Csc}[a+bx] (cd \operatorname{Sin}[bx] + d^2x \operatorname{Sin}[bx])}{8b^2} \right) \end{aligned}$$

**Problem 325: Result more than twice size of optimal antiderivative.**

$$\int (c+dx) \operatorname{Csc}[a+bx]^3 \operatorname{Sec}[a+bx]^3 dx$$

Optimal (type 4, 110 leaves, 7 steps):

$$\begin{aligned} & -\frac{4(c+dx) \operatorname{ArcTanh}\left[e^{2i(a+bx)}\right]}{b} - \frac{d \operatorname{Csc}[2a+2bx]}{b^2} - \\ & \frac{2(c+dx) \operatorname{Cot}[2a+2bx] \operatorname{Csc}[2a+2bx]}{b} + \frac{id \operatorname{PolyLog}\left[2, -e^{2i(a+bx)}\right]}{b^2} - \frac{id \operatorname{PolyLog}\left[2, e^{2i(a+bx)}\right]}{b^2} \end{aligned}$$

Result (type 4, 236 leaves):

$$\begin{aligned} & -\frac{d \operatorname{Cot}[a+bx]}{2b^2} - \frac{c \operatorname{Csc}[a+bx]^2}{2b} + \frac{d(2a - 2(a+bx)) \operatorname{Csc}[a+bx]^2}{4b^2} - \frac{2c \operatorname{Log}[\cos[a+bx]]}{b} + \frac{2c \operatorname{Log}[\sin[a+bx]]}{b} - \frac{2ad \operatorname{Log}[\tan[a+bx]]}{b^2} + \\ & \frac{1}{b^2} d(2(a+bx) (\operatorname{Log}[1 - e^{2i(a+bx)}] - \operatorname{Log}[1 + e^{2i(a+bx)}]) + id (\operatorname{PolyLog}[2, -e^{2i(a+bx)}] - \operatorname{PolyLog}[2, e^{2i(a+bx)}])) + \\ & \frac{c \operatorname{Sec}[a+bx]^2}{2b} + \frac{d(-2a + 2(a+bx)) \operatorname{Sec}[a+bx]^2}{4b^2} - \frac{d \operatorname{Tan}[a+bx]}{2b^2} \end{aligned}$$

### Problem 331: Result more than twice size of optimal antiderivative.

$$\int \frac{x \sin[a + bx]}{\sqrt{\cos[a + bx]}} dx$$

Optimal (type 4, 33 leaves, 2 steps):

$$-\frac{2x\sqrt{\cos[a + bx]}}{b} + \frac{4\text{EllipticE}\left[\frac{1}{2}(a + bx), 2\right]}{b^2}$$

Result (type 4, 181 leaves):

$$\frac{1}{b^2 \sqrt{\frac{\cos[a + bx]}{1 + \cos[a + bx]}}} + 4 \left( \cos\left[\frac{1}{2}(a + bx)\right]^2 \right)^{3/2} \sqrt{\frac{\cos[a + bx]}{(1 + \cos[a + bx])^2}} \sqrt{\frac{1}{1 + \cos[a + bx]}} \left( 2\text{EllipticE}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(a + bx)\right]\right], -1\right] \sqrt{\sec\left[\frac{1}{2}(a + bx)\right]^2} - 2\text{EllipticF}\left[\text{ArcSin}\left[\text{Tan}\left[\frac{1}{2}(a + bx)\right]\right], -1\right] \sqrt{\sec\left[\frac{1}{2}(a + bx)\right]^2} + \sqrt{\cos[a + bx] \sec\left[\frac{1}{2}(a + bx)\right]^2} \left(-bx + 2\text{Tan}\left[\frac{1}{2}(a + bx)\right]\right) \right)$$

### Problem 340: Result more than twice size of optimal antiderivative.

$$\int x \sqrt{\sec[a + bx]} \sin[a + bx] dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$-\frac{2x}{b\sqrt{\sec[a + bx]}} + \frac{4\sqrt{\cos[a + bx]}\text{EllipticE}\left[\frac{1}{2}(a + bx), 2\right]\sqrt{\sec[a + bx]}}{b^2}$$

Result (type 4, 132 leaves):

$$\frac{1}{b^2 \sqrt{\sec[a + bx]}} 2 \left( -bx + \frac{2 \operatorname{EllipticE}[\operatorname{ArcSin}[\tan[\frac{1}{2}(a + bx)]]], -1] \sec[\frac{1}{2}(a + bx)]^2}{\sqrt{\cos[a + bx] \sec[\frac{1}{2}(a + bx)]^4}} - \frac{2 \operatorname{EllipticF}[\operatorname{ArcSin}[\tan[\frac{1}{2}(a + bx)]]], -1] \sec[\frac{1}{2}(a + bx)]^2}{\sqrt{\cos[a + bx] \sec[\frac{1}{2}(a + bx)]^4}} + 2 \tan[\frac{1}{2}(a + bx)] \right)$$

**Problem 342: Result more than twice size of optimal antiderivative.**

$$\int \frac{x \sin[a + bx]}{\sec[a + bx]^{3/2}} dx$$

Optimal (type 4, 80 leaves, 4 steps):

$$-\frac{2x}{5b \sec[a + bx]^{5/2}} + \frac{12 \sqrt{\cos[a + bx]} \operatorname{EllipticE}[\frac{1}{2}(a + bx), 2] \sqrt{\sec[a + bx]}}{25b^2} + \frac{4 \sin[a + bx]}{25b^2 \sec[a + bx]^{3/2}}$$

Result (type 4, 212 leaves):

$$\frac{\sqrt{\sec[a + bx]} \left( -\frac{1}{10} x \cos[a + bx] - \frac{1}{10} x \cos[3(a + bx)] + \frac{\sin[a + bx]}{25b} + \frac{\sin[3(a + bx)]}{25b} \right)}{b} + \frac{1}{25b^2} \cos[\frac{1}{2}(a + bx)]^2 \sqrt{\sec[a + bx]}$$

$$\left( 12 \operatorname{EllipticE}[\operatorname{ArcSin}[\tan[\frac{1}{2}(a + bx)]]], -1 \right) \sqrt{\cos[a + bx] \sec[\frac{1}{2}(a + bx)]^4} - 12 \operatorname{EllipticF}[\operatorname{ArcSin}[\tan[\frac{1}{2}(a + bx)]]], -1$$

$$\sqrt{\cos[a + bx] \sec[\frac{1}{2}(a + bx)]^4} + \left( -5a + 5(a + bx) - 12 \tan[\frac{1}{2}(a + bx)] \right) \left( -1 + \tan[\frac{1}{2}(a + bx)]^2 \right)$$

**Problem 345: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int x \cos[a + bx] \sin[a + bx]^{3/2} dx$$

Optimal (type 4, 65 leaves, 3 steps):

$$-\frac{12 \operatorname{EllipticE}\left[\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right), 2\right]}{25 b^2} + \frac{4 \cos[a + bx] \sin[a + bx]^{3/2}}{25 b^2} + \frac{2 x \sin[a + bx]^{5/2}}{5 b}$$

Result (type 4, 186 leaves):

$$-\frac{1}{25 b^2 \sqrt{\tan\left[\frac{1}{2}(a + bx)\right]}} \sqrt{\sin[a + bx]} \left( 12 (-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan\left[\frac{1}{2}(a + bx)\right]}\right], -1\right] \sqrt{\sec\left[\frac{1}{2}(a + bx)\right]^2} - \right. \\ \left. 12 (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan\left[\frac{1}{2}(a + bx)\right]}\right], -1\right] \sqrt{\sec\left[\frac{1}{2}(a + bx)\right]^2} + \right. \\ \left. \sqrt{\tan\left[\frac{1}{2}(a + bx)\right]} \left( -5 b x + 5 b x \cos[2(a + bx)] - 2 \sin[2(a + bx)] + 12 \tan\left[\frac{1}{2}(a + bx)\right] \right) \right)$$

**Problem 347:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x \cos[a + bx]}{\sqrt{\sin[a + bx]}} dx$$

Optimal (type 4, 38 leaves, 2 steps):

$$-\frac{4 \operatorname{EllipticE}\left[\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right), 2\right]}{b^2} + \frac{2 x \sqrt{\sin[a + bx]}}{b}$$

Result (type 4, 162 leaves):

$$-\frac{1}{b^2 \sqrt{\tan\left[\frac{1}{2}(a + bx)\right]}} 2 \sqrt{\sin[a + bx]} \left( 2 (-1)^{3/4} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan\left[\frac{1}{2}(a + bx)\right]}\right], -1\right] \sqrt{\sec\left[\frac{1}{2}(a + bx)\right]^2} - \right. \\ \left. 2 (-1)^{3/4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{\tan\left[\frac{1}{2}(a + bx)\right]}\right], -1\right] \sqrt{\sec\left[\frac{1}{2}(a + bx)\right]^2} + \sqrt{\tan\left[\frac{1}{2}(a + bx)\right]} \left( -b x + 2 \tan\left[\frac{1}{2}(a + bx)\right] \right) \right)$$

**Problem 356:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x \cos[a + bx] \sqrt{\csc[a + bx]} dx$$

Optimal (type 4, 58 leaves, 3 steps):

$$\frac{2x}{b\sqrt{\text{Csc}[a+bx]}} - \frac{4\sqrt{\text{Csc}[a+bx]}\text{EllipticE}\left[\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right), 2\right]\sqrt{\text{Sin}[a+bx]}}{b^2}$$

Result (type 4, 161 leaves):

$$\frac{1}{b^2\sqrt{\text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}} - 4\sqrt{\text{Csc}[a+bx]} \left( -2(-1)^{3/4}\text{EllipticE}\left[i\text{ArcSinh}\left[(-1)^{1/4}\sqrt{\text{Tan}\left[\frac{1}{2}(a+bx)\right]}\right], -1\right] + \right. \\ \left. 2(-1)^{3/4}\text{EllipticF}\left[i\text{ArcSinh}\left[(-1)^{1/4}\sqrt{\text{Tan}\left[\frac{1}{2}(a+bx)\right]}\right], -1\right] + \frac{(bx - 2\text{Tan}\left[\frac{1}{2}(a+bx)\right])\sqrt{\text{Tan}\left[\frac{1}{2}(a+bx)\right]}}{\sqrt{\text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}} \right) \sqrt{\text{Tan}\left[\frac{1}{2}(a+bx)\right]}$$

**Problem 358: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{x \text{Cos}[a+bx]}{\text{Csc}[a+bx]^{3/2}} dx$$

Optimal (type 4, 85 leaves, 4 steps):

$$\frac{2x}{5b\text{Csc}[a+bx]^{5/2}} + \frac{4\text{Cos}[a+bx]}{25b^2\text{Csc}[a+bx]^{3/2}} - \frac{12\sqrt{\text{Csc}[a+bx]}\text{EllipticE}\left[\frac{1}{2}\left(a - \frac{\pi}{2} + bx\right), 2\right]\sqrt{\text{Sin}[a+bx]}}{25b^2}$$

Result (type 4, 190 leaves):

$$\frac{1}{25 b^2 \sqrt{\text{Csc}[a + b x]}}$$

$$\left( 5 b x - 5 b x \text{Cos}[2(a + b x)] + 2 \text{Sin}[2(a + b x)] - \frac{12 (-1)^{3/4} \sqrt{2} \sqrt{\frac{1}{1 + \text{Cos}[a + b x]}} \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{\text{Tan}\left[\frac{1}{2}(a + b x)\right]}\right], -1\right]}{\sqrt{\text{Tan}\left[\frac{1}{2}(a + b x)\right]}} + \frac{12 (-1)^{3/4} \sqrt{2} \sqrt{\frac{1}{1 + \text{Cos}[a + b x]}} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{\text{Tan}\left[\frac{1}{2}(a + b x)\right]}\right], -1\right]}{\sqrt{\text{Tan}\left[\frac{1}{2}(a + b x)\right]}} - 12 \text{Tan}\left[\frac{1}{2}(a + b x)\right] \right)$$

**Problem 376: Result more than twice size of optimal antiderivative.**

$$\int (c + d x)^3 \text{Csc}[a + b x]^2 \text{Sin}[3 a + 3 b x] dx$$

Optimal (type 4, 255 leaves, 20 steps):

$$\begin{aligned} & - \frac{6 (c + d x)^3 \text{ArcTanh}\left[e^{i(a + b x)}\right]}{b} - \frac{24 d^2 (c + d x) \text{Cos}[a + b x]}{b^3} + \frac{4 (c + d x)^3 \text{Cos}[a + b x]}{b} + \frac{9 i d (c + d x)^2 \text{PolyLog}\left[2, -e^{i(a + b x)}\right]}{b^2} \\ & - \frac{9 i d (c + d x)^2 \text{PolyLog}\left[2, e^{i(a + b x)}\right]}{b^2} - \frac{18 d^2 (c + d x) \text{PolyLog}\left[3, -e^{i(a + b x)}\right]}{b^3} + \frac{18 d^2 (c + d x) \text{PolyLog}\left[3, e^{i(a + b x)}\right]}{b^3} \\ & - \frac{18 i d^3 \text{PolyLog}\left[4, -e^{i(a + b x)}\right]}{b^4} + \frac{18 i d^3 \text{PolyLog}\left[4, e^{i(a + b x)}\right]}{b^4} + \frac{24 d^3 \text{Sin}[a + b x]}{b^4} - \frac{12 d (c + d x)^2 \text{Sin}[a + b x]}{b^2} \end{aligned}$$

Result (type 4, 515 leaves):

$$\begin{aligned} & \frac{1}{b^4} \left( -6 b^3 c^3 \text{ArcTanh}\left[e^{i(a + b x)}\right] + 4 b^3 c^3 \text{Cos}[a + b x] - 24 b c d^2 \text{Cos}[a + b x] + 12 b^3 c^2 d x \text{Cos}[a + b x] - \right. \\ & 24 b d^3 x \text{Cos}[a + b x] + 12 b^3 c d^2 x^2 \text{Cos}[a + b x] + 4 b^3 d^3 x^3 \text{Cos}[a + b x] + 9 b^3 c^2 d x \text{Log}\left[1 - e^{i(a + b x)}\right] + 9 b^3 c d^2 x^2 \text{Log}\left[1 - e^{i(a + b x)}\right] + \\ & 3 b^3 d^3 x^3 \text{Log}\left[1 - e^{i(a + b x)}\right] - 9 b^3 c^2 d x \text{Log}\left[1 + e^{i(a + b x)}\right] - 9 b^3 c d^2 x^2 \text{Log}\left[1 + e^{i(a + b x)}\right] - 3 b^3 d^3 x^3 \text{Log}\left[1 + e^{i(a + b x)}\right] + \\ & 9 i b^2 d (c + d x)^2 \text{PolyLog}\left[2, -e^{i(a + b x)}\right] - 9 i b^2 d (c + d x)^2 \text{PolyLog}\left[2, e^{i(a + b x)}\right] - 18 b c d^2 \text{PolyLog}\left[3, -e^{i(a + b x)}\right] - \\ & 18 b d^3 x \text{PolyLog}\left[3, -e^{i(a + b x)}\right] + 18 b c d^2 \text{PolyLog}\left[3, e^{i(a + b x)}\right] + 18 b d^3 x \text{PolyLog}\left[3, e^{i(a + b x)}\right] - 18 i d^3 \text{PolyLog}\left[4, -e^{i(a + b x)}\right] + \\ & \left. 18 i d^3 \text{PolyLog}\left[4, e^{i(a + b x)}\right] - 12 b^2 c^2 d \text{Sin}[a + b x] + 24 d^3 \text{Sin}[a + b x] - 24 b^2 c d^2 x \text{Sin}[a + b x] - 12 b^2 d^3 x^2 \text{Sin}[a + b x] \right) \end{aligned}$$



### Problem 382: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^4 \operatorname{Sec}[a + bx] \operatorname{Sin}[3a + 3bx] dx$$

Optimal (type 4, 299 leaves, 20 steps):

$$\begin{aligned} & \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{id(c+dx)^5}{5d} + \frac{(c+dx)^4 \operatorname{Log}[1 + e^{2i(a+bx)}]}{b} - \\ & \frac{2id(c+dx)^3 \operatorname{PolyLog}[2, -e^{2i(a+bx)}]}{b^2} + \frac{3d^2(c+dx)^2 \operatorname{PolyLog}[3, -e^{2i(a+bx)}]}{b^3} + \\ & \frac{3id^3(c+dx) \operatorname{PolyLog}[4, -e^{2i(a+bx)}]}{b^4} - \frac{3d^4 \operatorname{PolyLog}[5, -e^{2i(a+bx)}]}{2b^5} - \frac{6d^3(c+dx) \operatorname{Cos}[a+bx] \operatorname{Sin}[a+bx]}{b^4} + \\ & \frac{4d(c+dx)^3 \operatorname{Cos}[a+bx] \operatorname{Sin}[a+bx]}{b^2} + \frac{3d^4 \operatorname{Sin}[a+bx]^2}{b^5} - \frac{6d^2(c+dx)^2 \operatorname{Sin}[a+bx]^2}{b^3} + \frac{2(c+dx)^4 \operatorname{Sin}[a+bx]^2}{b} \end{aligned}$$

Result (type 4, 2517 leaves):

$$\begin{aligned}
& -\frac{1}{2b^3}c^2d^2e^{-ia} \\
& \left( 2ib^2x^2 \left( 2be^{2ia}x + 3i(1+e^{2ia})\operatorname{Log}[1+e^{2i(a+bx)}] \right) + 6ib(1+e^{2ia})x \operatorname{PolyLog}[2, -e^{2i(a+bx)}] - 3(1+e^{2ia})\operatorname{PolyLog}[3, -e^{2i(a+bx)}] \right) \\
& \operatorname{Sec}[a] + icd^3e^{ia} \left( -x^4 + (1+e^{-2ia})x^4 - \frac{1}{2b^4}e^{-2ia}(1+e^{2ia}) \right. \\
& \left. (2b^4x^4 + 4ib^3x^3\operatorname{Log}[1+e^{2i(a+bx)}] + 6b^2x^2\operatorname{PolyLog}[2, -e^{2i(a+bx)}] + 6ibx\operatorname{PolyLog}[3, -e^{2i(a+bx)}] - 3\operatorname{PolyLog}[4, -e^{2i(a+bx)}]) \right) \operatorname{Sec}[a] + \\
& \frac{1}{5}id^4e^{ia} \left( -x^5 + (1+e^{-2ia})x^5 - \frac{1}{4b^5}e^{-2ia}(1+e^{2ia}) \left( 4b^5x^5 + 10ib^4x^4\operatorname{Log}[1+e^{2i(a+bx)}] + 20b^3x^3\operatorname{PolyLog}[2, -e^{2i(a+bx)}] + \right. \right. \\
& \left. \left. 30ib^2x^2\operatorname{PolyLog}[3, -e^{2i(a+bx)}] - 30bx\operatorname{PolyLog}[4, -e^{2i(a+bx)}] - 15i\operatorname{PolyLog}[5, -e^{2i(a+bx)}] \right) \right) \operatorname{Sec}[a] + \\
& \frac{c^4\operatorname{Sec}[a] \left( \operatorname{Cos}[a]\operatorname{Log}[\operatorname{Cos}[a]\operatorname{Cos}[bx] - \operatorname{Sin}[a]\operatorname{Sin}[bx]] + bx\operatorname{Sin}[a] \right)}{b \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} + \left( 2c^3d\operatorname{Csc}[a] \left( b^2e^{-i\operatorname{ArcTan}[\operatorname{Cot}[a] ]}x^2 - \frac{1}{\sqrt{1+\operatorname{Cot}[a]^2}} \right. \right. \\
& \left. \left. \operatorname{Cot}[a] \left( ibx \left( -\pi - 2\operatorname{ArcTan}[\operatorname{Cot}[a] ] \right) - \pi\operatorname{Log}[1+e^{-2ibx}] - 2(bx - \operatorname{ArcTan}[\operatorname{Cot}[a] ]) \operatorname{Log}[1 - e^{2i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a] ])}] \right) + \right. \right. \\
& \left. \left. \pi\operatorname{Log}[\operatorname{Cos}[bx]] - 2\operatorname{ArcTan}[\operatorname{Cot}[a] ] \operatorname{Log}[\operatorname{Sin}[bx - \operatorname{ArcTan}[\operatorname{Cot}[a] ]]] + i\operatorname{PolyLog}[2, e^{2i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a] ])}] \right) \right) \operatorname{Sec}[a] \Big/ \\
& \left( b^2\sqrt{\operatorname{Csc}[a]^2 \left( \operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2 \right)} \right) + \operatorname{Sec}[a] \left( \frac{\operatorname{Cos}[2a+2bx]}{40b^5} - \frac{i\operatorname{Sin}[2a+2bx]}{40b^5} \right) \\
& (-20b^4c^4\operatorname{Cos}[a] + 40ib^3c^3d\operatorname{Cos}[a] + 60b^2c^2d^2\operatorname{Cos}[a] - 60ibcd^3\operatorname{Cos}[a] - 30d^4\operatorname{Cos}[a] - 80b^4c^3dx\operatorname{Cos}[a] + 120ib^3c^2d^2x\operatorname{Cos}[a] + \\
& 120b^2c^3d^3x\operatorname{Cos}[a] - 60ib^4d^4x\operatorname{Cos}[a] - 120b^4c^2d^2x^2\operatorname{Cos}[a] + 120ib^3c^3d^3x^2\operatorname{Cos}[a] + 60b^2d^4x^2\operatorname{Cos}[a] - 80b^4c^3d^3x^3\operatorname{Cos}[a] + \\
& 40ib^3d^4x^3\operatorname{Cos}[a] - 20b^4d^4x^4\operatorname{Cos}[a] - 20ib^5c^4x\operatorname{Cos}[a+2bx] - 40ib^5c^3d^2x^2\operatorname{Cos}[a+2bx] - 40ib^5c^2d^2x^3\operatorname{Cos}[a+2bx] - \\
& 20ib^5c^3d^3x^4\operatorname{Cos}[a+2bx] - 4ib^5d^4x^5\operatorname{Cos}[a+2bx] + 20ib^5c^4x\operatorname{Cos}[3a+2bx] + 40ib^5c^3d^2x^2\operatorname{Cos}[3a+2bx] + \\
& 40ib^5c^2d^2x^3\operatorname{Cos}[3a+2bx] + 20ib^5c^3d^3x^4\operatorname{Cos}[3a+2bx] + 4ib^5d^4x^5\operatorname{Cos}[3a+2bx] - 10b^4c^4\operatorname{Cos}[3a+4bx] - \\
& 20ib^3c^3d\operatorname{Cos}[3a+4bx] + 30b^2c^2d^2\operatorname{Cos}[3a+4bx] + 30ibcd^3\operatorname{Cos}[3a+4bx] - 15d^4\operatorname{Cos}[3a+4bx] - 40b^4c^3dx\operatorname{Cos}[3a+4bx] - \\
& 60ib^3c^2d^2x\operatorname{Cos}[3a+4bx] + 60b^2c^3d^3x\operatorname{Cos}[3a+4bx] + 30ib^4d^4x\operatorname{Cos}[3a+4bx] - 60b^4c^2d^2x^2\operatorname{Cos}[3a+4bx] - \\
& 60ib^3c^3d^3x^2\operatorname{Cos}[3a+4bx] + 30b^2d^4x^2\operatorname{Cos}[3a+4bx] - 40b^4c^3d^3x^3\operatorname{Cos}[3a+4bx] - 20ib^3d^4x^3\operatorname{Cos}[3a+4bx] - \\
& 10b^4d^4x^4\operatorname{Cos}[3a+4bx] - 10b^4c^4\operatorname{Cos}[5a+4bx] - 20ib^3c^3d\operatorname{Cos}[5a+4bx] + 30b^2c^2d^2\operatorname{Cos}[5a+4bx] + 30ibcd^3\operatorname{Cos}[5a+4bx] - \\
& 15d^4\operatorname{Cos}[5a+4bx] - 40b^4c^3dx\operatorname{Cos}[5a+4bx] - 60ib^3c^2d^2x\operatorname{Cos}[5a+4bx] + 60b^2c^3d^3x\operatorname{Cos}[5a+4bx] + 30ib^4d^4x\operatorname{Cos}[5a+4bx] - \\
& 60b^4c^2d^2x^2\operatorname{Cos}[5a+4bx] - 60ib^3c^3d^3x^2\operatorname{Cos}[5a+4bx] + 30b^2d^4x^2\operatorname{Cos}[5a+4bx] - 40b^4c^3d^3x^3\operatorname{Cos}[5a+4bx] - \\
& 20ib^3d^4x^3\operatorname{Cos}[5a+4bx] - 10b^4d^4x^4\operatorname{Cos}[5a+4bx] + 20b^5c^4x\operatorname{Sin}[a+2bx] + 40b^5c^3d^2x^2\operatorname{Sin}[a+2bx] + 40b^5c^2d^2x^3\operatorname{Sin}[a+2bx] + \\
& 20b^5c^3d^3x^4\operatorname{Sin}[a+2bx] + 4b^5d^4x^5\operatorname{Sin}[a+2bx] - 20b^5c^4x\operatorname{Sin}[3a+2bx] - 40b^5c^3d^2x^2\operatorname{Sin}[3a+2bx] - 40b^5c^2d^2x^3\operatorname{Sin}[3a+2bx] - \\
& 20b^5c^3d^3x^4\operatorname{Sin}[3a+2bx] - 4b^5d^4x^5\operatorname{Sin}[3a+2bx] - 10ib^4c^4\operatorname{Sin}[3a+4bx] + 20b^3c^3d\operatorname{Sin}[3a+4bx] + 30ib^2c^2d^2\operatorname{Sin}[3a+4bx] - \\
& 30bcd^3\operatorname{Sin}[3a+4bx] - 15id^4\operatorname{Sin}[3a+4bx] - 40ib^4c^3dx\operatorname{Sin}[3a+4bx] + 60b^3c^2d^2x\operatorname{Sin}[3a+4bx] + 60ib^2c^3d^3x\operatorname{Sin}[3a+4bx] - \\
& 30b^4d^4x\operatorname{Sin}[3a+4bx] - 60ib^4c^2d^2x^2\operatorname{Sin}[3a+4bx] + 60b^3c^3d^3x^2\operatorname{Sin}[3a+4bx] + 30ib^2d^4x^2\operatorname{Sin}[3a+4bx] - \\
& 40ib^4c^3d^3x^3\operatorname{Sin}[3a+4bx] + 20b^3d^4x^3\operatorname{Sin}[3a+4bx] - 10ib^4d^4x^4\operatorname{Sin}[3a+4bx] - 10ib^4c^4\operatorname{Sin}[5a+4bx] + 20b^3c^3d\operatorname{Sin}[5a+4bx] + \\
& 30ib^2c^2d^2\operatorname{Sin}[5a+4bx] - 30bcd^3\operatorname{Sin}[5a+4bx] - 15id^4\operatorname{Sin}[5a+4bx] - 40ib^4c^3dx\operatorname{Sin}[5a+4bx] + 60b^3c^2d^2x\operatorname{Sin}[5a+4bx] + \\
& 60ib^2c^3d^3x\operatorname{Sin}[5a+4bx] - 30b^4d^4x\operatorname{Sin}[5a+4bx] - 60ib^4c^2d^2x^2\operatorname{Sin}[5a+4bx] + 60b^3c^3d^3x^2\operatorname{Sin}[5a+4bx] + \\
& 30ib^2d^4x^2\operatorname{Sin}[5a+4bx] - 40ib^4c^3d^3x^3\operatorname{Sin}[5a+4bx] + 20b^3d^4x^3\operatorname{Sin}[5a+4bx] - 10ib^4d^4x^4\operatorname{Sin}[5a+4bx])
\end{aligned}$$

### Problem 383: Result more than twice size of optimal antiderivative.

$$\int (c + dx)^3 \operatorname{Sec}[a + bx] \operatorname{Sin}[3a + 3bx] dx$$

Optimal (type 4, 242 leaves, 19 steps):

$$\begin{aligned} & \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3 \operatorname{Log}[1 + e^{2i(a+bx)}]}{b} - \frac{3id(c+dx)^2 \operatorname{PolyLog}[2, -e^{2i(a+bx)}]}{2b^2} + \\ & \frac{3d^2(c+dx) \operatorname{PolyLog}[3, -e^{2i(a+bx)}]}{2b^3} + \frac{3id^3 \operatorname{PolyLog}[4, -e^{2i(a+bx)}]}{4b^4} - \frac{3d^3 \operatorname{Cos}[a+bx] \operatorname{Sin}[a+bx]}{2b^4} + \\ & \frac{3d(c+dx)^2 \operatorname{Cos}[a+bx] \operatorname{Sin}[a+bx]}{b^2} - \frac{3d^2(c+dx) \operatorname{Sin}[a+bx]^2}{b^3} + \frac{2(c+dx)^3 \operatorname{Sin}[a+bx]^2}{b} \end{aligned}$$

Result (type 4, 1733 leaves):

$$\begin{aligned}
& -\frac{1}{4b^3} c d^2 e^{-i a} \\
& \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] \right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] \right) \\
& \operatorname{Sec} [a] + \frac{1}{4} i d^3 e^{i a} \left( -x^4 + \left( 1 + e^{-2 i a} \right) x^4 - \frac{1}{2 b^4} e^{-2 i a} \left( 1 + e^{2 i a} \right) \right. \\
& \left. \left( 2 b^4 x^4 + 4 i b^3 x^3 \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] + 6 b^2 x^2 \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] + 6 i b x \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] - 3 \operatorname{PolyLog} \left[ 4, -e^{2 i (a+b x)} \right] \right) \right) \operatorname{Sec} [a] + \\
& \frac{c^3 \operatorname{Sec} [a] \left( \operatorname{Cos} [a] \operatorname{Log} \left[ \operatorname{Cos} [a] \operatorname{Cos} [b x] - \operatorname{Sin} [a] \operatorname{Sin} [b x] \right] + b x \operatorname{Sin} [a] \right)}{b \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} + \left( 3 c^2 d \operatorname{Csc} [a] \left( b^2 e^{-i \operatorname{ArcTan} [\operatorname{Cot} [a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot} [a]^2}} \right. \right. \\
& \left. \left. \operatorname{Cot} [a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) - \pi \operatorname{Log} \left[ 1 + e^{-2 i b x} \right] - 2 \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) \operatorname{Log} \left[ 1 - e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a]])} \right] + \right. \right. \right. \\
& \left. \left. \left. \pi \operatorname{Log} [\operatorname{Cos} [b x]] - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \operatorname{Log} [\operatorname{Sin} [b x - \operatorname{ArcTan} [\operatorname{Cot} [a]]] ] + i \operatorname{PolyLog} \left[ 2, e^{2 i (b x - \operatorname{ArcTan} [\operatorname{Cot} [a]])} \right] \right) \right) \operatorname{Sec} [a] \right) / \\
& \left( 2 b^2 \sqrt{\operatorname{Csc} [a]^2 \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} \right) + \operatorname{Sec} [a] \left( \frac{\operatorname{Cos} [2 a + 2 b x]}{16 b^4} - \frac{i \operatorname{Sin} [2 a + 2 b x]}{16 b^4} \right) \\
& \left( -8 b^3 c^3 \operatorname{Cos} [a] + 12 i b^2 c^2 d \operatorname{Cos} [a] + 12 b c d^2 \operatorname{Cos} [a] - 6 i d^3 \operatorname{Cos} [a] - 24 b^3 c^2 d x \operatorname{Cos} [a] + 24 i b^2 c d^2 x \operatorname{Cos} [a] + 12 b d^3 x \operatorname{Cos} [a] - \right. \\
& 24 b^3 c d^2 x^2 \operatorname{Cos} [a] + 12 i b^2 d^3 x^2 \operatorname{Cos} [a] - 8 b^3 d^3 x^3 \operatorname{Cos} [a] - 8 i b^4 c^3 x \operatorname{Cos} [a + 2 b x] - 12 i b^4 c^2 d x^2 \operatorname{Cos} [a + 2 b x] - \\
& 8 i b^4 c d^2 x^3 \operatorname{Cos} [a + 2 b x] - 2 i b^4 d^3 x^4 \operatorname{Cos} [a + 2 b x] + 8 i b^4 c^3 x \operatorname{Cos} [3 a + 2 b x] + 12 i b^4 c^2 d x^2 \operatorname{Cos} [3 a + 2 b x] + \\
& 8 i b^4 c d^2 x^3 \operatorname{Cos} [3 a + 2 b x] + 2 i b^4 d^3 x^4 \operatorname{Cos} [3 a + 2 b x] - 4 b^3 c^3 \operatorname{Cos} [3 a + 4 b x] - 6 i b^2 c^2 d \operatorname{Cos} [3 a + 4 b x] + 6 b c d^2 \operatorname{Cos} [3 a + 4 b x] + \\
& 3 i d^3 \operatorname{Cos} [3 a + 4 b x] - 12 b^3 c^2 d x \operatorname{Cos} [3 a + 4 b x] - 12 i b^2 c d^2 x \operatorname{Cos} [3 a + 4 b x] + 6 b d^3 x \operatorname{Cos} [3 a + 4 b x] - 12 b^3 c d^2 x^2 \operatorname{Cos} [3 a + 4 b x] - \\
& 6 i b^2 d^3 x^2 \operatorname{Cos} [3 a + 4 b x] - 4 b^3 d^3 x^3 \operatorname{Cos} [3 a + 4 b x] - 4 b^3 c^3 \operatorname{Cos} [5 a + 4 b x] - 6 i b^2 c^2 d \operatorname{Cos} [5 a + 4 b x] + 6 b c d^2 \operatorname{Cos} [5 a + 4 b x] + \\
& 3 i d^3 \operatorname{Cos} [5 a + 4 b x] - 12 b^3 c^2 d x \operatorname{Cos} [5 a + 4 b x] - 12 i b^2 c d^2 x \operatorname{Cos} [5 a + 4 b x] + 6 b d^3 x \operatorname{Cos} [5 a + 4 b x] - 12 b^3 c d^2 x^2 \operatorname{Cos} [5 a + 4 b x] - \\
& 6 i b^2 d^3 x^2 \operatorname{Cos} [5 a + 4 b x] - 4 b^3 d^3 x^3 \operatorname{Cos} [5 a + 4 b x] + 8 b^4 c^3 x \operatorname{Sin} [a + 2 b x] + 12 b^4 c^2 d x^2 \operatorname{Sin} [a + 2 b x] + 8 b^4 c d^2 x^3 \operatorname{Sin} [a + 2 b x] + \\
& 2 b^4 d^3 x^4 \operatorname{Sin} [a + 2 b x] - 8 b^4 c^3 x \operatorname{Sin} [3 a + 2 b x] - 12 b^4 c^2 d x^2 \operatorname{Sin} [3 a + 2 b x] - 8 b^4 c d^2 x^3 \operatorname{Sin} [3 a + 2 b x] - 2 b^4 d^3 x^4 \operatorname{Sin} [3 a + 2 b x] - \\
& 4 i b^3 c^3 \operatorname{Sin} [3 a + 4 b x] + 6 b^2 c^2 d \operatorname{Sin} [3 a + 4 b x] + 6 i b c d^2 \operatorname{Sin} [3 a + 4 b x] - 3 d^3 \operatorname{Sin} [3 a + 4 b x] - 12 i b^3 c^2 d x \operatorname{Sin} [3 a + 4 b x] + \\
& 12 b^2 c d^2 x \operatorname{Sin} [3 a + 4 b x] + 6 i b d^3 x \operatorname{Sin} [3 a + 4 b x] - 12 i b^3 c d^2 x^2 \operatorname{Sin} [3 a + 4 b x] + 6 b^2 d^3 x^2 \operatorname{Sin} [3 a + 4 b x] - 4 i b^3 d^3 x^3 \operatorname{Sin} [3 a + 4 b x] - \\
& 4 i b^3 c^3 \operatorname{Sin} [5 a + 4 b x] + 6 b^2 c^2 d \operatorname{Sin} [5 a + 4 b x] + 6 i b c d^2 \operatorname{Sin} [5 a + 4 b x] - 3 d^3 \operatorname{Sin} [5 a + 4 b x] - 12 i b^3 c^2 d x \operatorname{Sin} [5 a + 4 b x] + \\
& 12 b^2 c d^2 x \operatorname{Sin} [5 a + 4 b x] + 6 i b d^3 x \operatorname{Sin} [5 a + 4 b x] - 12 i b^3 c d^2 x^2 \operatorname{Sin} [5 a + 4 b x] + 6 b^2 d^3 x^2 \operatorname{Sin} [5 a + 4 b x] - 4 i b^3 d^3 x^3 \operatorname{Sin} [5 a + 4 b x] \left. \right)
\end{aligned}$$

### Problem 384: Result more than twice size of optimal antiderivative.

$$\int (c + d x)^2 \operatorname{Sec} [a + b x] \operatorname{Sin} [3 a + 3 b x] dx$$

Optimal (type 4, 173 leaves, 14 steps):

$$\begin{aligned}
& -\frac{2 c d x}{b} - \frac{d^2 x^2}{b} - \frac{i (c + d x)^3}{3 d} + \frac{(c + d x)^2 \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right]}{b} - \frac{i d (c + d x) \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right]}{b^2} + \\
& \frac{d^2 \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right]}{2 b^3} + \frac{2 d (c + d x) \operatorname{Cos} [a + b x] \operatorname{Sin} [a + b x]}{b^2} - \frac{d^2 \operatorname{Sin} [a + b x]^2}{b^3} + \frac{2 (c + d x)^2 \operatorname{Sin} [a + b x]^2}{b}
\end{aligned}$$

Result (type 4, 523 leaves):

$$\begin{aligned}
 & -\frac{1}{12 b^3} \\
 & d^2 e^{-i a} \left( 2 i b^2 x^2 \left( 2 b e^{2 i a} x + 3 i \left( 1 + e^{2 i a} \right) \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right] \right) + 6 i b \left( 1 + e^{2 i a} \right) x \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right] - 3 \left( 1 + e^{2 i a} \right) \operatorname{PolyLog} \left[ 3, -e^{2 i (a+b x)} \right] \right) \\
 & \operatorname{Sec} [a] + \frac{c^2 \operatorname{Sec} [a] \left( \operatorname{Cos} [a] \operatorname{Log} \left[ \operatorname{Cos} [a] \operatorname{Cos} [b x] - \operatorname{Sin} [a] \operatorname{Sin} [b x] \right] + b x \operatorname{Sin} [a] \right)}{b \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} + \\
 & \left( c d \operatorname{Csc} [a] \left( b^2 e^{-i \operatorname{ArcTan} [\operatorname{Cot} [a]]} x^2 - \frac{1}{\sqrt{1 + \operatorname{Cot} [a]^2}} \operatorname{Cot} [a] \left( i b x \left( -\pi - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) - \pi \operatorname{Log} \left[ 1 + e^{-2 i b x} \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 2 \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right) \operatorname{Log} \left[ 1 - e^{2 i \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right)} \right] + \pi \operatorname{Log} [\operatorname{Cos} [b x]] - 2 \operatorname{ArcTan} [\operatorname{Cot} [a]] \operatorname{Log} [\operatorname{Sin} [b x - \operatorname{ArcTan} [\operatorname{Cot} [a]]]] \right) \right) + \right. \\
 & \quad \left. i \operatorname{PolyLog} \left[ 2, e^{2 i \left( b x - \operatorname{ArcTan} [\operatorname{Cot} [a]] \right)} \right] \right) \operatorname{Sec} [a] \right) / \left( b^2 \sqrt{\operatorname{Csc} [a]^2 \left( \operatorname{Cos} [a]^2 + \operatorname{Sin} [a]^2 \right)} \right) - \frac{1}{2 b^3} \\
 & \operatorname{Cos} [2 b x] \left( 2 b^2 c^2 \operatorname{Cos} [2 a] - d^2 \operatorname{Cos} [2 a] + 4 b^2 c d x \operatorname{Cos} [2 a] + 2 b^2 d^2 x^2 \operatorname{Cos} [2 a] - 2 b c d \operatorname{Sin} [2 a] - 2 b d^2 x \operatorname{Sin} [2 a] \right) + \\
 & \frac{1}{2 b^3} \\
 & \left( 2 b c d \operatorname{Cos} [2 a] + 2 b d^2 x \operatorname{Cos} [2 a] + 2 b^2 c^2 \operatorname{Sin} [2 a] - d^2 \operatorname{Sin} [2 a] + 4 b^2 c d x \operatorname{Sin} [2 a] + 2 b^2 d^2 x^2 \operatorname{Sin} [2 a] \right) \operatorname{Sin} [2 b x] - \\
 & \frac{1}{3} x \left( 3 c^2 + 3 c d x + d^2 x^2 \right) \operatorname{Tan} [a]
 \end{aligned}$$

Problem 385: Result more than twice size of optimal antiderivative.

$$\int (c + d x) \operatorname{Sec} [a + b x] \operatorname{Sin} [3 a + 3 b x] dx$$

Optimal (type 4, 107 leaves, 13 steps):

$$-\frac{d x}{b} - \frac{i (c + d x)^2}{2 d} + \frac{(c + d x) \operatorname{Log} \left[ 1 + e^{2 i (a+b x)} \right]}{b} - \frac{i d \operatorname{PolyLog} \left[ 2, -e^{2 i (a+b x)} \right]}{2 b^2} + \frac{d \operatorname{Cos} [a + b x] \operatorname{Sin} [a + b x]}{b^2} + \frac{2 (c + d x) \operatorname{Sin} [a + b x]^2}{b}$$

Result (type 4, 257 leaves):

$$\begin{aligned}
& -\frac{c \operatorname{Cos}[2(a+bx)]}{b} + \frac{c \operatorname{Log}[\operatorname{Cos}[a+bx]]}{b} + \left( d \operatorname{Csc}[a] \left( b^2 e^{-i \operatorname{ArcTan}[\operatorname{Cot}[a]]} x^2 - \frac{1}{\sqrt{1+\operatorname{Cot}[a]^2}} \right. \right. \\
& \left. \left. \operatorname{Cot}[a] (i b x (-\pi - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]]) - \pi \operatorname{Log}[1 + e^{-2i b x}] - 2(bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \operatorname{Log}[1 - e^{2i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}]) \right. \right. \\
& \left. \left. + \pi \operatorname{Log}[\operatorname{Cos}[bx]] - 2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{Log}[\operatorname{Sin}[bx - \operatorname{ArcTan}[\operatorname{Cot}[a]]]] + i \operatorname{PolyLog}[2, e^{2i(bx - \operatorname{ArcTan}[\operatorname{Cot}[a])}] \right) \right) \operatorname{Sec}[a] \Big/ \\
& \left( 2 b^2 \sqrt{\operatorname{Csc}[a]^2 (\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2)} \right) - \frac{d \operatorname{Cos}[2bx] (2bx \operatorname{Cos}[2a] - \operatorname{Sin}[2a])}{2 b^2} + \frac{d (\operatorname{Cos}[2a] + 2bx \operatorname{Sin}[2a]) \operatorname{Sin}[2bx]}{2 b^2} - \frac{1}{2} \\
& d \\
& x^2 \\
& \operatorname{Tan}[a]
\end{aligned}$$

**Problem 389: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^3 \operatorname{Sec}[a+bx]^2 \operatorname{Sin}[3a+3bx] dx$$

Optimal (type 4, 230 leaves, 19 steps):

$$\begin{aligned}
& -\frac{6 i d (c+dx)^2 \operatorname{ArcTan}[e^{i(a+bx)}]}{b^2} + \frac{24 d^2 (c+dx) \operatorname{Cos}[a+bx]}{b^3} - \frac{4 (c+dx)^3 \operatorname{Cos}[a+bx]}{b} + \\
& \frac{6 i d^2 (c+dx) \operatorname{PolyLog}[2, -i e^{i(a+bx)}]}{b^3} - \frac{6 i d^2 (c+dx) \operatorname{PolyLog}[2, i e^{i(a+bx)}]}{b^3} - \frac{6 d^3 \operatorname{PolyLog}[3, -i e^{i(a+bx)}]}{b^4} + \\
& \frac{6 d^3 \operatorname{PolyLog}[3, i e^{i(a+bx)}]}{b^4} - \frac{(c+dx)^3 \operatorname{Sec}[a+bx]}{b} - \frac{24 d^3 \operatorname{Sin}[a+bx]}{b^4} + \frac{12 d (c+dx)^2 \operatorname{Sin}[a+bx]}{b^2}
\end{aligned}$$

Result (type 4, 532 leaves):

$$\begin{aligned}
& -\frac{1}{b^4} \operatorname{Sec}[a+bx] (3 b^3 c^3 - 12 b c d^2 + 9 b^3 c^2 d x - 12 b d^3 x + 9 b^3 c d^2 x^2 + 3 b^3 d^3 x^3 + 6 i b^2 c^2 d \operatorname{ArcTan}[e^{i(a+bx)}] \operatorname{Cos}[a+bx] + \\
& 2 b^3 c^3 \operatorname{Cos}[2(a+bx)] - 12 b c d^2 \operatorname{Cos}[2(a+bx)] + 6 b^3 c^2 d x \operatorname{Cos}[2(a+bx)] - 12 b d^3 x \operatorname{Cos}[2(a+bx)] + 6 b^3 c d^2 x^2 \operatorname{Cos}[2(a+bx)] + \\
& 2 b^3 d^3 x^3 \operatorname{Cos}[2(a+bx)] - 6 b^2 c d^2 x \operatorname{Cos}[a+bx] \operatorname{Log}[1 - i e^{i(a+bx)}] - 3 b^2 d^3 x^2 \operatorname{Cos}[a+bx] \operatorname{Log}[1 - i e^{i(a+bx)}] + \\
& 6 b^2 c d^2 x \operatorname{Cos}[a+bx] \operatorname{Log}[1 + i e^{i(a+bx)}] + 3 b^2 d^3 x^2 \operatorname{Cos}[a+bx] \operatorname{Log}[1 + i e^{i(a+bx)}] - 6 i b d^2 (c+dx) \operatorname{Cos}[a+bx] \operatorname{PolyLog}[2, -i e^{i(a+bx)}] + \\
& 6 i b d^2 (c+dx) \operatorname{Cos}[a+bx] \operatorname{PolyLog}[2, i e^{i(a+bx)}] + 6 d^3 \operatorname{Cos}[a+bx] \operatorname{PolyLog}[3, -i e^{i(a+bx)}] - 6 d^3 \operatorname{Cos}[a+bx] \operatorname{PolyLog}[3, i e^{i(a+bx)}] - \\
& 6 b^2 c^2 d \operatorname{Sin}[2(a+bx)] + 12 d^3 \operatorname{Sin}[2(a+bx)] - 12 b^2 c d^2 x \operatorname{Sin}[2(a+bx)] - 6 b^2 d^3 x^2 \operatorname{Sin}[2(a+bx)])
\end{aligned}$$

**Problem 390: Result more than twice size of optimal antiderivative.**

$$\int (c+dx)^2 \operatorname{Sec}[a+bx]^2 \operatorname{Sin}[3a+3bx] dx$$

Optimal (type 4, 147 leaves, 15 steps):

$$-\frac{4 i d (c+d x) \operatorname{ArcTan}\left[e^{i(a+b x)}\right]}{b^2} + \frac{8 d^2 \operatorname{Cos}[a+b x]}{b^3} - \frac{4(c+d x)^2 \operatorname{Cos}[a+b x]}{b} +$$

$$\frac{2 i d^2 \operatorname{PolyLog}\left[2, -i e^{i(a+b x)}\right]}{b^3} - \frac{2 i d^2 \operatorname{PolyLog}\left[2, i e^{i(a+b x)}\right]}{b^3} - \frac{(c+d x)^2 \operatorname{Sec}[a+b x]}{b} + \frac{8 d(c+d x) \operatorname{Sin}[a+b x]}{b^2}$$

Result (type 4, 542 leaves):

$$-\frac{(c+d x)^2 \operatorname{Sec}[a]}{b} - \frac{1}{b^3} 4 \operatorname{Cos}[b x] \left( b^2 c^2 \operatorname{Cos}[a] - 2 d^2 \operatorname{Cos}[a] + 2 b^2 c d x \operatorname{Cos}[a] + b^2 d^2 x^2 \operatorname{Cos}[a] - 2 b c d \operatorname{Sin}[a] - 2 b d^2 x \operatorname{Sin}[a] \right) +$$

$$\frac{4 i c d \operatorname{ArcTan}\left[\frac{-i \operatorname{Sin}[a] - i \operatorname{Cos}[a] \operatorname{Tan}\left[\frac{b x}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{b^2 \sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} + \frac{1}{b^3}$$

$$2 d^2 \left( -\frac{1}{\sqrt{1 + \operatorname{Cot}[a]^2}} \operatorname{Csc}[a] \left( (b x - \operatorname{ArcTan}[\operatorname{Cot}[a]]) \left( \operatorname{Log}\left[1 - e^{i(b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}\right] - \operatorname{Log}\left[1 + e^{i(b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}\right] \right) + \right.$$

$$\left. i \left( \operatorname{PolyLog}\left[2, -e^{i(b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}\right] - \operatorname{PolyLog}\left[2, e^{i(b x - \operatorname{ArcTan}[\operatorname{Cot}[a]])}\right] \right) + \frac{2 \operatorname{ArcTan}[\operatorname{Cot}[a]] \operatorname{ArcTanh}\left[\frac{\operatorname{Sin}[a] + \operatorname{Cos}[a] \operatorname{Tan}\left[\frac{b x}{2}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}}\right]}{\sqrt{\operatorname{Cos}[a]^2 + \operatorname{Sin}[a]^2}} \right) +$$

$$\frac{1}{b^3} 4 \left( 2 b c d \operatorname{Cos}[a] + 2 b d^2 x \operatorname{Cos}[a] + b^2 c^2 \operatorname{Sin}[a] - 2 d^2 \operatorname{Sin}[a] + 2 b^2 c d x \operatorname{Sin}[a] + b^2 d^2 x^2 \operatorname{Sin}[a] \right) \operatorname{Sin}[b x] +$$

$$\frac{-c^2 \operatorname{Sin}\left[\frac{b x}{2}\right] - 2 c d x \operatorname{Sin}\left[\frac{b x}{2}\right] - d^2 x^2 \operatorname{Sin}\left[\frac{b x}{2}\right]}{b \left( \operatorname{Cos}\left[\frac{a}{2}\right] - \operatorname{Sin}\left[\frac{a}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{a}{2} + \frac{b x}{2}\right] - \operatorname{Sin}\left[\frac{a}{2} + \frac{b x}{2}\right] \right)} +$$

$$\frac{c^2 \operatorname{Sin}\left[\frac{b x}{2}\right] + 2 c d x \operatorname{Sin}\left[\frac{b x}{2}\right] + d^2 x^2 \operatorname{Sin}\left[\frac{b x}{2}\right]}{b \left( \operatorname{Cos}\left[\frac{a}{2}\right] + \operatorname{Sin}\left[\frac{a}{2}\right] \right) \left( \operatorname{Cos}\left[\frac{a}{2} + \frac{b x}{2}\right] + \operatorname{Sin}\left[\frac{a}{2} + \frac{b x}{2}\right] \right)}$$

**Problem 397: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Cos}[2 x] \operatorname{Sec}[x]^3 dx$$

Optimal (type 4, 67 leaves, 19 steps):

$$-3 i x \operatorname{ArcTan}\left[e^{i x}\right] + \frac{3}{2} i \operatorname{PolyLog}\left[2, -i e^{i x}\right] - \frac{3}{2} i \operatorname{PolyLog}\left[2, i e^{i x}\right] + \frac{\operatorname{Sec}[x]}{2} - \frac{1}{2} x \operatorname{Sec}[x] \operatorname{Tan}[x]$$

Result (type 4, 146 leaves):

$$\frac{1}{4} \left( 6 x \operatorname{Log}\left[1 - i e^{i x}\right] - 6 x \operatorname{Log}\left[1 + i e^{i x}\right] + 6 i \operatorname{PolyLog}\left[2, -i e^{i x}\right] - \right. \\ \left. 6 i \operatorname{PolyLog}\left[2, i e^{i x}\right] + \frac{2 \operatorname{Sin}\left[\frac{x}{2}\right]}{\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]} + \frac{x}{\left(\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right)^2} - \frac{2 \operatorname{Sin}\left[\frac{x}{2}\right]}{\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]} + \frac{x}{-1 + \operatorname{Sin}[x]} \right)$$

## Test results for the 9 problems in "4.7.4 x^m (a+b trig^n)^p.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{a + b \operatorname{Sin}[x]^2} dx$$

Optimal (type 4, 203 leaves, 9 steps):

$$-\frac{i x \operatorname{Log}\left[1 - \frac{b e^{2 i x}}{2 a + b - 2 \sqrt{a} \sqrt{a + b}}\right]}{2 \sqrt{a} \sqrt{a + b}} + \frac{i x \operatorname{Log}\left[1 - \frac{b e^{2 i x}}{2 a + b + 2 \sqrt{a} \sqrt{a + b}}\right]}{2 \sqrt{a} \sqrt{a + b}} - \frac{\operatorname{PolyLog}\left[2, \frac{b e^{2 i x}}{2 a + b - 2 \sqrt{a} \sqrt{a + b}}\right]}{4 \sqrt{a} \sqrt{a + b}} + \frac{\operatorname{PolyLog}\left[2, \frac{b e^{2 i x}}{2 a + b + 2 \sqrt{a} \sqrt{a + b}}\right]}{4 \sqrt{a} \sqrt{a + b}}$$

Result (type 4, 545 leaves):



$$\begin{aligned}
& \frac{1}{4\sqrt{-a(a+b)}} \left( 4x \operatorname{ArcTanh}\left[\frac{a \operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] - 2 \operatorname{ArcCos}\left[1 + \frac{2a}{b}\right] \operatorname{ArcTanh}\left[\frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[1 + \frac{2a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{a \operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] + 2i \operatorname{ArcTanh}\left[\frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a}\right] \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-a(a+b)} e^{-ix}}{\sqrt{-b} \sqrt{2a+b-b\cos[2x]}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[1 + \frac{2a}{b}\right] + 2i \left( \operatorname{ArcTanh}\left[\frac{a \operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] - \operatorname{ArcTanh}\left[\frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{-a(a+b)} e^{ix}}{\sqrt{-b} \sqrt{2a+b-b\cos[2x]}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[1 + \frac{2a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a}\right] \right) \operatorname{Log}\left[\frac{2a(a+b-i\sqrt{-a(a+b)})(1-i\operatorname{Tan}[x])}{b(a+\sqrt{-a(a+b)}\operatorname{Tan}[x])}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[1 + \frac{2a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a}\right] \right) \operatorname{Log}\left[\frac{2a(a+b+i\sqrt{-a(a+b)})(1+i\operatorname{Tan}[x])}{b(a+\sqrt{-a(a+b)}\operatorname{Tan}[x])}\right] + \right. \\
& \left. i \left( \operatorname{PolyLog}\left[2, \frac{(2a+b-2i\sqrt{-a(a+b)})(-a+\sqrt{-a(a+b)}\operatorname{Tan}[x])}{b(a+\sqrt{-a(a+b)}\operatorname{Tan}[x])}\right] - \operatorname{PolyLog}\left[2, \frac{(2a+b+2i\sqrt{-a(a+b)})(-a+\sqrt{-a(a+b)}\operatorname{Tan}[x])}{b(a+\sqrt{-a(a+b)}\operatorname{Tan}[x])}\right] \right) \right)
\end{aligned}$$

**Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{a+b\cos[x]^2} dx$$

Optimal (type 4, 203 leaves, 9 steps):

$$-\frac{i x \operatorname{Log}\left[1 + \frac{b e^{2ix}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right]}{2\sqrt{a}\sqrt{a+b}} + \frac{i x \operatorname{Log}\left[1 + \frac{b e^{2ix}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right]}{2\sqrt{a}\sqrt{a+b}} - \frac{\operatorname{PolyLog}\left[2, -\frac{b e^{2ix}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right]}{4\sqrt{a}\sqrt{a+b}} + \frac{\operatorname{PolyLog}\left[2, -\frac{b e^{2ix}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right]}{4\sqrt{a}\sqrt{a+b}}$$

Result (type 4, 532 leaves):

$$\begin{aligned}
& \frac{1}{4\sqrt{-a(a+b)}} \left( 4x \operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] + 2 \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] - 2i \left( \operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] + \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-a(a+b)}e^{-ix}}{\sqrt{b}\sqrt{2a+b+b\cos[2x]}}\right] + \right. \\
& \left. \left( \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] + 2i \left( \operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] + \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-a(a+b)}e^{ix}}{\sqrt{b}\sqrt{2a+b+b\cos[2x]}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] \right) \operatorname{Log}\left[\frac{2(a+b)(-ia + \sqrt{-a(a+b)})(-i + \operatorname{Tan}[x])}{b(a+b + \sqrt{-a(a+b)})\operatorname{Tan}[x]}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] \right) \operatorname{Log}\left[\frac{2(a+b)(ia + \sqrt{-a(a+b)})(i + \operatorname{Tan}[x])}{b(a+b + \sqrt{-a(a+b)})\operatorname{Tan}[x]}\right] + \right. \\
& \left. i \left( \operatorname{PolyLog}\left[2, \frac{(2a+b - 2i\sqrt{-a(a+b)})\left(a+b - \sqrt{-a(a+b)}\operatorname{Tan}[x]\right)}{b(a+b + \sqrt{-a(a+b)})\operatorname{Tan}[x]}\right] - \right. \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{(2a+b + 2i\sqrt{-a(a+b)})\left(a+b - \sqrt{-a(a+b)}\operatorname{Tan}[x]\right)}{b(a+b + \sqrt{-a(a+b)})\operatorname{Tan}[x]}\right] \right) \right)
\end{aligned}$$

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Test results for the 330 problems in "4.7.5 x^m trig(a+b log(c x^n))^p.m"

Problem 26: Unable to integrate problem.

$$\int x^m \operatorname{Sin}\left[a + \sqrt{-\frac{(1+m)^2}{n^2}} \operatorname{Log}[cx^n]\right] dx$$

Optimal (type 3, 133 leaves, 3 steps):

$$-\frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}n}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{4\sqrt{-\frac{(1+m)^2}{n^2}n}} + \frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}n}}{1+m}}(1+m)x^{1+m}(cx^n)^{-\frac{1+m}{n}}\text{Log}[x]}{2\sqrt{-\frac{(1+m)^2}{n^2}n}}$$

Result (type 8, 30 leaves):

$$\int x^m \text{Sin}\left[a + \sqrt{-\frac{(1+m)^2}{n^2}} \text{Log}[cx^n]\right] dx$$

Problem 27: Unable to integrate problem.

$$\int x^2 \text{Sin}\left[a + 3\sqrt{-\frac{1}{n^2}} \text{Log}[cx^n]\right] dx$$

Optimal (type 3, 88 leaves, 3 steps):

$$\frac{1}{12} e^{-a\sqrt{-\frac{1}{n^2}n}} \sqrt{-\frac{1}{n^2}n} x^3 (cx^n)^{3/n} - \frac{1}{2} e^{a\sqrt{-\frac{1}{n^2}n}} \sqrt{-\frac{1}{n^2}n} x^3 (cx^n)^{-3/n} \text{Log}[x]$$

Result (type 8, 26 leaves):

$$\int x^2 \text{Sin}\left[a + 3\sqrt{-\frac{1}{n^2}} \text{Log}[cx^n]\right] dx$$

Problem 28: Unable to integrate problem.

$$\int x \text{Sin}\left[a + 2\sqrt{-\frac{1}{n^2}} \text{Log}[cx^n]\right] dx$$

Optimal (type 3, 88 leaves, 3 steps):

$$\frac{1}{8} e^{-a\sqrt{-\frac{1}{n^2}n}} \sqrt{-\frac{1}{n^2}n} x^2 (cx^n)^{2/n} - \frac{1}{2} e^{a\sqrt{-\frac{1}{n^2}n}} \sqrt{-\frac{1}{n^2}n} x^2 (cx^n)^{-2/n} \text{Log}[x]$$

Result (type 8, 24 leaves):

$$\int x \operatorname{Sin}\left[a + 2 \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]}\right] dx$$

Problem 29: Unable to integrate problem.

$$\int \operatorname{Sin}\left[a + \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]}\right] dx$$

Optimal (type 3, 82 leaves, 3 steps):

$$\frac{1}{4} e^{-a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} x (c x^n)^{\frac{1}{n}} - \frac{1}{2} e^{a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} x (c x^n)^{-1/n} \operatorname{Log}[x]$$

Result (type 8, 21 leaves):

$$\int \operatorname{Sin}\left[a + \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]}\right] dx$$

Problem 31: Unable to integrate problem.

$$\int \frac{\operatorname{Sin}\left[a + \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]}\right]}{x^2} dx$$

Optimal (type 3, 86 leaves, 3 steps):

$$\frac{e^{a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{-1/n}}{4 x} + \frac{e^{-a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{\frac{1}{n}} \operatorname{Log}[x]}{2 x}$$

Result (type 8, 25 leaves):

$$\int \frac{\operatorname{Sin}\left[a + \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]}\right]}{x^2} dx$$

### Problem 32: Unable to integrate problem.

$$\int \frac{\sin \left[ a + 2 \sqrt{-\frac{1}{n^2} \operatorname{Log} [c x^n]} \right]}{x^3} dx$$

Optimal (type 3, 88 leaves, 3 steps):

$$\frac{e^{a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{-2/n}}{8 x^2} + \frac{e^{-a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{2/n} \operatorname{Log} [x]}{2 x^2}$$

Result (type 8, 26 leaves):

$$\int \frac{\sin \left[ a + 2 \sqrt{-\frac{1}{n^2} \operatorname{Log} [c x^n]} \right]}{x^3} dx$$

### Problem 33: Unable to integrate problem.

$$\int x^m \sin \left[ a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2} \operatorname{Log} [c x^n]} \right]^2 dx$$

Optimal (type 3, 117 leaves, 3 steps):

$$\frac{x^{1+m}}{2(1+m)} - \frac{e^{-\frac{2a \sqrt{-\frac{(1+m)^2}{n^2} n}}{1+m}} x^{1+m} (c x^n)^{\frac{1+m}{n}}}{8(1+m)} - \frac{1}{4} e^{\frac{2a \sqrt{-\frac{(1+m)^2}{n^2} n}}{1+m}} x^{1+m} (c x^n)^{-\frac{1+m}{n}} \operatorname{Log} [x]$$

Result (type 8, 35 leaves):

$$\int x^m \sin \left[ a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2} \operatorname{Log} [c x^n]} \right]^2 dx$$

### Problem 34: Unable to integrate problem.

$$\int x^2 \sin \left[ a + \frac{3}{2} \sqrt{-\frac{1}{n^2} \operatorname{Log} [c x^n]} \right]^2 dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$\frac{x^3}{6} - \frac{1}{24} e^{-2a} \sqrt{-\frac{1}{n^2}} x^3 (c x^n)^{3/n} - \frac{1}{4} e^{2a} \sqrt{-\frac{1}{n^2}} x^3 (c x^n)^{-3/n} \text{Log}[x]$$

Result (type 8, 30 leaves):

$$\int x^2 \text{Sin}\left[a + \frac{3}{2} \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^2 dx$$

**Problem 35: Unable to integrate problem.**

$$\int x \text{Sin}\left[a + \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^2 dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$\frac{x^2}{4} - \frac{1}{16} e^{-2a} \sqrt{-\frac{1}{n^2}} x^2 (c x^n)^{2/n} - \frac{1}{4} e^{2a} \sqrt{-\frac{1}{n^2}} x^2 (c x^n)^{-2/n} \text{Log}[x]$$

Result (type 8, 25 leaves):

$$\int x \text{Sin}\left[a + \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^2 dx$$

**Problem 36: Unable to integrate problem.**

$$\int \text{Sin}\left[a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^2 dx$$

Optimal (type 3, 68 leaves, 3 steps):

$$\frac{x}{2} - \frac{1}{8} e^{-2a} \sqrt{-\frac{1}{n^2}} x (c x^n)^{\frac{1}{n}} - \frac{1}{4} e^{2a} \sqrt{-\frac{1}{n^2}} x (c x^n)^{-1/n} \text{Log}[x]$$

Result (type 8, 26 leaves):

$$\int \text{Sin}\left[a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^2 dx$$

### Problem 38: Unable to integrate problem.

$$\int \frac{\sin\left[a + \frac{1}{2} \sqrt{-\frac{1}{n^2} \log[cx^n]}\right]^2}{x^2} dx$$

Optimal (type 3, 74 leaves, 3 steps):

$$-\frac{1}{2x} + \frac{e^{2a\sqrt{-\frac{1}{n^2}n}}(cx^n)^{-1/n}}{8x} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}n}}(cx^n)^{\frac{1}{n}}\log[x]}{4x}$$

Result (type 8, 30 leaves):

$$\int \frac{\sin\left[a + \frac{1}{2} \sqrt{-\frac{1}{n^2} \log[cx^n]}\right]^2}{x^2} dx$$

### Problem 39: Unable to integrate problem.

$$\int \frac{\sin\left[a + \sqrt{-\frac{1}{n^2} \log[cx^n]}\right]^2}{x^3} dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$-\frac{1}{4x^2} + \frac{e^{2a\sqrt{-\frac{1}{n^2}n}}(cx^n)^{-2/n}}{16x^2} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}n}}(cx^n)^{2/n}\log[x]}{4x^2}$$

Result (type 8, 27 leaves):

$$\int \frac{\sin\left[a + \sqrt{-\frac{1}{n^2} \log[cx^n]}\right]^2}{x^3} dx$$

### Problem 41: Unable to integrate problem.

$$\int x^2 \sin\left[a + \sqrt{-\frac{1}{n^2} \log[cx^n]}\right]^3 dx$$

Optimal (type 3, 172 leaves, 3 steps):

$$-\frac{3}{16} e^{a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n x^3 (c x^n)^{-1/n}} + \frac{3}{32} e^{-a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n x^3 (c x^n)^{\frac{1}{n}}} -$$

$$\frac{1}{48} e^{-3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n x^3 (c x^n)^{3/n}} + \frac{1}{8} e^{3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n x^3 (c x^n)^{-3/n}} \text{Log}[x]$$

Result (type 8, 27 leaves):

$$\int x^2 \text{Sin}\left[a + \sqrt{-\frac{1}{n^2} \text{Log}[c x^n]}\right]^3 dx$$

Problem 42: Unable to integrate problem.

$$\int x \text{Sin}\left[a + \frac{2}{3} \sqrt{-\frac{1}{n^2} \text{Log}[c x^n]}\right]^3 dx$$

Optimal (type 3, 178 leaves, 3 steps):

$$-\frac{9}{32} e^{a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n x^2 (c x^n)^{-2/3}} + \frac{9}{64} e^{-a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n x^2 (c x^n)^{2/3}} -$$

$$\frac{1}{32} e^{-3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n x^2 (c x^n)^{2/n}} + \frac{1}{8} e^{3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n x^2 (c x^n)^{-2/n}} \text{Log}[x]$$

Result (type 8, 28 leaves):

$$\int x \text{Sin}\left[a + \frac{2}{3} \sqrt{-\frac{1}{n^2} \text{Log}[c x^n]}\right]^3 dx$$

Problem 43: Unable to integrate problem.

$$\int \text{Sin}\left[a + \frac{1}{3} \sqrt{-\frac{1}{n^2} \text{Log}[c x^n]}\right]^3 dx$$

Optimal (type 3, 168 leaves, 3 steps):



$$-\frac{9}{16} e^{a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} x (c x^n)^{-\frac{1}{3}/n} + \frac{9}{32} e^{-a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} x (c x^n)^{\frac{1}{3}/n} - \frac{1}{16} e^{-3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} x (c x^n)^{\frac{1}{n}} + \frac{1}{8} e^{3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} x (c x^n)^{-1/n} \text{Log}[x]$$

Result (type 8, 26 leaves):

$$\int \text{Sin}\left[a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^3 dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{\text{Sin}\left[a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^3}{x^2} dx$$

Optimal (type 3, 176 leaves, 3 steps):

$$-\frac{e^{3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{-1/n}}{16 x} + \frac{9 e^{a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{-\frac{1}{3}/n}}{32 x} - \frac{9 e^{-a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{\frac{1}{3}/n}}{16 x} - \frac{e^{-3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{\frac{1}{n}} \text{Log}[x]}{8 x}$$

Result (type 8, 30 leaves):

$$\int \frac{\text{Sin}\left[a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^3}{x^2} dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{\text{Sin}\left[a + \frac{2}{3} \sqrt{-\frac{1}{n^2}} \text{Log}[c x^n]\right]^3}{x^3} dx$$

Optimal (type 3, 178 leaves, 3 steps):

$$-\frac{e^{3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{-2/n}}{32 x^2} + \frac{9 e^{a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{-\frac{2}{3}/n}}{64 x^2} - \frac{9 e^{-a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{\frac{2}{3}/n}}{32 x^2} - \frac{e^{-3a \sqrt{-\frac{1}{n^2} n}} \sqrt{-\frac{1}{n^2} n} (c x^n)^{2/n} \text{Log}[x]}{8 x^2}$$

Result (type 8, 30 leaves):

$$\int \frac{\sin \left[ a + \frac{2}{3} \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]} \right]^3}{x^3} dx$$

**Problem 47: Unable to integrate problem.**

$$\int x^m \sin \left[ a + \frac{1}{2} \sqrt{-(1+m)^2} \operatorname{Log}[c x^2] \right] dx$$

Optimal (type 3, 112 leaves, 3 steps):

$$-\frac{e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}}} x^{1+m} (c x^2)^{\frac{1+m}{2}}}{4 \sqrt{-(1+m)^2}} + \frac{e^{\frac{a \sqrt{-(1+m)^2}}{1+m}} (1+m) x^{1+m} (c x^2)^{\frac{1}{2}(-1-m)} \operatorname{Log}[x]}{2 \sqrt{-(1+m)^2}}$$

Result (type 8, 30 leaves):

$$\int x^m \sin \left[ a + \frac{1}{2} \sqrt{-(1+m)^2} \operatorname{Log}[c x^2] \right] dx$$

**Problem 49: Unable to integrate problem.**

$$\int x^m \sin \left[ a + \frac{1}{4} \sqrt{-(1+m)^2} \operatorname{Log}[c x^2] \right]^2 dx$$

Optimal (type 3, 106 leaves, 3 steps):

$$\frac{x^{1+m}}{2(1+m)} - \frac{e^{\frac{2a(1+m)}{\sqrt{-(1+m)^2}}} x^{1+m} (c x^2)^{\frac{1+m}{2}}}{8(1+m)} - \frac{1}{4} e^{\frac{-2a(1+m)}{\sqrt{-(1+m)^2}}} x^{1+m} (c x^2)^{\frac{1}{2}(-1-m)} \operatorname{Log}[x]$$

Result (type 8, 32 leaves):

$$\int x^m \sin \left[ a + \frac{1}{4} \sqrt{-(1+m)^2} \operatorname{Log}[c x^2] \right]^2 dx$$

**Problem 51: Unable to integrate problem.**

$$\int x^m \sin \left[ a + \frac{1}{6} \sqrt{-(1+m)^2} \operatorname{Log}[c x^2] \right]^3 dx$$

Optimal (type 3, 218 leaves, 3 steps):

$$\frac{9 e^{\frac{a\sqrt{-(1+m)^2}}{1+m}} x^{1+m} (c x^2)^{\frac{1}{6}(-1-m)}}{16 \sqrt{-(1+m)^2}} - \frac{9 e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}}}}{32 \sqrt{-(1+m)^2}} x^{1+m} (c x^2)^{\frac{1+m}{6}} + \frac{e^{\frac{3a(1+m)}{\sqrt{-(1+m)^2}}}}{16 \sqrt{-(1+m)^2}} x^{1+m} (c x^2)^{\frac{1+m}{2}} - \frac{e^{-\frac{3a(1+m)}{\sqrt{-(1+m)^2}}}}{8 \sqrt{-(1+m)^2}} (1+m) x^{1+m} (c x^2)^{\frac{1}{2}(-1-m)} \text{Log}[x]$$

Result (type 8, 32 leaves):

$$\int x^m \text{Sin}\left[a + \frac{1}{6} \sqrt{-(1+m)^2} \text{Log}[c x^2]\right]^3 dx$$

**Problem 72: Result unnecessarily involves imaginary or complex numbers.**

$$\int (e x)^m \text{Sin}[d (a + b \text{Log}[c x^n])]^2 dx$$

Optimal (type 3, 154 leaves, 2 steps):

$$\frac{2 b^2 d^2 n^2 (e x)^{1+m}}{e (1+m) \left( (1+m)^2 + 4 b^2 d^2 n^2 \right)} - \frac{2 b d n (e x)^{1+m} \text{Cos}[d (a + b \text{Log}[c x^n])] \text{Sin}[d (a + b \text{Log}[c x^n])]}{e \left( (1+m)^2 + 4 b^2 d^2 n^2 \right)} + \frac{(1+m) (e x)^{1+m} \text{Sin}[d (a + b \text{Log}[c x^n])]^2}{e \left( (1+m)^2 + 4 b^2 d^2 n^2 \right)}$$

Result (type 3, 102 leaves):

$$-\left( \left( x (e x)^m \left( -1 - 2 m - m^2 - 4 b^2 d^2 n^2 + (1+m)^2 \text{Cos}[2 d (a + b \text{Log}[c x^n])] + 2 b d (1+m) n \text{Sin}[2 d (a + b \text{Log}[c x^n])] \right) \right) \right) / \left( 2 (1+m) (1+m - 2 i b d n) (1+m + 2 i b d n) \right)$$

**Problem 75: Result more than twice size of optimal antiderivative.**

$$\int (e x)^m \sqrt{\text{Sin}[d (a + b \text{Log}[c x^n])]} dx$$

Optimal (type 5, 149 leaves, 3 steps):

$$\left( 2 (e x)^{1+m} \text{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{2 i + 2 i m + b d n}{4 b d n}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, e^{2 i a d} (c x^n)^{2 i b d}\right] \sqrt{\text{Sin}[d (a + b \text{Log}[c x^n])]} \right) / \left( e (2 + 2 m - i b d n) \sqrt{1 - e^{2 i a d} (c x^n)^{2 i b d}} \right)$$

Result (type 5, 582 leaves):

$$\begin{aligned}
& \left( 2 b d e^{i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} n x^{1-i b d n} (e x)^m \sqrt{2 - 2 e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n}} \right. \\
& \left( (2 + 2 m - i b d n) x^{2 i b d n} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, -\frac{2 i + 2 i m - 7 b d n}{4 b d n}, e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] - \right. \\
& \left. \left. (2 + 2 m + 3 i b d n) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b d n}{4 b d n}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] \right) \right) / \\
& \left( (2 + 2 m - i b d n) (2 + 2 m + 3 i b d n) (-2 - 2 m + i b d n + e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} (2 + 2 m + i b d n)) \right. \\
& \left. \sqrt{-i e^{-i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{-i b d n} (-1 + e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n})} \right) + \\
& \left( 2 x (e x)^m \operatorname{Sin}[d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \sqrt{\operatorname{Sin}[b d n \operatorname{Log}[x] + d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]} \right) / \\
& \left( b d n \operatorname{Cos}[d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 \operatorname{Sin}[d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 m \operatorname{Sin}[d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \right) \Big)
\end{aligned}$$

### Problem 77: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^m}{\operatorname{Sin}[d (a+b \operatorname{Log}[c x^n])]^{3/2}} dx$$

Optimal (type 5, 150 leaves, 3 steps):

$$\begin{aligned}
& \left( 2 (e x)^{1+m} \left( 1 - e^{2 i a d} (c x^n)^{2 i b d} \right)^{3/2} \operatorname{Hypergeometric2F1} \left[ \frac{3}{2}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, -\frac{2 i + 2 i m - 7 b d n}{4 b d n}, e^{2 i a d} (c x^n)^{2 i b d} \right] \right) / \\
& \left( e (2 + 2 m + 3 i b d n) \operatorname{Sin}[d (a+b \operatorname{Log}[c x^n])]^{3/2} \right)
\end{aligned}$$

Result (type 5, 2040 leaves):

$$\begin{aligned}
& \left( 4 i x^{1-i b d n} (e x)^m \sqrt{2 - 2 e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n}} \right. \\
& \left( (2 + 2 m - i b d n) x^{2 i b d n} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, -\frac{2 i + 2 i m - 7 b d n}{4 b d n}, e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] - \right. \\
& \left. \left. (2 + 2 m + 3 i b d n) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b d n}{4 b d n}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] \right) \right) / \\
& \left( b d n (2 + 2 m - i b d n) (2 + 2 m + 3 i b d n) \sqrt{-i e^{-i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{-i b d n} (-1 + e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n})} \right. \\
& \left. \left. (b d n \operatorname{Cos}[d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 \operatorname{Sin}[d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 m \operatorname{Sin}[d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \right) \right) \Big) + \\
& \left( 8 i m x^{1-i b d n} (e x)^m \sqrt{2 - 2 e^{2 i d (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n}} \right.
\end{aligned}$$

$$\begin{aligned}
& \left( (2 + 2m - i b d n) x^{2 i b d n} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, -\frac{2 i + 2 i m - 7 b d n}{4 b d n}, e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] - \right. \\
& \quad \left. (2 + 2m + 3 i b d n) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b d n}{4 b d n}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] \right) / \\
& \left( b d n (2 + 2m - i b d n) (2 + 2m + 3 i b d n) \sqrt{-i e^{-i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{-i b d n} (-1 + e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n})} \right. \\
& \quad \left. (b d n \operatorname{Cos}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) + 2 \operatorname{Sin}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 m \operatorname{Sin}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \right) \Bigg) + \\
& \left( 4 i m^2 x^{1-i b d n} (e x)^m \sqrt{2 - 2 e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n}} \right. \\
& \quad \left( (2 + 2m - i b d n) x^{2 i b d n} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, -\frac{2 i + 2 i m - 7 b d n}{4 b d n}, e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] - \right. \\
& \quad \left. (2 + 2m + 3 i b d n) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b d n}{4 b d n}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] \right) / \\
& \left( b d n (2 + 2m - i b d n) (2 + 2m + 3 i b d n) \sqrt{-i e^{-i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{-i b d n} (-1 + e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n})} \right. \\
& \quad \left. (b d n \operatorname{Cos}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) + 2 \operatorname{Sin}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 m \operatorname{Sin}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \right) \Bigg) + \\
& \left( i b d n x^{1-i b d n} (e x)^m \sqrt{2 - 2 e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n}} \right. \\
& \quad \left( (2 + 2m - i b d n) x^{2 i b d n} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, -\frac{2 i + 2 i m - 7 b d n}{4 b d n}, e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] - \right. \\
& \quad \left. (2 + 2m + 3 i b d n) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b d n}{4 b d n}, -\frac{2 i + 2 i m - 3 b d n}{4 b d n}, e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n} \right] \right) / \\
& \left( (2 + 2m - i b d n) (2 + 2m + 3 i b d n) \sqrt{-i e^{-i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{-i b d n} (-1 + e^{2 i d (a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b d n})} \right. \\
& \quad \left. (b d n \operatorname{Cos}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) + 2 \operatorname{Sin}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 m \operatorname{Sin}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \right) \Bigg) + \\
& x^{-m} (e x)^m \left( \frac{1}{b d n} 2 x^{1+m} \operatorname{Csc}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \operatorname{Csc}[b d n \operatorname{Log}[x] + d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \operatorname{Sin}[b d n \operatorname{Log}[x]] - \right. \\
& \quad \left. (2 x^{1+m} \operatorname{Csc}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) \right) / \\
& \quad \left( b d n \operatorname{Cos}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 \operatorname{Sin}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 2 m \operatorname{Sin}[d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \right) \Bigg) \\
& \sqrt{\operatorname{Sin}[b d n \operatorname{Log}[x] + d(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]}
\end{aligned}$$

### Problem 89: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[a + b \operatorname{Log}[c x^n]]}{x} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$\frac{\sin[a + b \operatorname{Log}[c x^n]]}{b n}$$

Result (type 3, 37 leaves):

$$\frac{\cos[b \operatorname{Log}[c x^n]] \sin[a]}{b n} + \frac{\cos[a] \sin[b \operatorname{Log}[c x^n]]}{b n}$$

### Problem 104: Unable to integrate problem.

$$\int x^m \cos\left[a + \sqrt{-\frac{(1+m)^2}{n^2}} \operatorname{Log}[c x^n]\right] dx$$

Optimal (type 3, 101 leaves, 3 steps):

$$\frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}} n} x^{1+m} (c x^n)^{\frac{1+m}{n}}}{4(1+m)} + \frac{1}{2} e^{a \frac{\sqrt{-\frac{(1+m)^2}{n^2}} n}{1+m}} x^{1+m} (c x^n)^{-\frac{1+m}{n}} \operatorname{Log}[x]$$

Result (type 8, 30 leaves):

$$\int x^m \cos\left[a + \sqrt{-\frac{(1+m)^2}{n^2}} \operatorname{Log}[c x^n]\right] dx$$

### Problem 105: Unable to integrate problem.

$$\int \cos\left[a + \sqrt{-\frac{1}{n^2}} \operatorname{Log}[c x^n]\right] dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$\frac{1}{4} e^{-a \sqrt{-\frac{1}{n^2}} n} x (c x^n)^{\frac{1}{n}} + \frac{1}{2} e^{a \sqrt{-\frac{1}{n^2}} n} x (c x^n)^{-1/n} \operatorname{Log}[x]$$

Result (type 8, 21 leaves):

$$\int \cos \left[ a + \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]} \right] dx$$

Problem 106: Unable to integrate problem.

$$\int x^m \cos \left[ a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2} \operatorname{Log}[c x^n]} \right]^2 dx$$

Optimal (type 3, 117 leaves, 3 steps):

$$\frac{x^{1+m}}{2(1+m)} + \frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{1+m}} x^{1+m} (c x^n)^{\frac{1+m}{n}}}{8(1+m)} + \frac{1}{4} e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{1+m}} x^{1+m} (c x^n)^{-\frac{1+m}{n}} \operatorname{Log}[x]$$

Result (type 8, 35 leaves):

$$\int x^m \cos \left[ a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2} \operatorname{Log}[c x^n]} \right]^2 dx$$

Problem 107: Unable to integrate problem.

$$\int \cos \left[ a + \frac{1}{2} \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]} \right]^2 dx$$

Optimal (type 3, 68 leaves, 3 steps):

$$\frac{x}{2} + \frac{1}{8} e^{-2a\sqrt{-\frac{1}{n^2}}} x (c x^n)^{\frac{1}{n}} + \frac{1}{4} e^{2a\sqrt{-\frac{1}{n^2}}} x (c x^n)^{-1/n} \operatorname{Log}[x]$$

Result (type 8, 26 leaves):

$$\int \cos \left[ a + \frac{1}{2} \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]} \right]^2 dx$$

### Problem 109: Unable to integrate problem.

$$\int \cos \left[ a + \frac{1}{3} \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]} \right]^3 dx$$

Optimal (type 3, 128 leaves, 3 steps):

$$\frac{9}{16} e^{a \sqrt{-\frac{1}{n^2} n}} x (c x^n)^{-\frac{1}{3}/n} + \frac{9}{32} e^{-a \sqrt{-\frac{1}{n^2} n}} x (c x^n)^{\frac{1}{3}/n} + \frac{1}{16} e^{-3a \sqrt{-\frac{1}{n^2} n}} x (c x^n)^{\frac{1}{n}} + \frac{1}{8} e^{3a \sqrt{-\frac{1}{n^2} n}} x (c x^n)^{-1/n} \operatorname{Log}[x]$$

Result (type 8, 26 leaves):

$$\int \cos \left[ a + \frac{1}{3} \sqrt{-\frac{1}{n^2} \operatorname{Log}[c x^n]} \right]^3 dx$$

### Problem 110: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\cos [a + b \operatorname{Log}[c x^n]]} dx$$

Optimal (type 5, 110 leaves, 3 steps):

$$\frac{2 x \sqrt{\cos [a + b \operatorname{Log}[c x^n]]} \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -\frac{2 i + b n}{4 b n}, \frac{1}{4} \left( 3 - \frac{2 i}{b n} \right), -e^{2 i a} (c x^n)^{2 i b} \right]}{(2 - i b n) \sqrt{1 + e^{2 i a} (c x^n)^{2 i b}}}$$

Result (type 5, 438 leaves):

$$\begin{aligned} & \left( 2 i \sqrt{2} b e^{-i (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} n x^{1-i b n} \left( (2 i + b n) \left( 1 + e^{2 i (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n} \right) + (-2 i - b n + e^{2 i (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}) (-2 i + b n) \right) \right. \\ & \quad \left. \sqrt{1 + e^{2 i (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + b n}{4 b n}, \frac{3}{4} - \frac{i}{2 b n}, -e^{2 i (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n} \right] \right) / \\ & \quad \left( (4 + b^2 n^2) (-2 i - b n + e^{2 i (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}) (-2 i + b n) \sqrt{e^{-i (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{-i b n} \left( 1 + e^{2 i (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n} \right)} \right) - \\ & \quad \frac{2 x \cos [a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \sqrt{\cos [a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]} }{-2 \cos [a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + b n \sin [a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]} \end{aligned}$$



### Problem 112: Result more than twice size of optimal antiderivative.

$$\int \cos [a + b \operatorname{Log}[c x^n]]^{3/2} dx$$

Optimal (type 5, 109 leaves, 3 steps):

$$\frac{2 x \cos [a + b \operatorname{Log}[c x^n]]^{3/2} \operatorname{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right), \frac{1}{4}\left(1 - \frac{2i}{bn}\right), -e^{2ia} (c x^n)^{2ib}\right]}{(2 - 3i bn) \left(1 + e^{2ia} (c x^n)^{2ib}\right)^{3/2}}$$

Result (type 5, 220 leaves):

$$\frac{6i\sqrt{2} b^2 \sqrt{1 + e^{2i(a+b \operatorname{Log}[c x^n])}} n^2 x \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, -e^{2i(a+b \operatorname{Log}[c x^n])}\right]}{\sqrt{e^{-i(a+b \operatorname{Log}[c x^n])} \left(1 + e^{2i(a+b \operatorname{Log}[c x^n])}\right)} (-2i + bn) (-2i + 3bn) (2i + 3bn)} +$$

$$\frac{2x \sqrt{\cos [a + b \operatorname{Log}[c x^n]]} (2 \cos [a + b \operatorname{Log}[c x^n]] + 3bn \sin [a + b \operatorname{Log}[c x^n]])}{4 + 9b^2 n^2}$$

### Problem 114: Result more than twice size of optimal antiderivative.

$$\int \cos [a + b \operatorname{Log}[c x^n]]^{5/2} dx$$

Optimal (type 5, 110 leaves, 3 steps):

$$\frac{2 x \cos [a + b \operatorname{Log}[c x^n]]^{5/2} \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right), -\frac{2i+bn}{4bn}, -e^{2ia} (c x^n)^{2ib}\right]}{(2 - 5i bn) \left(1 + e^{2ia} (c x^n)^{2ib}\right)^{5/2}}$$

Result (type 5, 681 leaves):

$$\begin{aligned}
& \left( 30 i \sqrt{2} b^3 e^{-i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} n^3 x^{1-i b n} \left( (2 i + b n) \left( 1 + e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{2 i b n} \right) + (-2 i - b n + e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} (-2 i + b n) \right) \right. \\
& \quad \left. \sqrt{1 + e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{2 i b n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2 i + b n}{4 b n}, \frac{3}{4} - \frac{i}{2 b n}, -e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{2 i b n}\right] \right) \Bigg) / \\
& \left( (-2 i + 5 b n) (2 i + 5 b n) (4 + b^2 n^2) (-2 i - b n + e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} (-2 i + b n) \right) \\
& \quad \left. \sqrt{e^{-i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{-i b n} \left( 1 + e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{2 i b n} \right)} + \sqrt{\operatorname{Cos}[a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]} \right) \\
& \left( - \left( (2 x (2 \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + 15 b^2 n^2 \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] - b n \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]) \right) / \right. \\
& \quad \left( (-2 i + 5 b n) (2 i + 5 b n) (-2 \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + b n \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]) \right) \Bigg) + \\
& \left( x \operatorname{Sin}[2 b n \operatorname{Log}[x]] (5 b n \operatorname{Cos}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] - 2 \operatorname{Sin}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) \right) / \left( (-2 i + 5 b n) (2 i + 5 b n) \right) + \\
& \left( x \operatorname{Cos}[2 b n \operatorname{Log}[x]] (2 \operatorname{Cos}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 5 b n \operatorname{Sin}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) \right) / \left( (-2 i + 5 b n) (2 i + 5 b n) \right)
\end{aligned}$$

**Problem 118: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Cos}[a + b \operatorname{Log}[c x^n]]^{3/2}} dx$$

Optimal (type 5, 109 leaves, 3 steps):

$$\frac{2 x \left( 1 + e^{2 i a} (c x^n)^{2 i b} \right)^{3/2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1}{4} \left( 3 - \frac{2 i}{b n} \right), \frac{1}{4} \left( 7 - \frac{2 i}{b n} \right), -e^{2 i a} (c x^n)^{2 i b}\right]}{(2 + 3 i b n) \operatorname{Cos}[a + b \operatorname{Log}[c x^n]]^{3/2}}$$

Result (type 5, 847 leaves):

$$\begin{aligned}
& - \left( \left( 4 \sqrt{2} e^{-2i(a+b(-n\log[x]+\log[cx^n]))} x^{1-ibn} \left( (2i+bn) \left( 1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn} \right) + (-2i-bn + e^{2i(a+b(-n\log[x]+\log[cx^n]))} (-2i+bn)) \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i+bn}{4bn}, \frac{3}{4} - \frac{i}{2bn}, -e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn} \right] \right) \right) \right) / \\
& \left( bn(4+b^2n^2) \sqrt{e^{-i(a+b(-n\log[x]+\log[cx^n]))} x^{-ibn} \left( 1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn} \right)} \right. \\
& \quad \left. (-2\cos[a+b(-n\log[x]+\log[cx^n]))] + bn\sin[a+b(-n\log[x]+\log[cx^n]))] \right) \Bigg) - \\
& \left( \sqrt{2} b e^{-2i(a+b(-n\log[x]+\log[cx^n]))} n x^{1-ibn} \left( (2i+bn) \left( 1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn} \right) + (-2i-bn + e^{2i(a+b(-n\log[x]+\log[cx^n]))} (-2i+bn)) \right. \right. \\
& \quad \left. \left. \left. \sqrt{1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i+bn}{4bn}, \frac{3}{4} - \frac{i}{2bn}, -e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn} \right] \right) \right) \right) / \\
& \left( (4+b^2n^2) \sqrt{e^{-i(a+b(-n\log[x]+\log[cx^n]))} x^{-ibn} \left( 1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn} \right)} \right. \\
& \quad \left. (-2\cos[a+b(-n\log[x]+\log[cx^n]))] + bn\sin[a+b(-n\log[x]+\log[cx^n]))] \right) \Bigg) + \\
& \sqrt{\cos[a+bn\log[x]+b(-n\log[x]+\log[cx^n]))]} \\
& \left( \frac{1}{bn} 2x \sec[a+b(-n\log[x]+\log[cx^n))] \sec[a+bn\log[x]+b(-n\log[x]+\log[cx^n))] \sin[bn\log[x]] + \right. \\
& \quad \left. \frac{2x \sec[a+b(-n\log[x]+\log[cx^n))]}{-2\cos[a+b(-n\log[x]+\log[cx^n))] + bn\sin[a+b(-n\log[x]+\log[cx^n))]} \right)
\end{aligned}$$

**Problem 123: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^m \cos[a+b\log[cx^n]]^4 dx$$

Optimal (type 3, 266 leaves, 3 steps):

$$\begin{aligned}
& \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2+4b^2n^2\right)\left((1+m)^2+16b^2n^2\right)} + \frac{12b^2(1+m)n^2x^{1+m}\cos[a+b\log[cx^n]]^2}{\left((1+m)^2+4b^2n^2\right)\left((1+m)^2+16b^2n^2\right)} + \frac{(1+m)x^{1+m}\cos[a+b\log[cx^n]]^4}{(1+m)^2+16b^2n^2} + \\
& \frac{24b^3n^3x^{1+m}\cos[a+b\log[cx^n]]\sin[a+b\log[cx^n]]}{\left((1+m)^2+4b^2n^2\right)\left((1+m)^2+16b^2n^2\right)} + \frac{4bnx^{1+m}\cos[a+b\log[cx^n]]^3\sin[a+b\log[cx^n]]}{(1+m)^2+16b^2n^2}
\end{aligned}$$

Result (type 3, 435 leaves):

$$\frac{3 x^{1+m}}{8 (1+m)} - \left( x^{1+m} \operatorname{Sin}[2 b n \operatorname{Log}[x]] \right. \\ \left. \left( -2 b n \operatorname{Cos}[2 a + 2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) + \operatorname{Sin}[2 a + 2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) + m \operatorname{Sin}[2 a + 2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \left. \right) / \\ \left( 2 (1+m-2 i b n) (1+m+2 i b n) \right) + \left( x^{1+m} \operatorname{Cos}[2 b n \operatorname{Log}[x]] \right. \\ \left. \left( \operatorname{Cos}[2 a + 2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) + m \operatorname{Cos}[2 a + 2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) + 2 b n \operatorname{Sin}[2 a + 2 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \left. \right) / \\ \left( 2 (1+m-2 i b n) (1+m+2 i b n) \right) - \left( x^{1+m} \operatorname{Sin}[4 b n \operatorname{Log}[x]] \right. \\ \left. \left( -4 b n \operatorname{Cos}[4 a + 4 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) + \operatorname{Sin}[4 a + 4 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) + m \operatorname{Sin}[4 a + 4 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \left. \right) / \\ \left( 8 (1+m-4 i b n) (1+m+4 i b n) \right) + \left( x^{1+m} \operatorname{Cos}[4 b n \operatorname{Log}[x]] \right. \\ \left. \left( \operatorname{Cos}[4 a + 4 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) \right) + \\ m \operatorname{Cos}[4 a + 4 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + 4 b n \operatorname{Sin}[4 a + 4 b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \left. \right) / \left( 8 (1+m-4 i b n) (1+m+4 i b n) \right)$$

**Problem 125: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^m \operatorname{Cos}[a + b \operatorname{Log}[c x^n]]^2 dx$$

Optimal (type 3, 120 leaves, 2 steps):

$$\frac{2 b^2 n^2 x^{1+m}}{(1+m) \left( (1+m)^2 + 4 b^2 n^2 \right)} + \frac{(1+m) x^{1+m} \operatorname{Cos}[a + b \operatorname{Log}[c x^n]]^2}{(1+m)^2 + 4 b^2 n^2} + \frac{2 b n x^{1+m} \operatorname{Cos}[a + b \operatorname{Log}[c x^n]] \operatorname{Sin}[a + b \operatorname{Log}[c x^n]]}{(1+m)^2 + 4 b^2 n^2}$$

Result (type 3, 91 leaves):

$$\frac{x^{1+m} \left( 1 + 2 m + m^2 + 4 b^2 n^2 + (1+m)^2 \operatorname{Cos}[2 (a + b \operatorname{Log}[c x^n])] + 2 b (1+m) n \operatorname{Sin}[2 (a + b \operatorname{Log}[c x^n])] \right)}{2 (1+m) (1+m-2 i b n) (1+m+2 i b n)}$$

**Problem 128: Result more than twice size of optimal antiderivative.**

$$\int x^m \sqrt{\operatorname{Cos}[a + b \operatorname{Log}[c x^n]]} dx$$

Optimal (type 5, 129 leaves, 3 steps):

$$\frac{2 x^{1+m} \sqrt{\operatorname{Cos}[a + b \operatorname{Log}[c x^n]]} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{2i+2im+bn}{4bn}, -\frac{2i+2im-3bn}{4bn}, -e^{2ia} (c x^n)^{2ib}\right]}{(2+2m-ibn) \sqrt{1+e^{2ia} (c x^n)^{2ib}}}$$

Result (type 5, 529 leaves):

$$\begin{aligned}
& - \left( \left( 2 b e^{i(a+b(-n \log[x] + \log[c x^n]))} n x^{1+m-i b n} \sqrt{2 + 2 e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n}} \right. \right. \\
& \quad \left( (2 i + 2 i m + b n) x^{2 i b n} \text{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -\frac{2 i + 2 i m - 7 b n}{4 b n}, -e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n} \right] + \right. \\
& \quad \left. \left. (-2 i - 2 i m + 3 b n) \text{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b n}{4 b n}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n} \right] \right) \right) / \\
& \quad \left( (2 + 2 m - i b n) (2 + 2 m + 3 i b n) (2 + 2 m - i b n + e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n}) (2 + 2 m + i b n) \right) \\
& \quad \left. \sqrt{e^{-i(a+b(-n \log[x] + \log[c x^n]))} x^{-i b n} (1 + e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n})} \right) \right) + \\
& \quad \left( 2 x^{1+m} \cos[a + b(-n \log[x] + \log[c x^n])] \sqrt{\cos[a + b n \log[x] + b(-n \log[x] + \log[c x^n])]} \right) / \\
& \quad \left( 2 \cos[a + b(-n \log[x] + \log[c x^n])] + 2 m \cos[a + b(-n \log[x] + \log[c x^n])] - b n \sin[a + b(-n \log[x] + \log[c x^n])] \right)
\end{aligned}$$

**Problem 130: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^m}{\cos[a + b \log[c x^n]]^{3/2}} dx$$

Optimal (type 5, 130 leaves, 3 steps):

$$\frac{2 x^{1+m} (1 + e^{2 i a} (c x^n)^{2 i b})^{3/2} \text{Hypergeometric2F1} \left[ \frac{3}{2}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -\frac{2 i + 2 i m - 7 b n}{4 b n}, -e^{2 i a} (c x^n)^{2 i b} \right]}{(2 + 2 m + 3 i b n) \cos[a + b \log[c x^n]]^{3/2}}$$

Result (type 5, 1822 leaves):

$$\begin{aligned}
& - \left( \left( 4 i x^{1+m-i b n} \sqrt{2 + 2 e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n}} \right. \right. \\
& \quad \left( (2 + 2 m - i b n) x^{2 i b n} \text{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -\frac{2 i + 2 i m - 7 b n}{4 b n}, -e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n} \right] - \right. \\
& \quad \left. (2 + 2 m + 3 i b n) \text{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b n}{4 b n}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n} \right] \right) \right) / \\
& \quad \left( b n (2 + 2 m - i b n) (2 + 2 m + 3 i b n) \sqrt{e^{-i(a+b(-n \log[x] + \log[c x^n]))} x^{-i b n} (1 + e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n})} \right. \\
& \quad \left. \left. (-2 \cos[a + b(-n \log[x] + \log[c x^n])] - 2 m \cos[a + b(-n \log[x] + \log[c x^n])] + b n \sin[a + b(-n \log[x] + \log[c x^n])]) \right) \right) - \\
& \quad \left( 8 i m x^{1+m-i b n} \sqrt{2 + 2 e^{2 i(a+b(-n \log[x] + \log[c x^n]))} x^{2 i b n}} \right.
\end{aligned}$$

$$\begin{aligned}
& \left( (2 + 2m - ibn) x^{2ibn} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i + 2im - 3bn}{4bn}, -\frac{2i + 2im - 7bn}{4bn}, -e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn} \right] - \right. \\
& \left. (2 + 2m + 3ibn) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i + 2im + bn}{4bn}, -\frac{2i + 2im - 3bn}{4bn}, -e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn} \right] \right) / \\
& \left( bn(2 + 2m - ibn)(2 + 2m + 3ibn) \sqrt{e^{-i(a+b(-\log[x] + \log[cx^n]))} x^{-ibn} (1 + e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn})} \right. \\
& \left. (-2 \cos[a + b(-\log[x] + \log[cx^n])] - 2m \cos[a + b(-\log[x] + \log[cx^n])] + bn \sin[a + b(-\log[x] + \log[cx^n])]) \right) - \\
& \left( 4im^2 x^{1+m-ibn} \sqrt{2 + 2e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn}} \right. \\
& \left. (2 + 2m - ibn) x^{2ibn} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i + 2im - 3bn}{4bn}, -\frac{2i + 2im - 7bn}{4bn}, -e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn} \right] - \right. \\
& \left. (2 + 2m + 3ibn) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i + 2im + bn}{4bn}, -\frac{2i + 2im - 3bn}{4bn}, -e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn} \right] \right) / \\
& \left( bn(2 + 2m - ibn)(2 + 2m + 3ibn) \sqrt{e^{-i(a+b(-\log[x] + \log[cx^n]))} x^{-ibn} (1 + e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn})} \right. \\
& \left. (-2 \cos[a + b(-\log[x] + \log[cx^n])] - 2m \cos[a + b(-\log[x] + \log[cx^n])] + bn \sin[a + b(-\log[x] + \log[cx^n])]) \right) - \\
& \left( ibn x^{1+m-ibn} \sqrt{2 + 2e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn}} \right. \\
& \left. (2 + 2m - ibn) x^{2ibn} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i + 2im - 3bn}{4bn}, -\frac{2i + 2im - 7bn}{4bn}, -e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn} \right] - \right. \\
& \left. (2 + 2m + 3ibn) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i + 2im + bn}{4bn}, -\frac{2i + 2im - 3bn}{4bn}, -e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn} \right] \right) / \\
& \left( (2 + 2m - ibn)(2 + 2m + 3ibn) \sqrt{e^{-i(a+b(-\log[x] + \log[cx^n]))} x^{-ibn} (1 + e^{2i(a+b(-\log[x] + \log[cx^n]))} x^{2ibn})} \right. \\
& \left. (-2 \cos[a + b(-\log[x] + \log[cx^n])] - 2m \cos[a + b(-\log[x] + \log[cx^n])] + bn \sin[a + b(-\log[x] + \log[cx^n])]) \right) + \\
& \sqrt{\cos[a + bn \log[x] + b(-\log[x] + \log[cx^n])]} \left( \frac{1}{bn} 2x^{1+m} \operatorname{Sec}[a + b(-\log[x] + \log[cx^n])] \right. \\
& \left. \operatorname{Sec}[a + bn \log[x] + b(-\log[x] + \log[cx^n])] \sin[bn \log[x]] - (2x^{1+m} \operatorname{Sec}[a + b(-\log[x] + \log[cx^n])]) \right) / \\
& \left( 2 \cos[a + b(-\log[x] + \log[cx^n])] + 2m \cos[a + b(-\log[x] + \log[cx^n])] - bn \sin[a + b(-\log[x] + \log[cx^n]) \right) \right)
\end{aligned}$$

**Problem 131:** Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{\cos[a + b \operatorname{Log}[c x^n]]^{5/2}} dx$$

Optimal (type 5, 130 leaves, 3 steps):

$$\frac{2 x^{1+m} \left(1 + e^{2 i a} (c x^n)^{2 i b}\right)^{5/2} \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, -\frac{2 i+2 i m-5 b n}{4 b n}, -\frac{2 i+2 i m-9 b n}{4 b n}, -e^{2 i a} (c x^n)^{2 i b}\right]}{(2 + 2 m + 5 i b n) \cos[a + b \operatorname{Log}[c x^n]]^{5/2}}$$

Result (type 5, 263 leaves):

$$\begin{aligned} & \left( x^{1+m} \left( -4 (1+m) \cos[a + b \operatorname{Log}[c x^n]] + \right. \right. \\ & \left. \left( (2 + 2 m - i b n) \sqrt{1 + e^{2 i a} (c x^n)^{2 i b}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{-2 i - 2 i m + b n}{4 b n}, -\frac{2 i + 2 i m - 5 b n}{4 b n}, -e^{2 i (a+b \operatorname{Log}[c x^n])}\right] \right) \right. \\ & \left. \left. \left( e^{-i a} \left( (1 + e^{2 i a}) \cos[b \operatorname{Log}[c x^n]] + i (-1 + e^{2 i a}) \sin[b \operatorname{Log}[c x^n]] \right) \right)^{3/2} \right) \right) / \\ & \left( \sqrt{e^{-i a} (c x^n)^{-i b} + e^{i a} (c x^n)^{i b}} + 2 b n \sin[a + b \operatorname{Log}[c x^n]] \right) / \left( 3 b^2 n^2 \cos[a + b \operatorname{Log}[c x^n]]^{3/2} \right) \end{aligned}$$

**Problem 135: Result more than twice size of optimal antiderivative.**

$$\int x^3 \operatorname{Tan}[a + i \operatorname{Log}[x]] dx$$

Optimal (type 3, 47 leaves, 5 steps):

$$-i e^{2 i a} x^2 + \frac{i x^4}{4} + i e^{4 i a} \operatorname{Log}[e^{2 i a} + x^2]$$

Result (type 3, 132 leaves):

$$\begin{aligned} & \frac{i x^4}{4} - i x^2 \cos[2 a] + \operatorname{ArcTan}\left[\frac{(1+x^2) \cos[a]}{\sin[a] - x^2 \sin[a]}\right] \cos[4 a] + \frac{1}{2} i \cos[4 a] \operatorname{Log}[1 + x^4 + 2 x^2 \cos[2 a]] + \\ & x^2 \sin[2 a] + i \operatorname{ArcTan}\left[\frac{(1+x^2) \cos[a]}{\sin[a] - x^2 \sin[a]}\right] \sin[4 a] - \frac{1}{2} \operatorname{Log}[1 + x^4 + 2 x^2 \cos[2 a]] \sin[4 a] \end{aligned}$$

**Problem 137: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Tan}[a + i \operatorname{Log}[x]] dx$$

Optimal (type 3, 33 leaves, 5 steps):

$$\frac{i x^2}{2} - i e^{2 i a} \operatorname{Log}\left[e^{2 i a} + x^2\right]$$

Result (type 3, 114 leaves):

$$\frac{i x^2}{2} - \operatorname{ArcTan}\left[\frac{(1+x^2) \cos[a]}{\sin[a] - x^2 \sin[a]}\right] \cos[2 a] - \frac{1}{2} i \cos[2 a] \operatorname{Log}\left[1+x^4+2 x^2 \cos[2 a]\right] -$$

$$i \operatorname{ArcTan}\left[\frac{(1+x^2) \cos[a]}{\sin[a] - x^2 \sin[a]}\right] \sin[2 a] + \frac{1}{2} \operatorname{Log}\left[1+x^4+2 x^2 \cos[2 a]\right] \sin[2 a]$$

**Problem 141: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[a + i \operatorname{Log}[x]]}{x^3} dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$\frac{i}{2 x^2} - i e^{-2 i a} \operatorname{Log}\left[1 + \frac{e^{2 i a}}{x^2}\right]$$

Result (type 3, 132 leaves):

$$\frac{i}{2 x^2} - \operatorname{ArcTan}\left[\frac{(1+x^2) \cos[a]}{\sin[a] - x^2 \sin[a]}\right] \cos[2 a] + 2 i \cos[2 a] \operatorname{Log}[x] - \frac{1}{2} i \cos[2 a] \operatorname{Log}\left[1+x^4+2 x^2 \cos[2 a]\right] +$$

$$i \operatorname{ArcTan}\left[\frac{(1+x^2) \cos[a]}{\sin[a] - x^2 \sin[a]}\right] \sin[2 a] + 2 \operatorname{Log}[x] \sin[2 a] - \frac{1}{2} \operatorname{Log}\left[1+x^4+2 x^2 \cos[2 a]\right] \sin[2 a]$$

**Problem 143: Result more than twice size of optimal antiderivative.**

$$\int x^3 \operatorname{Tan}[a + i \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$2 e^{2 i a} x^2 - \frac{x^4}{4} - \frac{2 e^{6 i a}}{e^{2 i a} + x^2} - 4 e^{4 i a} \operatorname{Log}\left[e^{2 i a} + x^2\right]$$

Result (type 3, 155 leaves):

$$-\frac{x^4}{4} + 2 x^2 \cos[2 a] - 4 i \operatorname{ArcTan}\left[\frac{(1+x^2) \cot[a]}{-1+x^2}\right] \cos[4 a] - 2 \cos[4 a] \operatorname{Log}\left[1+x^4+2 x^2 \cos[2 a]\right] + 2 i x^2 \sin[2 a] +$$

$$4 \operatorname{ArcTan}\left[\frac{(1+x^2) \cot[a]}{-1+x^2}\right] \sin[4 a] - 2 i \operatorname{Log}\left[1+x^4+2 x^2 \cos[2 a]\right] \sin[4 a] - \frac{2 (\cos[5 a] + i \sin[5 a])}{(1+x^2) \cos[a] - i (-1+x^2) \sin[a]}$$



### Problem 145: Result more than twice size of optimal antiderivative.

$$\int x \tan[a + i \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$-\frac{x^2}{2} + \frac{2 e^{4 i a}}{e^{2 i a} + x^2} + 2 e^{2 i a} \operatorname{Log}[e^{2 i a} + x^2]$$

Result (type 3, 135 leaves):

$$-\frac{x^2}{2} + 2 i \operatorname{ArcTan}\left[\frac{(1+x^2) \operatorname{Cot}[a]}{-1+x^2}\right] \cos[2 a] + \cos[2 a] \operatorname{Log}[1+x^4+2 x^2 \cos[2 a]] -$$

$$2 \operatorname{ArcTan}\left[\frac{(1+x^2) \operatorname{Cot}[a]}{-1+x^2}\right] \sin[2 a] + i \operatorname{Log}[1+x^4+2 x^2 \cos[2 a]] \sin[2 a] + \frac{2 \cos[3 a] + 2 i \sin[3 a]}{(1+x^2) \cos[a] - i(-1+x^2) \sin[a]}$$

### Problem 149: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan[a + i \operatorname{Log}[x]]^2}{x^3} dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$-\frac{2 e^{-2 i a}}{1 + \frac{e^{2 i a}}{x^2}} + \frac{1}{2 x^2} - 2 e^{-2 i a} \operatorname{Log}\left[1 + \frac{e^{2 i a}}{x^2}\right]$$

Result (type 3, 150 leaves):

$$\frac{1}{2 x^2} - 2 i \operatorname{ArcTan}\left[\frac{(1+x^2) \operatorname{Cot}[a]}{-1+x^2}\right] \cos[2 a] + 4 \cos[2 a] \operatorname{Log}[x] - \cos[2 a] \operatorname{Log}[1+x^4+2 x^2 \cos[2 a]] +$$

$$\frac{2 \cos[a] - 2 i \sin[a]}{(1+x^2) \cos[a] - i(-1+x^2) \sin[a]} - 2 \operatorname{ArcTan}\left[\frac{(1+x^2) \operatorname{Cot}[a]}{-1+x^2}\right] \sin[2 a] - 4 i \operatorname{Log}[x] \sin[2 a] + i \operatorname{Log}[1+x^4+2 x^2 \cos[2 a]] \sin[2 a]$$

### Problem 151: Result more than twice size of optimal antiderivative.

$$\int (e x)^m \tan[a + i \operatorname{Log}[x]]^2 dx$$

Optimal (type 5, 77 leaves, 5 steps):

$$-\frac{x(e^x)^m}{1+m} + \frac{2x(e^x)^m}{1+\frac{e^{2ia}}{x^2}} - 2x(e^x)^m \text{Hypergeometric2F1}\left[1, \frac{1}{2}(-1-m), \frac{1-m}{2}, -\frac{e^{2ia}}{x^2}\right]$$

Result (type 5, 172 leaves):

$$\frac{1}{(\cos[a] + i \sin[a])^2} x (e^x)^m \left( \frac{2x^2 \text{Hypergeometric2F1}\left[2, \frac{3+m}{2}, \frac{5+m}{2}, -x^2(\cos[2a] - i \sin[2a])\right]}{3+m} - \frac{x^4 \text{Hypergeometric2F1}\left[2, \frac{5+m}{2}, \frac{7+m}{2}, -x^2(\cos[2a] - i \sin[2a])\right] (\cos[a] - i \sin[a])^2}{5+m} - \frac{\text{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, -x^2(\cos[2a] - i \sin[2a])\right] (\cos[2a] + i \sin[2a])}{1+m} \right)$$

**Problem 153: Result more than twice size of optimal antiderivative.**

$$\int \tan[a + b \log[x]]^p dx$$

Optimal (type 6, 142 leaves, 4 steps):

$$x(1 - e^{2ia}x^{2ib})^{-p} \left( \frac{i(1 - e^{2ia}x^{2ib})}{1 + e^{2ia}x^{2ib}} \right)^p (1 + e^{2ia}x^{2ib})^p \text{AppellF1}\left[-\frac{i}{2b}, -p, p, 1 - \frac{i}{2b}, e^{2ia}x^{2ib}, -e^{2ia}x^{2ib}\right]$$

Result (type 6, 330 leaves):

$$\left( (-i + 2b)x \left( -\frac{i(-1 + e^{2ia}x^{2ib})}{1 + e^{2ia}x^{2ib}} \right)^p \text{AppellF1}\left[-\frac{i}{2b}, -p, p, 1 - \frac{i}{2b}, e^{2ia}x^{2ib}, -e^{2ia}x^{2ib}\right] \right) / \left( -2be^{2ia}px^{2ib} \text{AppellF1}\left[1 - \frac{i}{2b}, 1 - p, p, 2 - \frac{i}{2b}, e^{2ia}x^{2ib}, -e^{2ia}x^{2ib}\right] - 2be^{2ia}px^{2ib} \text{AppellF1}\left[1 - \frac{i}{2b}, -p, 1 + p, 2 - \frac{i}{2b}, e^{2ia}x^{2ib}, -e^{2ia}x^{2ib}\right] + (-i + 2b) \text{AppellF1}\left[-\frac{i}{2b}, -p, p, 1 - \frac{i}{2b}, e^{2ia}x^{2ib}, -e^{2ia}x^{2ib}\right] \right)$$

**Problem 154: Result more than twice size of optimal antiderivative.**

$$\int (e^x)^m \tan[a + b \log[x]]^p dx$$

Optimal (type 6, 162 leaves, 4 steps):

$$\frac{1}{e(1+m)} (e^x)^{1+m} (1 - e^{2ia}x^{2ib})^{-p} \left( \frac{i(1 - e^{2ia}x^{2ib})}{1 + e^{2ia}x^{2ib}} \right)^p (1 + e^{2ia}x^{2ib})^p \text{AppellF1}\left[-\frac{i(1+m)}{2b}, -p, p, 1 - \frac{i(1+m)}{2b}, e^{2ia}x^{2ib}, -e^{2ia}x^{2ib}\right]$$

Result (type 6, 351 leaves):

$$\left( (1 + 2 i b + m) x (e x)^m \left( -\frac{i (-1 + e^{2 i a} x^{2 i b})}{1 + e^{2 i a} x^{2 i b}} \right)^p \text{AppellF1} \left[ -\frac{i (1 + m)}{2 b}, -p, p, 1 - \frac{i (1 + m)}{2 b}, e^{2 i a} x^{2 i b}, -e^{2 i a} x^{2 i b} \right] \right) /$$

$$\left( (1 + m) \left( (1 + 2 i b + m) \text{AppellF1} \left[ -\frac{i (1 + m)}{2 b}, -p, p, 1 - \frac{i (1 + m)}{2 b}, e^{2 i a} x^{2 i b}, -e^{2 i a} x^{2 i b} \right] - 2 i b e^{2 i a} p x^{2 i b} \left( \text{AppellF1} \left[ 1 - \frac{i (1 + m)}{2 b}, 1 - p, p, 2 - \frac{i (1 + m)}{2 b}, e^{2 i a} x^{2 i b}, -e^{2 i a} x^{2 i b} \right] + \text{AppellF1} \left[ 1 - \frac{i (1 + m)}{2 b}, -p, 1 + p, 2 - \frac{i (1 + m)}{2 b}, e^{2 i a} x^{2 i b}, -e^{2 i a} x^{2 i b} \right] \right) \right) \right)$$

Problem 158: Result more than twice size of optimal antiderivative.

$$\int x^3 \text{Tan} [d (a + b \text{Log} [c x^n])] dx$$

Optimal (type 5, 71 leaves, 4 steps):

$$-\frac{i x^4}{4} + \frac{1}{2} i x^4 \text{Hypergeometric2F1} \left[ 1, -\frac{2 i}{b d n}, 1 - \frac{2 i}{b d n}, -e^{2 i a d} (c x^n)^{2 i b d} \right]$$

Result (type 5, 146 leaves):

$$\frac{1}{-8 - 4 i b d n} x^4 \left( 2 i e^{2 i d (a + b \text{Log} [c x^n])} \text{Hypergeometric2F1} \left[ 1, 1 - \frac{2 i}{b d n}, 2 - \frac{2 i}{b d n}, -e^{2 i d (a + b \text{Log} [c x^n])} \right] + (-2 i + b d n) \text{Hypergeometric2F1} \left[ 1, -\frac{2 i}{b d n}, 1 - \frac{2 i}{b d n}, -e^{2 i d (a + b \text{Log} [c x^n])} \right] \right)$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int x^2 \text{Tan} [d (a + b \text{Log} [c x^n])] dx$$

Optimal (type 5, 75 leaves, 4 steps):

$$-\frac{i x^3}{3} + \frac{2}{3} i x^3 \text{Hypergeometric2F1} \left[ 1, -\frac{3 i}{2 b d n}, 1 - \frac{3 i}{2 b d n}, -e^{2 i a d} (c x^n)^{2 i b d} \right]$$

Result (type 5, 155 leaves):

$$\frac{1}{-9 - 6 i b d n} x^3 \left( 3 i e^{2 i d (a + b \text{Log} [c x^n])} \text{Hypergeometric2F1} \left[ 1, 1 - \frac{3 i}{2 b d n}, 2 - \frac{3 i}{2 b d n}, -e^{2 i d (a + b \text{Log} [c x^n])} \right] + (-3 i + 2 b d n) \text{Hypergeometric2F1} \left[ 1, -\frac{3 i}{2 b d n}, 1 - \frac{3 i}{2 b d n}, -e^{2 i d (a + b \text{Log} [c x^n])} \right] \right)$$

### Problem 160: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Tan}[d (a + b \operatorname{Log}[c x^n])] dx$$

Optimal (type 5, 69 leaves, 4 steps):

$$-\frac{i x^2}{2} + i x^2 \operatorname{Hypergeometric2F1}\left[1, -\frac{i}{b d n}, 1 - \frac{i}{b d n}, -e^{2 i a d} (c x^n)^{2 i b d}\right]$$

Result (type 5, 146 leaves):

$$\frac{1}{-2 - 2 i b d n} x^2 \left( i e^{2 i d (a + b \operatorname{Log}[c x^n])} \operatorname{Hypergeometric2F1}\left[1, 1 - \frac{i}{b d n}, 2 - \frac{i}{b d n}, -e^{2 i d (a + b \operatorname{Log}[c x^n])}\right] + (-i + b d n) \operatorname{Hypergeometric2F1}\left[1, -\frac{i}{b d n}, 1 - \frac{i}{b d n}, -e^{2 i d (a + b \operatorname{Log}[c x^n])}\right] \right)$$

### Problem 161: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Tan}[d (a + b \operatorname{Log}[c x^n])] dx$$

Optimal (type 5, 67 leaves, 4 steps):

$$-i x + 2 i x \operatorname{Hypergeometric2F1}\left[1, -\frac{i}{2 b d n}, 1 - \frac{i}{2 b d n}, -e^{2 i a d} (c x^n)^{2 i b d}\right]$$

Result (type 5, 154 leaves):

$$\frac{1}{-i + 2 b d n} x \left( -e^{2 i a d} (c x^n)^{2 i b d} \operatorname{Hypergeometric2F1}\left[1, 1 - \frac{i}{2 b d n}, 2 - \frac{i}{2 b d n}, -e^{2 i d (a + b \operatorname{Log}[c x^n])}\right] + (1 + 2 i b d n) \operatorname{Hypergeometric2F1}\left[1, -\frac{i}{2 b d n}, 1 - \frac{i}{2 b d n}, -e^{2 i d (a + b \operatorname{Log}[c x^n])}\right] \right)$$

### Problem 163: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tan}[d (a + b \operatorname{Log}[c x^n])]}{x^2} dx$$

Optimal (type 5, 71 leaves, 4 steps):

$$\frac{i}{x} - \frac{2 i \operatorname{Hypergeometric2F1}\left[1, \frac{i}{2 b d n}, 1 + \frac{i}{2 b d n}, -e^{2 i a d} (c x^n)^{2 i b d}\right]}{x}$$

Result (type 5, 153 leaves):

$$\frac{1}{(\frac{i}{2} + 2 b d n) x} \left( -e^{2 i d (a+b \operatorname{Log}[c x^n])} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{i}{2 b d n}, 2 + \frac{i}{2 b d n}, -e^{2 i d (a+b \operatorname{Log}[c x^n])}\right] + \right. \\ \left. (1 - 2 i b d n) \operatorname{Hypergeometric2F1}\left[1, \frac{i}{2 b d n}, 1 + \frac{i}{2 b d n}, -e^{2 i d (a+b \operatorname{Log}[c x^n])}\right] \right)$$

**Problem 164: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Tan}[d (a + b \operatorname{Log}[c x^n])]}{x^3} dx$$

Optimal (type 5, 69 leaves, 4 steps):

$$\frac{i}{2 x^2} - \frac{i \operatorname{Hypergeometric2F1}\left[1, \frac{i}{b d n}, 1 + \frac{i}{b d n}, -e^{2 i a d} (c x^n)^{2 i b d}\right]}{x^2}$$

Result (type 5, 147 leaves):

$$\frac{1}{2 (i + b d n) x^2} \left( -e^{2 i d (a+b \operatorname{Log}[c x^n])} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{i}{b d n}, 2 + \frac{i}{b d n}, -e^{2 i d (a+b \operatorname{Log}[c x^n])}\right] + \right. \\ \left. (1 - i b d n) \operatorname{Hypergeometric2F1}\left[1, \frac{i}{b d n}, 1 + \frac{i}{b d n}, -e^{2 i d (a+b \operatorname{Log}[c x^n])}\right] \right)$$

**Problem 178: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Tan}[d (a + b \operatorname{Log}[c x^n])]^p dx$$

Optimal (type 6, 190 leaves, 5 steps):

$$x \left(1 - e^{2 i a d} (c x^n)^{2 i b d}\right)^{-p} \left( \frac{i \left(1 - e^{2 i a d} (c x^n)^{2 i b d}\right)^p}{1 + e^{2 i a d} (c x^n)^{2 i b d}} \right)^p \left(1 + e^{2 i a d} (c x^n)^{2 i b d}\right)^p \\ \operatorname{AppellF1}\left[-\frac{i}{2 b d n}, -p, p, 1 - \frac{i}{2 b d n}, e^{2 i a d} (c x^n)^{2 i b d}, -e^{2 i a d} (c x^n)^{2 i b d}\right]$$

Result (type 6, 458 leaves):

$$\left( (-i + 2bdn) x \left( -\frac{i(-1 + e^{2iad}(cx^n)^{2ibd})}{1 + e^{2iad}(cx^n)^{2ibd}} \right)^p \text{AppellF1}\left[-\frac{i}{2bdn}, -p, p, 1 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] \right) /$$

$$\left( -2bd e^{2iad} n p (cx^n)^{2ibd} \text{AppellF1}\left[1 - \frac{i}{2bdn}, 1 - p, p, 2 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] - \right.$$

$$2bd e^{2iad} n p (cx^n)^{2ibd} \text{AppellF1}\left[1 - \frac{i}{2bdn}, -p, 1 + p, 2 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] +$$

$$\left. (-i + 2bdn) \text{AppellF1}\left[-\frac{i}{2bdn}, -p, p, 1 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] \right)$$

**Problem 179: Result more than twice size of optimal antiderivative.**

$$\int (ex)^m \text{Tan}[d(a + b \text{Log}[cx^n])]^p dx$$

Optimal (type 6, 210 leaves, 5 steps):

$$\frac{1}{e(1+m)} (ex)^{1+m} (1 - e^{2iad}(cx^n)^{2ibd})^{-p} \left( \frac{i(1 - e^{2iad}(cx^n)^{2ibd})}{1 + e^{2iad}(cx^n)^{2ibd}} \right)^p$$

$$(1 + e^{2iad}(cx^n)^{2ibd})^p \text{AppellF1}\left[-\frac{i(1+m)}{2bdn}, -p, p, 1 - \frac{i(1+m)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right]$$

Result (type 6, 496 leaves):

$$\left( (1+m+2ibd) x (ex)^m \left( -\frac{i(-1 + e^{2iad}(cx^n)^{2ibd})}{1 + e^{2iad}(cx^n)^{2ibd}} \right)^p \text{AppellF1}\left[-\frac{i(1+m)}{2bdn}, -p, p, 1 - \frac{i(1+m)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] \right) /$$

$$\left( (1+m) \left( (1+m+2ibd) \text{AppellF1}\left[-\frac{i(1+m)}{2bdn}, -p, p, -\frac{i(1+m+2ibd)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] - \right.$$

$$2ibd e^{2iad} n p (cx^n)^{2ibd} \left( \text{AppellF1}\left[-\frac{i(1+m+2ibd)}{2bdn}, 1-p, p, -\frac{i(1+m+4ibd)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] + \right.$$

$$\left. \left. \left. \text{AppellF1}\left[-\frac{i(1+m+2ibd)}{2bdn}, -p, 1+p, -\frac{i(1+m+4ibd)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] \right) \right) \right)$$

**Problem 186: Result more than twice size of optimal antiderivative.**

$$\int x^3 \text{Cot}[a + i \text{Log}[x]] dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-i e^{2ia} x^2 - \frac{i x^4}{4} - i e^{4ia} \operatorname{Log}[e^{2ia} - x^2]$$

Result (type 3, 137 leaves):

$$-\frac{i x^4}{4} - i x^2 \cos[2a] - \operatorname{ArcTan}\left[\frac{(-1+x^2)\cos[a]}{-\sin[a] - x^2 \sin[a]}\right] \cos[4a] - \frac{1}{2} i \cos[4a] \operatorname{Log}[1+x^4 - 2x^2 \cos[2a]] +$$

$$x^2 \sin[2a] - i \operatorname{ArcTan}\left[\frac{(-1+x^2)\cos[a]}{-\sin[a] - x^2 \sin[a]}\right] \sin[4a] + \frac{1}{2} \operatorname{Log}[1+x^4 - 2x^2 \cos[2a]] \sin[4a]$$

**Problem 188: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Cot}[a + i \operatorname{Log}[x]] dx$$

Optimal (type 3, 35 leaves, 5 steps):

$$-\frac{i x^2}{2} - i e^{2ia} \operatorname{Log}[e^{2ia} - x^2]$$

Result (type 3, 118 leaves):

$$-\frac{i x^2}{2} - \operatorname{ArcTan}\left[\frac{(-1+x^2)\cos[a]}{-\sin[a] - x^2 \sin[a]}\right] \cos[2a] - \frac{1}{2} i \cos[2a] \operatorname{Log}[1+x^4 - 2x^2 \cos[2a]] -$$

$$i \operatorname{ArcTan}\left[\frac{(-1+x^2)\cos[a]}{-\sin[a] - x^2 \sin[a]}\right] \sin[2a] + \frac{1}{2} \operatorname{Log}[1+x^4 - 2x^2 \cos[2a]] \sin[2a]$$

**Problem 192: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cot}[a + i \operatorname{Log}[x]]}{x^3} dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{i}{2x^2} - i e^{-2ia} \operatorname{Log}\left[1 - \frac{e^{2ia}}{x^2}\right]$$

Result (type 3, 136 leaves):

$$-\frac{i}{2x^2} - \operatorname{ArcTan}\left[\frac{(-1+x^2)\cos[a]}{-\sin[a] - x^2 \sin[a]}\right] \cos[2a] + 2i \cos[2a] \operatorname{Log}[x] - \frac{1}{2} i \cos[2a] \operatorname{Log}[1+x^4 - 2x^2 \cos[2a]] +$$

$$i \operatorname{ArcTan}\left[\frac{(-1+x^2)\cos[a]}{-\sin[a] - x^2 \sin[a]}\right] \sin[2a] + 2 \operatorname{Log}[x] \sin[2a] - \frac{1}{2} \operatorname{Log}[1+x^4 - 2x^2 \cos[2a]] \sin[2a]$$

### Problem 194: Result more than twice size of optimal antiderivative.

$$\int x^3 \operatorname{Cot}[a + i \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 67 leaves, 5 steps):

$$-2 e^{2 i a} x^2 - \frac{x^4}{4} - \frac{2 e^{6 i a}}{e^{2 i a} - x^2} - 4 e^{4 i a} \operatorname{Log}[e^{2 i a} - x^2]$$

Result (type 3, 162 leaves):

$$\begin{aligned} & -\frac{x^4}{4} - 2 x^2 \operatorname{Cos}[2 a] + 4 i \operatorname{ArcTan}\left[\frac{\operatorname{Cot}[a] - x^2 \operatorname{Cot}[a]}{1 + x^2}\right] \operatorname{Cos}[4 a] - 2 \operatorname{Cos}[4 a] \operatorname{Log}[1 + x^4 - 2 x^2 \operatorname{Cos}[2 a]] - 2 i x^2 \operatorname{Sin}[2 a] - \\ & 4 \operatorname{ArcTan}\left[\frac{\operatorname{Cot}[a] - x^2 \operatorname{Cot}[a]}{1 + x^2}\right] \operatorname{Sin}[4 a] - 2 i \operatorname{Log}[1 + x^4 - 2 x^2 \operatorname{Cos}[2 a]] \operatorname{Sin}[4 a] + \frac{2 \operatorname{Cos}[5 a] + 2 i \operatorname{Sin}[5 a]}{(-1 + x^2) \operatorname{Cos}[a] - i (1 + x^2) \operatorname{Sin}[a]} \end{aligned}$$

### Problem 196: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Cot}[a + i \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$-\frac{x^2}{2} - \frac{2 e^{4 i a}}{e^{2 i a} - x^2} - 2 e^{2 i a} \operatorname{Log}[e^{2 i a} - x^2]$$

Result (type 3, 142 leaves):

$$\begin{aligned} & -\frac{x^2}{2} + 2 i \operatorname{ArcTan}\left[\frac{\operatorname{Cot}[a] - x^2 \operatorname{Cot}[a]}{1 + x^2}\right] \operatorname{Cos}[2 a] - \operatorname{Cos}[2 a] \operatorname{Log}[1 + x^4 - 2 x^2 \operatorname{Cos}[2 a]] - \\ & 4 \operatorname{ArcTan}\left[\frac{\operatorname{Cot}[a] - x^2 \operatorname{Cot}[a]}{1 + x^2}\right] \operatorname{Cos}[a] \operatorname{Sin}[a] - i \operatorname{Log}[1 + x^4 - 2 x^2 \operatorname{Cos}[2 a]] \operatorname{Sin}[2 a] + \frac{2 \operatorname{Cos}[3 a] + 2 i \operatorname{Sin}[3 a]}{(-1 + x^2) \operatorname{Cos}[a] - i (1 + x^2) \operatorname{Sin}[a]} \end{aligned}$$

### Problem 200: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cot}[a + i \operatorname{Log}[x]]^2}{x^3} dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$\frac{2 e^{-2 i a}}{1 - \frac{e^{2 i a}}{x^2}} + \frac{1}{2 x^2} + 2 e^{-2 i a} \operatorname{Log}\left[1 - \frac{e^{2 i a}}{x^2}\right]$$



Result (type 3, 153 leaves):

$$\frac{1}{2x^2} + \cos[2a] \left( -4 \log[x] + \log[1+x^4 - 2x^2 \cos[2a]] \right) + \frac{2 \cos[a]}{(-1+x^2) \cos[a] - i(1+x^2) \sin[a]} + \frac{2 \sin[a]}{i(-1+x^2) \cos[a] + (1+x^2) \sin[a]} +$$

$$\operatorname{ArcTan}\left[\frac{\cot[a] - x^2 \cot[a]}{1+x^2}\right] \left( -2i \cos[2a] - 4 \cos[a] \sin[a] \right) + 4i \log[x] \sin[2a] - i \log[1+x^4 - 2x^2 \cos[2a]] \sin[2a]$$

**Problem 202: Result more than twice size of optimal antiderivative.**

$$\int (ex)^m \cot[a + i \log[x]]^2 dx$$

Optimal (type 5, 77 leaves, 5 steps):

$$-\frac{x(ex)^m}{1+m} + \frac{2x(ex)^m}{1-\frac{e^{2ia}}{x^2}} - 2x(ex)^m \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{e^{2ia}}{x^2}\right]$$

Result (type 5, 169 leaves):

$$\frac{1}{(\cos[a] + i \sin[a])^2} x (ex)^m \left( -\frac{2x^2 \operatorname{Hypergeometric2F1}\left[2, \frac{3+m}{2}, \frac{5+m}{2}, x^2 (\cos[2a] - i \sin[2a])\right]}{3+m} - \frac{x^4 \operatorname{Hypergeometric2F1}\left[2, \frac{5+m}{2}, \frac{7+m}{2}, x^2 (\cos[2a] - i \sin[2a])\right] (\cos[a] - i \sin[a])^2}{5+m} - \frac{\operatorname{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, x^2 (\cos[2a] - i \sin[2a])\right] (\cos[2a] + i \sin[2a])}{1+m} \right)$$

**Problem 204: Result more than twice size of optimal antiderivative.**

$$\int \cot[a + b \log[x]]^p dx$$

Optimal (type 6, 142 leaves, 4 steps):

$$x(1 - e^{2ia} x^{2ib})^p (1 + e^{2ia} x^{2ib})^{-p} \left( -\frac{i(1 + e^{2ia} x^{2ib})}{1 - e^{2ia} x^{2ib}} \right)^p \operatorname{AppellF1}\left[-\frac{i}{2b}, p, -p, 1 - \frac{i}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib}\right]$$

Result (type 6, 330 leaves):

$$\left( (-i + 2b) x \left( \frac{i(1 + e^{2ia} x^{2ib})}{-1 + e^{2ia} x^{2ib}} \right)^p \text{AppellF1} \left[ -\frac{i}{2b}, p, -p, 1 - \frac{i}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right] \right) /$$

$$\left( 2b e^{2ia} p x^{2ib} \text{AppellF1} \left[ 1 - \frac{i}{2b}, p, 1 - p, 2 - \frac{i}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right] + \right.$$

$$\left. 2b e^{2ia} p x^{2ib} \text{AppellF1} \left[ 1 - \frac{i}{2b}, 1 + p, -p, 2 - \frac{i}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right] + (-i + 2b) \text{AppellF1} \left[ -\frac{i}{2b}, p, -p, 1 - \frac{i}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right] \right)$$

**Problem 205: Result more than twice size of optimal antiderivative.**

$$\int (ex)^m \text{Cot}[a + b \text{Log}[x]]^p dx$$

Optimal (type 6, 162 leaves, 4 steps):

$$\frac{1}{e(1+m)} (ex)^{1+m} (1 - e^{2ia} x^{2ib})^p (1 + e^{2ia} x^{2ib})^{-p} \left( -\frac{i(1 + e^{2ia} x^{2ib})}{1 - e^{2ia} x^{2ib}} \right)^p \text{AppellF1} \left[ -\frac{i(1+m)}{2b}, p, -p, 1 - \frac{i(1+m)}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right]$$

Result (type 6, 351 leaves):

$$\left( (1 + 2ib + m) x (ex)^m \left( \frac{i(1 + e^{2ia} x^{2ib})}{-1 + e^{2ia} x^{2ib}} \right)^p \text{AppellF1} \left[ -\frac{i(1+m)}{2b}, p, -p, 1 - \frac{i(1+m)}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right] \right) /$$

$$\left( (1+m) \left( (1 + 2ib + m) \text{AppellF1} \left[ -\frac{i(1+m)}{2b}, p, -p, 1 - \frac{i(1+m)}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right] + 2ib e^{2ia} p x^{2ib} \left( \text{AppellF1} \left[ 1 - \frac{i(1+m)}{2b}, p, \right. \right. \right. \right.$$

$$\left. \left. \left. 1 - p, 2 - \frac{i(1+m)}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right] + \text{AppellF1} \left[ 1 - \frac{i(1+m)}{2b}, 1 + p, -p, 2 - \frac{i(1+m)}{2b}, e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right] \right) \right) \right)$$

**Problem 209: Result more than twice size of optimal antiderivative.**

$$\int x^3 \text{Cot}[d(a + b \text{Log}[c x^n])] dx$$

Optimal (type 5, 70 leaves, 4 steps):

$$\frac{i x^4}{4} - \frac{1}{2} i x^4 \text{Hypergeometric2F1} \left[ 1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, e^{2iad} (c x^n)^{2ibd} \right]$$

Result (type 5, 220 leaves):

$$\begin{aligned}
& - \frac{1}{-8i + 4bdn} x^4 \left( 2 e^{2id(a+b\log[cx^n])} \text{Hypergeometric2F1}\left[1, 1 - \frac{2i}{bdn}, 2 - \frac{2i}{bdn}, e^{2id(a+b\log[cx^n])}\right] + \right. \\
& \quad \left. (-2i + bdn) \left( \text{Cot}[d(a+b\log[cx^n])] - \text{Cot}[d(a-bn\log[x] + b\log[cx^n])] + i \text{Hypergeometric2F1}\left[1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, e^{2id(a+b\log[cx^n])}\right] + \right. \right. \\
& \quad \left. \left. \text{Csc}[d(a+b\log[cx^n])] \text{Csc}[d(a-bn\log[x] + b\log[cx^n])] \text{Sin}[bdn\log[x]] \right) \right) \Big)
\end{aligned}$$

**Problem 210: Result more than twice size of optimal antiderivative.**

$$\int x^2 \text{Cot}[d(a+b\log[cx^n])] dx$$

Optimal (type 5, 74 leaves, 4 steps):

$$\frac{i x^3}{3} - \frac{2}{3} i x^3 \text{Hypergeometric2F1}\left[1, -\frac{3i}{2bdn}, 1 - \frac{3i}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right]$$

Result (type 5, 229 leaves):

$$\begin{aligned}
& - \frac{1}{-9i + 6bdn} x^3 \left( 3 e^{2id(a+b\log[cx^n])} \text{Hypergeometric2F1}\left[1, 1 - \frac{3i}{2bdn}, 2 - \frac{3i}{2bdn}, e^{2id(a+b\log[cx^n])}\right] + (-3i + 2bdn) \right. \\
& \quad \left( \text{Cot}[d(a+b\log[cx^n])] - \text{Cot}[d(a-bn\log[x] + b\log[cx^n])] + i \text{Hypergeometric2F1}\left[1, -\frac{3i}{2bdn}, 1 - \frac{3i}{2bdn}, e^{2id(a+b\log[cx^n])}\right] + \right. \\
& \quad \left. \left. \text{Csc}[d(a+b\log[cx^n])] \text{Csc}[d(a-bn\log[x] + b\log[cx^n])] \text{Sin}[bdn\log[x]] \right) \right) \Big)
\end{aligned}$$

**Problem 211: Result more than twice size of optimal antiderivative.**

$$\int x \text{Cot}[d(a+b\log[cx^n])] dx$$

Optimal (type 5, 68 leaves, 4 steps):

$$\frac{i x^2}{2} - i x^2 \text{Hypergeometric2F1}\left[1, -\frac{i}{bdn}, 1 - \frac{i}{bdn}, e^{2iad}(cx^n)^{2ibd}\right]$$

Result (type 5, 219 leaves):

$$\begin{aligned}
& - \frac{1}{-2i + 2bdn} x^2 \left( e^{2id(a+b\log[cx^n])} \text{Hypergeometric2F1}\left[1, 1 - \frac{i}{bdn}, 2 - \frac{i}{bdn}, e^{2id(a+b\log[cx^n])}\right] + \right. \\
& \quad \left. (-i + bdn) \left( \text{Cot}[d(a+b\log[cx^n])] - \text{Cot}[d(a-bn\log[x] + b\log[cx^n])] + i \text{Hypergeometric2F1}\left[1, -\frac{i}{bdn}, 1 - \frac{i}{bdn}, e^{2id(a+b\log[cx^n])}\right] + \right. \right. \\
& \quad \left. \left. \text{Csc}[d(a+b\log[cx^n])] \text{Csc}[d(a-bn\log[x] + b\log[cx^n])] \text{Sin}[bdn\log[x]] \right) \right) \Big)
\end{aligned}$$

### Problem 212: Result more than twice size of optimal antiderivative.

$$\int \text{Cot} [d (a + b \text{Log} [c x^n])] dx$$

Optimal (type 5, 66 leaves, 4 steps):

$$i x - 2 i x \text{Hypergeometric2F1} \left[ 1, -\frac{i}{2 b d n}, 1 - \frac{i}{2 b d n}, e^{2 i a d} (c x^n)^{2 i b d} \right]$$

Result (type 5, 338 leaves):

$$\begin{aligned} & x \text{Cot} [d (a + b (-n \text{Log} [x] + \text{Log} [c x^n]))] - \text{Csc} [d (a + b (-n \text{Log} [x] + \text{Log} [c x^n]))] \left( x \text{Csc} [d (a + b \text{Log} [c x^n])] \text{Sin} [b d n \text{Log} [x]] - \right. \\ & \frac{1}{-i + 2 b d n} e^{-\frac{a+b(-n \text{Log}[x]+\text{Log}[c x^n])}{b n}} \left( -e^{\frac{(1+2 i b d n)(a+b \text{Log}[c x^n])}{b n}} \text{Hypergeometric2F1} \left[ 1, 1 - \frac{i}{2 b d n}, 2 - \frac{i}{2 b d n}, e^{2 i d (a+b \text{Log}[c x^n])} \right] - \right. \\ & \left. \left. e^{\frac{a}{b n} + \frac{-n \text{Log}[x]+\text{Log}[c x^n]}{n}} (-i + 2 b d n) x \left( \text{Cot} [d (a + b n \text{Log} [x] + b (-n \text{Log} [x] + \text{Log} [c x^n]))] + \right. \right. \\ & \left. \left. i \text{Hypergeometric2F1} \left[ 1, -\frac{i}{2 b d n}, 1 - \frac{i}{2 b d n}, e^{2 i d (a+b n \text{Log}[x]+b(-n \text{Log}[x]+\text{Log}[c x^n]))} \right] \right) \text{Sin} [d (a + b (-n \text{Log} [x] + \text{Log} [c x^n]))] \right) \end{aligned}$$

### Problem 214: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot} [d (a + b \text{Log} [c x^n])]}{x^2} dx$$

Optimal (type 5, 70 leaves, 4 steps):

$$-\frac{i}{x} + \frac{2 i \text{Hypergeometric2F1} \left[ 1, \frac{i}{2 b d n}, 1 + \frac{i}{2 b d n}, e^{2 i a d} (c x^n)^{2 i b d} \right]}{x}$$

Result (type 5, 217 leaves):

$$\begin{aligned} & \frac{1}{x} \left( \text{Cot} [d (a + b \text{Log} [c x^n])] - \text{Cot} [d (a - b n \text{Log} [x] + b \text{Log} [c x^n])] - \frac{e^{2 i d (a+b \text{Log}[c x^n])} \text{Hypergeometric2F1} \left[ 1, 1 + \frac{i}{2 b d n}, 2 + \frac{i}{2 b d n}, e^{2 i d (a+b \text{Log}[c x^n])} \right]}{i + 2 b d n} + \right. \\ & \left. i \text{Hypergeometric2F1} \left[ 1, \frac{i}{2 b d n}, 1 + \frac{i}{2 b d n}, e^{2 i d (a+b \text{Log}[c x^n])} \right] + \text{Csc} [d (a + b \text{Log} [c x^n])] \text{Csc} [d (a - b n \text{Log} [x] + b \text{Log} [c x^n])] \text{Sin} [b d n \text{Log} [x]] \right) \end{aligned}$$

### Problem 215: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[d(a + b \text{Log}[c x^n])]}{x^3} dx$$

Optimal (type 5, 68 leaves, 4 steps):

$$-\frac{i}{2x^2} + \frac{i \text{Hypergeometric2F1}\left[1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, e^{2iad}(cx^n)^{2ibd}\right]}{x^2}$$

Result (type 5, 211 leaves):

$$\frac{1}{2x^2} \left( \text{Cot}[d(a + b \text{Log}[c x^n])] - \text{Cot}[d(a - b n \text{Log}[x] + b \text{Log}[c x^n])] - \frac{e^{2iad(a+b \text{Log}[c x^n])} \text{Hypergeometric2F1}\left[1, 1 + \frac{i}{bdn}, 2 + \frac{i}{bdn}, e^{2iad(a+b \text{Log}[c x^n])}\right]}{i + bdn} + \right. \\ \left. i \text{Hypergeometric2F1}\left[1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, e^{2iad(a+b \text{Log}[c x^n])}\right] + \text{Csc}[d(a + b \text{Log}[c x^n])] \text{Csc}[d(a - b n \text{Log}[x] + b \text{Log}[c x^n])] \text{Sin}[bdn \text{Log}[x]] \right)$$

### Problem 228: Result more than twice size of optimal antiderivative.

$$\int (ex)^m \text{Cot}[d(a + b \text{Log}[c x^n])]^3 dx$$

Optimal (type 5, 350 leaves, 6 steps):

$$\frac{(i(1+m) - bdn)(1+m+2ibd)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} + \frac{(ex)^{1+m}(1+e^{2iad}(cx^n)^{2ibd})^2}{2bden(1-e^{2iad}(cx^n)^{2ibd})^2} + \frac{i e^{-2iad}(ex)^{1+m} \left( \frac{e^{2iad}(1+m-2ibd)}{n} + \frac{e^{4iad}(1+m+2ibd)(cx^n)^{2ibd}}{n} \right)}{2b^2d^2en(1-e^{2iad}(cx^n)^{2ibd})} - \\ \frac{i(1+2m+m^2-2b^2d^2n^2)(ex)^{1+m} \text{Hypergeometric2F1}\left[1, -\frac{i(1+m)}{2bdn}, 1 - \frac{i(1+m)}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right]}{b^2d^2e(1+m)n^2}$$

Result (type 5, 812 leaves):

$$\begin{aligned}
& - \frac{x (e x)^m \operatorname{Cot}\left[d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)\right]}{1+m} - \frac{x (e x)^m \operatorname{Csc}\left[b d n \operatorname{Log}[x]+d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)\right]^2}{2 b d n} + \\
& \frac{1}{1+m} e^{-i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)} x (e x)^m \operatorname{Csc}\left[d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)\right] \\
& \left(1+\left(-1+e^{2 i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)}\right) \operatorname{Hypergeometric2F1}\left[1,-\frac{i(1+m)}{2 b d n}, 1-\frac{i(1+m)}{2 b d n}, e^{2 i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)} x^{2 i b d n}\right]\right) - \\
& \frac{1}{2 b^2 d^2(1+m) n^2} e^{-i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)} x (e x)^m \operatorname{Csc}\left[d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)\right] \\
& \left(1+\left(-1+e^{2 i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)}\right) \operatorname{Hypergeometric2F1}\left[1,-\frac{i(1+m)}{2 b d n}, 1-\frac{i(1+m)}{2 b d n}, e^{2 i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)} x^{2 i b d n}\right]\right) - \\
& \frac{1}{b^2 d^2(1+m) n^2} e^{-i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)} m x (e x)^m \operatorname{Csc}\left[d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)\right] \\
& \left(1+\left(-1+e^{2 i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)}\right) \operatorname{Hypergeometric2F1}\left[1,-\frac{i(1+m)}{2 b d n}, 1-\frac{i(1+m)}{2 b d n}, e^{2 i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)} x^{2 i b d n}\right]\right) - \\
& \frac{1}{2 b^2 d^2(1+m) n^2} e^{-i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)} m^2 x (e x)^m \operatorname{Csc}\left[d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)\right] \\
& \left(1+\left(-1+e^{2 i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)}\right) \operatorname{Hypergeometric2F1}\left[1,-\frac{i(1+m)}{2 b d n}, 1-\frac{i(1+m)}{2 b d n}, e^{2 i d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)} x^{2 i b d n}\right]\right) + \frac{1}{2 b^2 d^2 n^2} \\
& (1+m) x (e x)^m \operatorname{Csc}\left[d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)\right] \operatorname{Csc}\left[b d n \operatorname{Log}[x]+d\left(a+b\left(-n \operatorname{Log}[x]+\operatorname{Log}\left[c x^n\right]\right)\right)\right] \operatorname{Sin}\left[b d n \operatorname{Log}[x]\right]
\end{aligned}$$

### Problem 229: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cot}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]^p dx$$

Optimal (type 6, 190 leaves, 5 steps):

$$\begin{aligned}
& x \left(1 - e^{2 i a d} (c x^n)^{2 i b d}\right)^p \left(1 + e^{2 i a d} (c x^n)^{2 i b d}\right)^{-p} \left(-\frac{i\left(1 + e^{2 i a d} (c x^n)^{2 i b d}\right)}{1 - e^{2 i a d} (c x^n)^{2 i b d}}\right)^p \\
& \operatorname{AppellF1}\left[-\frac{i}{2 b d n}, p, -p, 1 - \frac{i}{2 b d n}, e^{2 i a d} (c x^n)^{2 i b d}, -e^{2 i a d} (c x^n)^{2 i b d}\right]
\end{aligned}$$

Result (type 6, 458 leaves):

$$\left( (-i + 2bdn) x \left( \frac{i(1 + e^{2iad}(cx^n)^{2ibd})}{-1 + e^{2iad}(cx^n)^{2ibd}} \right)^p \text{AppellF1}\left[-\frac{i}{2bdn}, p, -p, 1 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] \right) /$$

$$\left( 2bd e^{2iad} n p (cx^n)^{2ibd} \text{AppellF1}\left[1 - \frac{i}{2bdn}, p, 1 - p, 2 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] + \right.$$

$$2bd e^{2iad} n p (cx^n)^{2ibd} \text{AppellF1}\left[1 - \frac{i}{2bdn}, 1 + p, -p, 2 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] +$$

$$\left. (-i + 2bdn) \text{AppellF1}\left[-\frac{i}{2bdn}, p, -p, 1 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] \right)$$

**Problem 230: Result more than twice size of optimal antiderivative.**

$$\int (ex)^m \text{Cot}[d(a + b \text{Log}[cx^n])]^p dx$$

Optimal (type 6, 210 leaves, 5 steps):

$$\frac{1}{e(1+m)} (ex)^{1+m} (1 - e^{2iad}(cx^n)^{2ibd})^p (1 + e^{2iad}(cx^n)^{2ibd})^{-p}$$

$$\left( -\frac{i(1 + e^{2iad}(cx^n)^{2ibd})}{1 - e^{2iad}(cx^n)^{2ibd}} \right)^p \text{AppellF1}\left[-\frac{i(1+m)}{2bdn}, p, -p, 1 - \frac{i(1+m)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right]$$

Result (type 6, 496 leaves):

$$\left( (1+m+2ibd) x (ex)^m \left( \frac{i(1 + e^{2iad}(cx^n)^{2ibd})}{-1 + e^{2iad}(cx^n)^{2ibd}} \right)^p \text{AppellF1}\left[-\frac{i(1+m)}{2bdn}, p, -p, 1 - \frac{i(1+m)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] \right) /$$

$$\left( (1+m) \left( (1+m+2ibd) \text{AppellF1}\left[-\frac{i(1+m)}{2bdn}, p, -p, -\frac{i(1+m+2ibd)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] + \right.$$

$$2ibd e^{2iad} n p (cx^n)^{2ibd} \left( \text{AppellF1}\left[-\frac{i(1+m+2ibd)}{2bdn}, p, 1-p, -\frac{i(1+m+4ibd)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] + \right.$$

$$\left. \left. \text{AppellF1}\left[-\frac{i(1+m+2ibd)}{2bdn}, 1+p, -p, -\frac{i(1+m+4ibd)}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right] \right) \right)$$

**Problem 240: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[a + b \text{Log}[cx^n]]}{x} dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$\frac{\text{ArcTanh}[\text{Sin}[a + b \text{Log}[c x^n]]]}{b n}$$

b n

Result (type 3, 94 leaves):

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{1}{2} b \text{Log}[c x^n]\right] - \text{Sin}\left[\frac{a}{2} + \frac{1}{2} b \text{Log}[c x^n]\right]\right]}{b n} + \frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{1}{2} b \text{Log}[c x^n]\right] + \text{Sin}\left[\frac{a}{2} + \frac{1}{2} b \text{Log}[c x^n]\right]\right]}{b n}$$

**Problem 254: Result more than twice size of optimal antiderivative.**

$$\int x \text{Sec}[a + b \text{Log}[c x^n]]^4 dx$$

Optimal (type 5, 79 leaves, 3 steps):

$$\frac{8 e^{4 i a} x^2 (c x^n)^{4 i b} \text{Hypergeometric2F1}\left[4, 2 - \frac{i}{b n}, 3 - \frac{i}{b n}, -e^{2 i a} (c x^n)^{2 i b}\right]}{1 + 2 i b n}$$

Result (type 5, 668 leaves):

$$\frac{1}{3 b^3 n^3} 2 (1 + b^2 n^2) x^2 \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Sec}[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Sin}[b n \text{Log}[x]] + x^2 \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Sec}[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])]^3 \text{Sin}[b n \text{Log}[x]]}{3 b n}$$

$$\frac{1}{3 b^3 n^3 (-2 - 2 i b n)} 4 x^2 \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])]$$

$$\left( i e^{2 i (a + b \text{Log}[c x^n])} \text{Cos}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Hypergeometric2F1}\left[1, 1 - \frac{i}{b n}, 2 - \frac{i}{b n}, -e^{2 i (a + b \text{Log}[c x^n])}\right] + (-i + b n) \left( \text{Cos}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Hypergeometric2F1}\left[1, -\frac{i}{b n}, 1 - \frac{i}{b n}, -e^{2 i (a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])}\right]\right) + i \text{Sin}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])]\right) \right) - \frac{1}{3 b n (-2 - 2 i b n)} 4 x^2 \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])]$$

$$\left( i e^{2 i (a + b \text{Log}[c x^n])} \text{Cos}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Hypergeometric2F1}\left[1, 1 - \frac{i}{b n}, 2 - \frac{i}{b n}, -e^{2 i (a + b \text{Log}[c x^n])}\right] + (-i + b n) \left( \text{Cos}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Hypergeometric2F1}\left[1, -\frac{i}{b n}, 1 - \frac{i}{b n}, -e^{2 i (a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])}\right]\right) + i \text{Sin}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])]\right) \right) +$$

$$\frac{1}{3 b^2 n^2} x^2 \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Sec}[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])]^2 (-\text{Cos}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] + b n \text{Sin}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])])$$



### Problem 255: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[a + b \text{Log}[c x^n]]^4 dx$$

Optimal (type 5, 85 leaves, 3 steps):

$$\frac{16 e^{4 i a} x (c x^n)^{4 i b} \text{Hypergeometric2F1}\left[4, \frac{1}{2} \left(4 - \frac{i}{b n}\right), \frac{1}{2} \left(6 - \frac{i}{b n}\right), -e^{2 i a} (c x^n)^{2 i b}\right]}{1 + 4 i b n}$$

Result (type 5, 517 leaves):

$$\begin{aligned} & \frac{1}{6 b^3 n^3} (1 + 4 b^2 n^2) x \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Sec}[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Sin}[b n \text{Log}[x]] + \\ & \frac{x \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Sec}[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])]^3 \text{Sin}[b n \text{Log}[x]]}{3 b n} \\ & \frac{1}{6 b^3 n^3 (-i + 2 b n)} e^{-\frac{a+b(-n \text{Log}[x] + \text{Log}[c x^n])}{b n}} (1 + 4 b^2 n^2) \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \\ & \left( -e^{(2 i + \frac{1}{b n})(a+b \text{Log}[c x^n])} \text{Cos}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Hypergeometric2F1}\left[1, 1 - \frac{i}{2 b n}, 2 - \frac{i}{2 b n}, -e^{2 i (a+b \text{Log}[c x^n])}\right] + e^{\frac{a}{b n} + \frac{-n \text{Log}[x] + \text{Log}[c x^n]}{n}} \right. \\ & \left. (1 + 2 i b n) x \left( \text{Cos}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \text{Hypergeometric2F1}\left[1, -\frac{i}{2 b n}, 1 - \frac{i}{2 b n}, -e^{2 i (a+b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])}\right] + \right. \right. \\ & \left. \left. i \text{Sin}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])]\right) \right) + \frac{1}{6 b^2 n^2} x \text{Sec}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] \\ & \text{Sec}[a + b n \text{Log}[x] + b (-n \text{Log}[x] + \text{Log}[c x^n])]^2 (-\text{Cos}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])] + 2 b n \text{Sin}[a + b (-n \text{Log}[x] + \text{Log}[c x^n])]) \end{aligned}$$

### Problem 257: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[a + b \text{Log}[c x^n]]^4}{x^2} dx$$

Optimal (type 5, 87 leaves, 3 steps):

$$\frac{16 e^{4 i a} (c x^n)^{4 i b} \text{Hypergeometric2F1}\left[4, \frac{1}{2} \left(4 + \frac{i}{b n}\right), \frac{1}{2} \left(6 + \frac{i}{b n}\right), -e^{2 i a} (c x^n)^{2 i b}\right]}{(1 - 4 i b n) x}$$

Result (type 5, 660 leaves):

$$\begin{aligned}
& \frac{1}{6 b^3 n^3 x} (1 + 4 b^2 n^2) \operatorname{Sec}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sec}[a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sin}[b n \operatorname{Log}[x]] + \\
& \frac{\operatorname{Sec}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sec}[a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]^3 \operatorname{Sin}[b n \operatorname{Log}[x]]}{3 b n x} + \frac{1}{6 b^3 n^3 x} \operatorname{Sec}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \\
& \left( -\frac{1}{i + 2 b n} e^{2 i (a + b \operatorname{Log}[c x^n])} \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{i}{2 b n}, 2 + \frac{i}{2 b n}, -e^{2 i (a + b \operatorname{Log}[c x^n])}\right] - \right. \\
& \quad i \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, \frac{i}{2 b n}, 1 + \frac{i}{2 b n}, -e^{2 i (a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}\right)] + \\
& \quad \left. \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) + \frac{1}{3 b n x} 2 \operatorname{Sec}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \\
& \left( -\frac{1}{i + 2 b n} e^{2 i (a + b \operatorname{Log}[c x^n])} \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{i}{2 b n}, 2 + \frac{i}{2 b n}, -e^{2 i (a + b \operatorname{Log}[c x^n])}\right] - \right. \\
& \quad i \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, \frac{i}{2 b n}, 1 + \frac{i}{2 b n}, -e^{2 i (a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}\right)] + \\
& \quad \left. \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) + \frac{1}{6 b^2 n^2 x} \operatorname{Sec}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \\
& \operatorname{Sec}[a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]^2 (\operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + 2 b n \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])])
\end{aligned}$$

**Problem 258: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[a + b \operatorname{Log}[c x^n]]^4}{x^3} dx$$

Optimal (type 5, 79 leaves, 3 steps):

$$-\frac{8 e^{4 i a} (c x^n)^{4 i b} \operatorname{Hypergeometric2F1}\left[4, 2 + \frac{i}{b n}, 3 + \frac{i}{b n}, -e^{2 i a} (c x^n)^{2 i b}\right]}{(1 - 2 i b n) x^2}$$

Result (type 5, 640 leaves):

$$\begin{aligned}
& \frac{1}{3 b^3 n^3 x^2} 2 (1 + b^2 n^2) \operatorname{Sec}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sec}[a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sin}[b n \operatorname{Log}[x]] + \\
& \frac{\operatorname{Sec}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sec}[a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]^3 \operatorname{Sin}[b n \operatorname{Log}[x]]}{3 b n x^2} + \\
& \frac{1}{3 b^3 n^3 x^2} 2 \operatorname{Sec}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \\
& \left( -\frac{1}{i + b n} e^{2 i (a + b \operatorname{Log}[c x^n])} \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{i}{b n}, 2 + \frac{i}{b n}, -e^{2 i (a + b \operatorname{Log}[c x^n])}\right] - \right. \\
& \quad i \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, \frac{i}{b n}, 1 + \frac{i}{b n}, -e^{2 i (a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}\right] + \\
& \quad \left. \operatorname{Sin}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])\right] \Big) + \frac{1}{3 b n x^2} 2 \operatorname{Sec}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \\
& \left( -\frac{1}{i + b n} e^{2 i (a + b \operatorname{Log}[c x^n])} \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{i}{b n}, 2 + \frac{i}{b n}, -e^{2 i (a + b \operatorname{Log}[c x^n])}\right] - \right. \\
& \quad i \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Hypergeometric2F1}\left[1, \frac{i}{b n}, 1 + \frac{i}{b n}, -e^{2 i (a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}\right] + \\
& \quad \left. \operatorname{Sin}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])\right] \Big) + \frac{1}{3 b^2 n^2 x^2} \operatorname{Sec}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \\
& \operatorname{Sec}[a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]^2 (\operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + b n \operatorname{Sin}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])])
\end{aligned}$$

**Problem 261: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Sec}[a + 2 \operatorname{Log}[c x^i]]^3 dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$\frac{e^{i a} (c x^i)^{2 i} x^2}{(1 + e^{2 i a} (c x^i)^{4 i})^2}$$

Result (type 3, 127 leaves):

$$\begin{aligned}
& -\frac{1}{4 x^4} \operatorname{Sec}[a + 2 \operatorname{Log}[c x^i]]^2 \left( (1 + 2 x^4) \operatorname{Cos}[a + 2 \operatorname{Log}[c x^i] - 2 i \operatorname{Log}[x]] + i (1 - 2 x^4) \operatorname{Sin}[a + 2 \operatorname{Log}[c x^i] - 2 i \operatorname{Log}[x]] \right) \\
& (\operatorname{Cos}[2 (a + 2 \operatorname{Log}[c x^i] - 2 i \operatorname{Log}[x])] + i \operatorname{Sin}[2 (a + 2 \operatorname{Log}[c x^i] - 2 i \operatorname{Log}[x])])
\end{aligned}$$

**Problem 262: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[a + 2 \operatorname{Log}[c x^{\frac{i}{2}}]]^3 dx$$

Optimal (type 3, 58 leaves, 3 steps):

$$\frac{1}{2} x \operatorname{Sec}\left[a + 2 \operatorname{Log}\left[c x^{\frac{i}{2}}\right]\right] - \frac{1}{2} i x \operatorname{Sec}\left[a + 2 \operatorname{Log}\left[c x^{\frac{i}{2}}\right]\right] \operatorname{Tan}\left[a + 2 \operatorname{Log}\left[c x^{\frac{i}{2}}\right]\right]$$

Result (type 3, 137 leaves):

$$-\frac{1}{2 x^2} \operatorname{Sec}\left[a + 2 \operatorname{Log}\left[c x^{\frac{i}{2}}\right]\right]^2 \left( (1 + 2 x^2) \operatorname{Cos}\left[a + 2 \operatorname{Log}\left[c x^{\frac{i}{2}}\right] - i \operatorname{Log}[x]\right] + i (1 - 2 x^2) \operatorname{Sin}\left[a + 2 \operatorname{Log}\left[c x^{\frac{i}{2}}\right] - i \operatorname{Log}[x]\right] \right) \\ \left( \operatorname{Cos}\left[2\left(a + 2 \operatorname{Log}\left[c x^{\frac{i}{2}}\right] - i \operatorname{Log}[x]\right)\right] + i \operatorname{Sin}\left[2\left(a + 2 \operatorname{Log}\left[c x^{\frac{i}{2}}\right] - i \operatorname{Log}[x]\right)\right] \right)$$

**Problem 263: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}\left[a + 2 \operatorname{Log}\left[c x^{-\frac{i}{2}}\right]\right]^3 dx$$

Optimal (type 3, 48 leaves, 3 steps):

$$\frac{2 e^{3 i a} \left(c x^{-\frac{i}{2}}\right)^{6 i} x}{\left(1 + e^{2 i a} \left(c x^{-\frac{i}{2}}\right)^{4 i}\right)^2}$$

Result (type 3, 139 leaves):

$$\frac{1}{4 x^2} \operatorname{Sec}\left[a + 2 \operatorname{Log}\left[c x^{-\frac{i}{2}}\right]\right]^2 \left( (1 + 2 x^2) \operatorname{Cos}\left[a + 2 \operatorname{Log}\left[c x^{-\frac{i}{2}}\right] + i \operatorname{Log}[x]\right] + i (-1 + 2 x^2) \operatorname{Sin}\left[a + 2 \operatorname{Log}\left[c x^{-\frac{i}{2}}\right] + i \operatorname{Log}[x]\right] \right) \\ \left( -2 \operatorname{Cos}\left[2\left(a + 2 \operatorname{Log}\left[c x^{-\frac{i}{2}}\right] + i \operatorname{Log}[x]\right)\right] + 2 i \operatorname{Sin}\left[2\left(a + 2 \operatorname{Log}\left[c x^{-\frac{i}{2}}\right] + i \operatorname{Log}[x]\right)\right] \right)$$

**Problem 268: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}\left[a + b \operatorname{Log}\left[c x^n\right]\right]^{3/2} dx$$

Optimal (type 5, 109 leaves, 3 steps):

$$\frac{1}{2 + 3 i b n} 2 x \left(1 + e^{2 i a} (c x^n)^{2 i b}\right)^{3/2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1}{4} \left(3 - \frac{2 i}{b n}\right), \frac{1}{4} \left(7 - \frac{2 i}{b n}\right), -e^{2 i a} (c x^n)^{2 i b}\right] \operatorname{Sec}\left[a + b \operatorname{Log}\left[c x^n\right]\right]^{3/2}$$

Result (type 5, 843 leaves):

$$\begin{aligned}
& - \left( \left( 4 \sqrt{2} e^{-2i(a+b(-n\log[x]+\log[cx^n]))} x^{1-ibn} \sqrt{\frac{e^{i(a+b(-n\log[x]+\log[cx^n]))} x^{ibn}}{1+e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}}} \right. \right. \\
& \quad \left. \left( (2i+bn) (1+e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}) + (-2i-bn+e^{2i(a+b(-n\log[x]+\log[cx^n]))} (-2i+bn)) \right) \right. \\
& \quad \left. \left. \sqrt{1+e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i+bn}{4bn}, \frac{3}{4} - \frac{i}{2bn}, -e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn} \right] \right) \right) / \\
& \quad \left. \left( (bn(4+b^2n^2) (-2\cos[a+b(-n\log[x]+\log[cx^n])]) + bn\sin[a+b(-n\log[x]+\log[cx^n])]) \right) \right) - \\
& \left( \sqrt{2} b e^{-2i(a+b(-n\log[x]+\log[cx^n]))} n x^{1-ibn} \sqrt{\frac{e^{i(a+b(-n\log[x]+\log[cx^n]))} x^{ibn}}{1+e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}}} \right. \\
& \quad \left. \left( (2i+bn) (1+e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}) + (-2i-bn+e^{2i(a+b(-n\log[x]+\log[cx^n]))} (-2i+bn)) \right) \right. \\
& \quad \left. \left. \sqrt{1+e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i+bn}{4bn}, \frac{3}{4} - \frac{i}{2bn}, -e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn} \right] \right) \right) / \\
& \quad \left( (4+b^2n^2) (-2\cos[a+b(-n\log[x]+\log[cx^n])]) + bn\sin[a+b(-n\log[x]+\log[cx^n])]) \right) + \\
& \sqrt{\sec[a+bn\log[x]+b(-n\log[x]+\log[cx^n])]} \\
& \left( \frac{2x\cos[bn\log[x]]}{-2\cos[a+b(-n\log[x]+\log[cx^n])]+bn\sin[a+b(-n\log[x]+\log[cx^n])]} - \right. \\
& \quad \left. \frac{4x\sin[bn\log[x]]}{bn(-2\cos[a+b(-n\log[x]+\log[cx^n])]+bn\sin[a+b(-n\log[x]+\log[cx^n])])} \right)
\end{aligned}$$

**Problem 272: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\sec[a+b\log[cx^n]]}} dx$$

Optimal (type 5, 110 leaves, 3 steps):

$$\frac{2x \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -\frac{2i+bn}{4bn}, \frac{1}{4} \left( 3 - \frac{2i}{bn} \right), -e^{2ia} (cx^n)^{2ib} \right]}{(2-i)bn \sqrt{1+e^{2ia} (cx^n)^{2ib}} \sqrt{\sec[a+b\log[cx^n]]}}$$

Result (type 5, 364 leaves):

$$\left( 2 i \sqrt{2} b e^{-i a} n x (c x^n)^{-i b} \sqrt{\frac{e^{i a} (c x^n)^{i b}}{1 + e^{2 i a} (c x^n)^{2 i b}}} \left( (2 i + b n) (1 + e^{2 i a} (c x^n)^{2 i b}) + \sqrt{1 + e^{2 i a} (c x^n)^{2 i b}} (-2 i - b n + e^{2 i a} (-2 i + b n) x^{-2 i b n} (c x^n)^{2 i b}) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2 i + b n}{4 b n}, \frac{3}{4} - \frac{i}{2 b n}, -e^{2 i a} (c x^n)^{2 i b}\right] \right) \right) /$$

$$\frac{\left( (4 + b^2 n^2) (-2 i - b n + e^{2 i a} (-2 i + b n) x^{-2 i b n} (c x^n)^{2 i b}) \right) - 2 x \operatorname{Cos}[a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]]}{\sqrt{\operatorname{Sec}[a + b \operatorname{Log}[c x^n]]} (-2 \operatorname{Cos}[a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]] + b n \operatorname{Sin}[a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]])}}$$

**Problem 276: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\operatorname{Sec}[a + b \operatorname{Log}[c x^n]]^{5/2}} dx$$

Optimal (type 5, 110 leaves, 3 steps):

$$\frac{2 x \operatorname{Hypergeometric2F1}\left[-\frac{5}{2}, \frac{1}{4} \left(-5 - \frac{2 i}{b n}\right), -\frac{2 i + b n}{4 b n}, -e^{2 i a} (c x^n)^{2 i b}\right]}{(2 - 5 i b n) (1 + e^{2 i a} (c x^n)^{2 i b})^{5/2} \operatorname{Sec}[a + b \operatorname{Log}[c x^n]]^{5/2}}$$

Result (type 5, 861 leaves):

$$\begin{aligned}
& \left( 30 i \sqrt{2} b^3 e^{-i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} n^3 x^{1-i b n} \sqrt{\frac{e^{i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{i b n}}{1+e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{2 i b n}}} \right. \\
& \quad \left. \left( (2 i+b n) \left( 1+e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{2 i b n} \right) + (-2 i-b n+e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} (-2 i+b n)) \right) \right. \\
& \quad \left. \sqrt{1+e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{2 i b n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2 i+b n}{4 b n}, \frac{3}{4}-\frac{i}{2 b n}, -e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} x^{2 i b n}\right] \right) \Bigg) / \\
& \quad \left( (-2 i+5 b n) (2 i+5 b n) (4+b^2 n^2) (-2 i-b n+e^{2 i(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))} (-2 i+b n)) \right) + \\
& \quad \sqrt{\operatorname{Sec}[a+b n \operatorname{Log}[x]+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n])]} \\
& \quad \left( -\left( (x \operatorname{Cos}[b n \operatorname{Log}[x]] (12+55 b^2 n^2+12 \operatorname{Cos}[2(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))]) + 65 b^2 n^2 \operatorname{Cos}[2(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))]) \right) \right. \\
& \quad \quad \left. + 4 b n \operatorname{Sin}[2(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))] \right) \Bigg) / \\
& \quad \left( 4(-2 i+5 b n) (2 i+5 b n) (-2 \operatorname{Cos}[a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n])] + b n \operatorname{Sin}[a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n])]) \right) \Bigg) + \\
& \quad \left( x \operatorname{Sin}[b n \operatorname{Log}[x]] (-16 b n-4 b n \operatorname{Cos}[2(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))] + 12 \operatorname{Sin}[2(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))]) \right. \\
& \quad \quad \left. + 65 b^2 n^2 \operatorname{Sin}[2(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))] \right) \Bigg) / \\
& \quad \left( 4(-2 i+5 b n) (2 i+5 b n) (-2 \operatorname{Cos}[a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n])] + b n \operatorname{Sin}[a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n])]) \right) \Bigg) + \\
& \quad \left( x \operatorname{Sin}[3 b n \operatorname{Log}[x]] (5 b n \operatorname{Cos}[3(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))] - 2 \operatorname{Sin}[3(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))]) \right) \Bigg) / \\
& \quad \left( 2(-2 i+5 b n) (2 i+5 b n) + \right. \\
& \quad \left. (x \operatorname{Cos}[3 b n \operatorname{Log}[x]] (2 \operatorname{Cos}[3(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))] + 5 b n \operatorname{Sin}[3(a+b(-n \operatorname{Log}[x]+\operatorname{Log}[c x^n]))]) \right) \Bigg) / \\
& \quad \left( 2(-2 i+5 b n) (2 i+5 b n) \right)
\end{aligned}$$

### Problem 282: Result more than twice size of optimal antiderivative.

$$\int x^m \operatorname{Sec}[a+b \operatorname{Log}[c x^n]]^{3/2} dx$$

Optimal (type 5, 130 leaves, 3 steps):

$$\frac{1}{2+2 m+3 i b n} 2 x^{1+m} \left( 1+e^{2 i a} (c x^n)^{2 i b} \right)^{3/2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -\frac{2 i+2 i m-3 b n}{4 b n}, -\frac{2 i+2 i m-7 b n}{4 b n}, -e^{2 i a} (c x^n)^{2 i b}\right] \operatorname{Sec}[a+b \operatorname{Log}[c x^n]]^{3/2}$$

Result (type 5, 470 leaves):

$$\begin{aligned}
& \left( \sqrt{2} x^{1+m-i b n} \left( - (4+8 m+4 m^2+b^2 n^2) x^{2 i b n} \sqrt{\frac{e^{i a} (c x^n)^{i b}}{1+e^{2 i a} (c x^n)^{2 i b}}} \sqrt{1+e^{2 i a} (c x^n)^{2 i b}} \right. \right. \\
& \quad \text{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i+2 i m-3 b n}{4 b n}, -\frac{2 i+2 i m-7 b n}{4 b n}, -e^{2 i a} (c x^n)^{2 i b} \right] + (2+2 m+3 i b n) \\
& \quad \left. \left( (2+2 m+i b n) \sqrt{\frac{e^{i a} (c x^n)^{i b}}{1+e^{2 i a} (c x^n)^{2 i b}}} \sqrt{1+e^{2 i a} (c x^n)^{2 i b}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i+2 i m+b n}{4 b n}, -\frac{2 i+2 i m-3 b n}{4 b n}, -e^{2 i a} (c x^n)^{2 i b} \right] - \right. \right. \\
& \quad \left. \left. i \sqrt{2} x^{i b n} \sqrt{\text{Sec}[a+b \text{Log}[c x^n]]} (b n \text{Cos}[b n \text{Log}[x]] - 2(1+m) \text{Sin}[b n \text{Log}[x]]) \right) \right) \Bigg) \Bigg) \Bigg) / \\
& (b n (-2 i-2 i m+3 b n) (-2(1+m) \text{Cos}[a-b n \text{Log}[x]+b \text{Log}[c x^n]] + b n \text{Sin}[a-b n \text{Log}[x]+b \text{Log}[c x^n]]))
\end{aligned}$$

**Problem 284: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^m}{\sqrt{\text{Sec}[a+b \text{Log}[c x^n]]}} dx$$

Optimal (type 5, 129 leaves, 3 steps):

$$\frac{2 x^{1+m} \text{Hypergeometric2F1} \left[ -\frac{1}{2}, -\frac{2 i+2 i m+b n}{4 b n}, -\frac{2 i+2 i m-3 b n}{4 b n}, -e^{2 i a} (c x^n)^{2 i b} \right]}{(2+2 m-i b n) \sqrt{1+e^{2 i a} (c x^n)^{2 i b}} \sqrt{\text{Sec}[a+b \text{Log}[c x^n]]}}$$

Result (type 5, 630 leaves):



$$\begin{aligned}
& - \left( 2 b e^{2 i (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} n x^{1+m} \right. \\
& \quad \left( (2 i + 2 i m + b n) x^{2 i b n} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -\frac{2 i + 2 i m - 7 b n}{4 b n}, -e^{2 i (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n} \right] + \right. \\
& \quad \left. (-2 i - 2 i m + 3 b n) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b n}{4 b n}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -e^{2 i (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n} \right] \right) \Bigg) / \\
& \left( (2 + 2 m - i b n) (2 + 2 m + 3 i b n) (2 + 2 m - i b n + e^{2 i (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} (2 + 2 m + i b n)) \right. \\
& \quad \left. \sqrt{1 + e^{2 i (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n}} \sqrt{\frac{e^{i (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{i b n}}{2 + 2 e^{2 i (a+b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n}}} \right) \Bigg) + \\
& \sqrt{\operatorname{Sec}[a + b n \operatorname{Log}[x] + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \left( (2 x^{1+m} \operatorname{Cos}[b n \operatorname{Log}[x]] \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right)^2 /} \\
& \quad (2 \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + 2 m \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] - b n \operatorname{Sin}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]) -} \\
& \quad (x^{1+m} \operatorname{Sin}[b n \operatorname{Log}[x]] \operatorname{Sin}[2 (a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])])]) /} \\
& \quad (2 \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + 2 m \operatorname{Cos}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] - b n \operatorname{Sin}[a + b (-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])])]) \Bigg)
\end{aligned}$$

**Problem 292: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Csc}[a + b \operatorname{Log}[c x^n]]}{x} dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}[\operatorname{Cos}[a + b \operatorname{Log}[c x^n]]]}{b n}$$

Result (type 3, 54 leaves):

$$-\frac{\operatorname{Log}[\operatorname{Cos}[\frac{a}{2} + \frac{1}{2} b \operatorname{Log}[c x^n]]]}{b n} + \frac{\operatorname{Log}[\operatorname{Sin}[\frac{a}{2} + \frac{1}{2} b \operatorname{Log}[c x^n]]]}{b n}$$

**Problem 297: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}[a + b \operatorname{Log}[c x^n]]^3 dx$$

Optimal (type 5, 84 leaves, 3 steps):

$$\frac{8 e^{3 i a} x (c x^n)^{3 i b} \text{Hypergeometric2F1}\left[3, \frac{1}{2} \left(3 - \frac{i}{b n}\right), \frac{1}{2} \left(5 - \frac{i}{b n}\right), e^{2 i a} (c x^n)^{2 i b}\right]}{i - 3 b n}$$

Result (type 5, 549 leaves):

$$\begin{aligned} & \frac{x \operatorname{Csc}\left[a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right]}{2 b^2 n^2} - \frac{x \operatorname{Csc}\left[\frac{1}{2} b n \operatorname{Log}[x] + \frac{1}{2} \left(a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right]^2}{8 b n} - \\ & \frac{e^{i \left(a + (-i + b n) \operatorname{Log}[x] + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)} \left(1 + b^2 n^2\right) \text{Hypergeometric2F1}\left[1, \frac{1}{2} - \frac{i}{2 b n}, \frac{3}{2} - \frac{i}{2 b n}, e^{2 i \left(a + b \operatorname{Log}[c x^n]\right)}\right]}{b^2 n^2 (-i + b n)} + \\ & \frac{x \operatorname{Sec}\left[\frac{1}{2} b n \operatorname{Log}[x] + \frac{1}{2} \left(a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right]^2}{8 b n} - \\ & \left(x \operatorname{Sec}\left[\frac{1}{2} b n \operatorname{Log}[x] + \frac{1}{2} \left(a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right] \operatorname{Sin}\left[\frac{1}{2} b n \operatorname{Log}[x]\right]\right) / \left(4 b^2 n^2 \left(\frac{1}{2} \operatorname{Cos}\left[\frac{1}{2} \left(-a - b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right]\right) + \right. \\ & \left. \frac{1}{2} \operatorname{Cos}\left[\frac{1}{2} \left(a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right] + \frac{1}{2} i \operatorname{Sin}\left[\frac{1}{2} \left(-a - b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right] + \frac{1}{2} i \operatorname{Sin}\left[\frac{1}{2} \left(a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right]\right) + \\ & \left(x \operatorname{Csc}\left[\frac{1}{2} b n \operatorname{Log}[x] + \frac{1}{2} \left(a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right] \operatorname{Sin}\left[\frac{1}{2} b n \operatorname{Log}[x]\right]\right) / \left(4 b^2 n^2 \left(\frac{1}{2} i \operatorname{Cos}\left[\frac{1}{2} \left(-a - b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right]\right) - \right. \\ & \left. \frac{1}{2} i \operatorname{Cos}\left[\frac{1}{2} \left(a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right] - \frac{1}{2} \operatorname{Sin}\left[\frac{1}{2} \left(-a - b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right] + \frac{1}{2} \operatorname{Sin}\left[\frac{1}{2} \left(a + b \left(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]\right)\right)\right]\right) \end{aligned}$$

**Problem 299: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Csc}\left[a + b \operatorname{Log}[c x^n]\right]^4 dx$$

Optimal (type 5, 84 leaves, 3 steps):

$$\frac{16 e^{4 i a} x (c x^n)^{4 i b} \text{Hypergeometric2F1}\left[4, \frac{1}{2} \left(4 - \frac{i}{b n}\right), \frac{1}{2} \left(6 - \frac{i}{b n}\right), e^{2 i a} (c x^n)^{2 i b}\right]}{1 + 4 i b n}$$

Result (type 5, 782 leaves):

$$\frac{1}{6 b^3 n^3} (1 + 4 b^2 n^2) x \operatorname{Csc}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Csc}[a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Sin}[b n \operatorname{Log}[x]] +$$

$$\frac{x \operatorname{Csc}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Csc}[a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]^3 \operatorname{Sin}[b n \operatorname{Log}[x]]}{3 b n}$$

$$\frac{1}{6 b^2 n^2} x \operatorname{Csc}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \operatorname{Csc}[a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]^2$$

$$(2 b n \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]) -$$

$$\frac{1}{6 b^3 n^3 (-i + 2 b n)} e^{-\frac{a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}{b n}} \operatorname{Csc}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] +$$

$$\left( e^{(2i + \frac{1}{b n})(a+b \operatorname{Log}[c x^n])} \operatorname{Hypergeometric2F1}\left[1, 1 - \frac{i}{2 b n}, 2 - \frac{i}{2 b n}, e^{2i(a+b \operatorname{Log}[c x^n])}\right] \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) +$$

$$e^{\frac{a}{b n} + \frac{-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]}{n}} (-i + 2 b n) x \left( \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) +$$

$$i \operatorname{Hypergeometric2F1}\left[1, -\frac{i}{2 b n}, 1 - \frac{i}{2 b n}, e^{2i(a+b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}\right] \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) - \frac{1}{3 b n (-i + 2 b n)}$$

$$2 e^{-\frac{a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])}{b n}} \operatorname{Csc}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \left( e^{(2i + \frac{1}{b n})(a+b \operatorname{Log}[c x^n])} \operatorname{Hypergeometric2F1}\left[1, 1 - \frac{i}{2 b n}, 2 - \frac{i}{2 b n}, e^{2i(a+b \operatorname{Log}[c x^n])}\right] \right)$$

$$\operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + e^{\frac{a}{b n} + \frac{-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]}{n}} (-i + 2 b n) x \left( \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right) +$$

$$i \operatorname{Hypergeometric2F1}\left[1, -\frac{i}{2 b n}, 1 - \frac{i}{2 b n}, e^{2i(a+b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))}\right] \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \right)$$

**Problem 303: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Csc}[a + 2 \operatorname{Log}[c x^i]]^3 dx$$

Optimal (type 3, 49 leaves, 3 steps):

$$-\frac{i e^{i a} (c x^i)^{2 i} x^2}{(1 - e^{2 i a} (c x^i)^{4 i})^2}$$

Result (type 3, 127 leaves):

$$\frac{1}{4 x^4} \operatorname{Csc}[a + 2 \operatorname{Log}[c x^i]]^2 (i(-1 + 2 x^4) \operatorname{Cos}[a + 2 \operatorname{Log}[c x^i] - 2 i \operatorname{Log}[x]] + (1 + 2 x^4) \operatorname{Sin}[a + 2 \operatorname{Log}[c x^i] - 2 i \operatorname{Log}[x]])$$

$$(\operatorname{Cos}[2(a + 2 \operatorname{Log}[c x^i] - 2 i \operatorname{Log}[x])] + i \operatorname{Sin}[2(a + 2 \operatorname{Log}[c x^i] - 2 i \operatorname{Log}[x])])$$

### Problem 304: Result more than twice size of optimal antiderivative.

$$\int \text{Csc} \left[ a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] \right]^3 dx$$

Optimal (type 3, 58 leaves, 3 steps):

$$\frac{1}{2} x \text{Csc} \left[ a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] \right] + \frac{1}{2} i x \text{Cot} \left[ a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] \right] \text{Csc} \left[ a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] \right]$$

Result (type 3, 137 leaves):

$$\frac{1}{2 x^2} \text{Csc} \left[ a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] \right]^2 \left( i \left( -1 + 2 x^2 \right) \text{Cos} \left[ a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] - i \text{Log} \left[ x \right] \right] + \left( 1 + 2 x^2 \right) \text{Sin} \left[ a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] - i \text{Log} \left[ x \right] \right] \right) \\ \left( \text{Cos} \left[ 2 \left( a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] - i \text{Log} \left[ x \right] \right) \right] + i \text{Sin} \left[ 2 \left( a + 2 \text{Log} \left[ c x^{\frac{i}{2}} \right] - i \text{Log} \left[ x \right] \right) \right] \right)$$

### Problem 305: Result more than twice size of optimal antiderivative.

$$\int \text{Csc} \left[ a + 2 \text{Log} \left[ c x^{-\frac{i}{2}} \right] \right]^3 dx$$

Optimal (type 3, 51 leaves, 3 steps):

$$\frac{2 i e^{3 i a} \left( c x^{-\frac{i}{2}} \right)^{6 i} x}{\left( 1 - e^{2 i a} \left( c x^{-\frac{i}{2}} \right)^{4 i} \right)^2}$$

Result (type 3, 137 leaves):

$$-\frac{1}{2 x^2} \text{Csc} \left[ a + 2 \text{Log} \left[ c x^{-\frac{i}{2}} \right] \right]^2 \left( \left( -1 + 2 x^2 \right) \text{Cos} \left[ a + 2 \text{Log} \left[ c x^{-\frac{i}{2}} \right] + i \text{Log} \left[ x \right] \right] + i \left( 1 + 2 x^2 \right) \text{Sin} \left[ a + 2 \text{Log} \left[ c x^{-\frac{i}{2}} \right] + i \text{Log} \left[ x \right] \right] \right) \\ \left( i \text{Cos} \left[ 2 \left( a + 2 \text{Log} \left[ c x^{-\frac{i}{2}} \right] + i \text{Log} \left[ x \right] \right) \right] + \text{Sin} \left[ 2 \left( a + 2 \text{Log} \left[ c x^{-\frac{i}{2}} \right] + i \text{Log} \left[ x \right] \right) \right] \right)$$

### Problem 310: Result more than twice size of optimal antiderivative.

$$\int \text{Csc} \left[ a + b \text{Log} \left[ c x^n \right] \right]^{3/2} dx$$

Optimal (type 5, 109 leaves, 3 steps):

$$\frac{1}{2 + 3 i b n} 2 x \left( 1 - e^{2 i a} \left( c x^n \right)^{2 i b} \right)^{3/2} \text{Csc} \left[ a + b \text{Log} \left[ c x^n \right] \right]^{3/2} \text{Hypergeometric2F1} \left[ \frac{3}{2}, \frac{1}{4} \left( 3 - \frac{2 i}{b n} \right), \frac{1}{4} \left( 7 - \frac{2 i}{b n} \right), e^{2 i a} \left( c x^n \right)^{2 i b} \right]$$

Result (type 5, 846 leaves):

$$\begin{aligned}
& \left( 4 \sqrt{2} e^{-2i(a+b(-n\log[x]+\log[cx^n]))} x^{1-ibn} \sqrt{\frac{i e^{i(a+b(-n\log[x]+\log[cx^n]))} x^{ibn}}{-1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}}} \right. \\
& \quad \left. \left( (2i + bn) (-1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}) + (2i + bn + e^{2i(a+b(-n\log[x]+\log[cx^n]))} (-2i + bn)) \right) \right. \\
& \quad \left. \sqrt{1 - e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2i + bn}{4bn}, \frac{3}{4} - \frac{i}{2bn}, e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}\right] \right) / \\
& \quad \left( bn(4 + b^2n^2) (bn \cos[a + b(-n\log[x] + \log[cx^n])] + 2 \sin[a + b(-n\log[x] + \log[cx^n])]) \right) + \\
& \left( \sqrt{2} b e^{-2i(a+b(-n\log[x]+\log[cx^n]))} n x^{1-ibn} \sqrt{\frac{i e^{i(a+b(-n\log[x]+\log[cx^n]))} x^{ibn}}{-1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}}} \right. \\
& \quad \left. \left( (2i + bn) (-1 + e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}) + (2i + bn + e^{2i(a+b(-n\log[x]+\log[cx^n]))} (-2i + bn)) \right) \right. \\
& \quad \left. \sqrt{1 - e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2i + bn}{4bn}, \frac{3}{4} - \frac{i}{2bn}, e^{2i(a+b(-n\log[x]+\log[cx^n]))} x^{2ibn}\right] \right) / \\
& \quad \left( (4 + b^2n^2) (bn \cos[a + b(-n\log[x] + \log[cx^n])] + 2 \sin[a + b(-n\log[x] + \log[cx^n])]) \right) + \\
& \sqrt{\operatorname{Csc}[a + bn \log[x] + b(-n\log[x] + \log[cx^n])]} \\
& \left( -\frac{2x \cos[bn \log[x]]}{bn \cos[a + b(-n\log[x] + \log[cx^n])] + 2 \sin[a + b(-n\log[x] + \log[cx^n])]} + \right. \\
& \quad \left. \frac{4x \sin[bn \log[x]]}{bn (bn \cos[a + b(-n\log[x] + \log[cx^n])] + 2 \sin[a + b(-n\log[x] + \log[cx^n])])} \right)
\end{aligned}$$

**Problem 314: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\operatorname{Csc}[a + b \log[cx^n]]}} dx$$

Optimal (type 5, 110 leaves, 3 steps):

$$\frac{2x \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{2i+bn}{4bn}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right]}{(2-ibn) \sqrt{1 - e^{2ia}(cx^n)^{2ib}} \sqrt{\operatorname{Csc}[a + b \log[cx^n]]}}$$

Result (type 5, 367 leaves):

$$\begin{aligned}
& 2x \left( - \left( \left( \frac{i \sqrt{2} b e^{-ia} n (c x^n)^{-ib}}{\sqrt{-1 + e^{2ia} (c x^n)^{2ib}}} \right) \right. \right. \\
& \quad \left. \left( (2i + bn) (-1 + e^{2ia} (c x^n)^{2ib}) + \sqrt{1 - e^{2ia} (c x^n)^{2ib}} (2i + bn + e^{2ia} (-2i + bn) x^{-2ibn} (c x^n)^{2ib}) \right. \right. \\
& \quad \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2i + bn}{4bn}, \frac{3}{4} - \frac{i}{2bn}, e^{2ia} (c x^n)^{2ib} \right] \right) \right) / \left( (4 + b^2 n^2) (-2 + ibn + e^{2ia} (2 + ibn) x^{-2ibn} (c x^n)^{2ib}) \right) + \\
& \left. \frac{\sin[a - bn \log[x] + b \log[cx^n]]}{\sqrt{\csc[a + b \log[cx^n]]} (bn \cos[a - bn \log[x] + b \log[cx^n]] + 2 \sin[a - bn \log[x] + b \log[cx^n]])} \right)
\end{aligned}$$

**Problem 318: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\csc[a + b \log[cx^n]]^{5/2}} dx$$

Optimal (type 5, 110 leaves, 3 steps):

$$\frac{2x \text{Hypergeometric2F1} \left[ -\frac{5}{2}, \frac{1}{4} \left( -5 - \frac{2i}{bn} \right), -\frac{2i + bn}{4bn}, e^{2ia} (c x^n)^{2ib} \right]}{(2 - 5ibn) (1 - e^{2ia} (c x^n)^{2ib})^{5/2} \csc[a + b \log[cx^n]]^{5/2}}$$

Result (type 5, 862 leaves):

$$\begin{aligned}
& - \left( \left( 30 i \sqrt{2} b^3 e^{-i(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} n^3 x^{1-i b n} \sqrt{\frac{i e^{i(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{i b n}}{-1 + e^{2 i(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n}}} \right. \right. \\
& \quad \left. \left( (2 i + b n) (-1 + e^{2 i(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n}) + (2 i + b n + e^{2 i(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} (-2 i + b n)) \right. \right. \\
& \quad \left. \left. \sqrt{1 - e^{2 i(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2 i + b n}{4 b n}, \frac{3}{4} - \frac{i}{2 b n}, e^{2 i(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} x^{2 i b n}\right] \right) \right) \Bigg) / \\
& \quad \left( (-2 + 5 i b n) (-2 i + 5 b n) (4 + b^2 n^2) (2 i + b n + e^{2 i(a+b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))} (-2 i + b n)) \right) \Bigg) + \\
& \sqrt{\operatorname{Csc}[a + b n \operatorname{Log}[x] + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] \left( - \left( (x \operatorname{Cos}[b n \operatorname{Log}[x]] (-12 - 55 b^2 n^2 + 12 \operatorname{Cos}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) \right) \right) + } \\
& \quad 65 b^2 n^2 \operatorname{Cos}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 4 b n \operatorname{Sin}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \Bigg) \Bigg) / \\
& \quad \left( 4 (-2 i + 5 b n) (2 i + 5 b n) (b n \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + 2 \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]) \Bigg) \Bigg) + \\
& \quad \left( x \operatorname{Sin}[b n \operatorname{Log}[x]] (16 b n - 4 b n \operatorname{Cos}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 12 \operatorname{Sin}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) \right) \Bigg) + \\
& \quad 65 b^2 n^2 \operatorname{Sin}[2(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] \Bigg) \Bigg) / \\
& \quad \left( 4 (-2 i + 5 b n) (2 i + 5 b n) (b n \operatorname{Cos}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])] + 2 \operatorname{Sin}[a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n])]) \Bigg) \Bigg) + \\
& \quad \left( x \operatorname{Cos}[3 b n \operatorname{Log}[x]] (5 b n \operatorname{Cos}[3(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] - 2 \operatorname{Sin}[3(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) \right) \Bigg) \Bigg) / \\
& \quad \left( 2 (-2 i + 5 b n) (2 i + 5 b n) \right) - \\
& \quad \left( x \operatorname{Sin}[3 b n \operatorname{Log}[x]] (2 \operatorname{Cos}[3(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))] + 5 b n \operatorname{Sin}[3(a + b(-n \operatorname{Log}[x] + \operatorname{Log}[c x^n]))]) \right) \Bigg) \Bigg) / \\
& \quad \left( 2 (-2 i + 5 b n) (2 i + 5 b n) \right)
\end{aligned}$$

**Problem 320: Result more than twice size of optimal antiderivative.**

$$\int (e x)^m \operatorname{Csc}[d(a + b \operatorname{Log}[c x^n])]^3 dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$\frac{8 e^{3 i a d} (e x)^{1+m} (c x^n)^{3 i b d} \operatorname{Hypergeometric2F1}\left[3, -\frac{i(1+m)-3 b d n}{2 b d n}, -\frac{i(1+m)-5 b d n}{2 b d n}, e^{2 i a d} (c x^n)^{2 i b d}\right]}{e(i(1+m) - 3 b d n)}$$

Result (type 5, 367 leaves):

$$\begin{aligned} & \frac{1}{8 b^2 d^2 n^2} x (e x)^m \left( -b d n \operatorname{Csc} \left[ \frac{1}{2} d (a + b \operatorname{Log}[c x^n]) \right]^2 - 4 (1+m) \operatorname{Csc} [d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])] \right) + \\ & b d n \operatorname{Sec} \left[ \frac{1}{2} d (a + b \operatorname{Log}[c x^n]) \right]^2 + 2 (1+m) \operatorname{Csc} \left[ \frac{1}{2} d (a + b \operatorname{Log}[c x^n]) \right] \operatorname{Csc} \left[ \frac{1}{2} d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \right] \operatorname{Sin} \left[ \frac{1}{2} b d n \operatorname{Log}[x] \right] - \\ & 2 (1+m) \operatorname{Sec} \left[ \frac{1}{2} d (a + b \operatorname{Log}[c x^n]) \right] \operatorname{Sec} \left[ \frac{1}{2} d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]) \right] \operatorname{Sin} \left[ \frac{1}{2} b d n \operatorname{Log}[x] \right] + 8 (1+m - i b d n) x^{i b d n} \operatorname{Hypergeometric2F1} \left[ 1, \right. \\ & \left. \frac{-i - i m + b d n}{2 b d n}, -\frac{i (1+m + 3 i b d n)}{2 b d n}, x^{2 i b d n} (\operatorname{Cos} [2 d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])] + i \operatorname{Sin} [2 d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])]) \right] \left. \right) \\ & \left( -i \operatorname{Cos} [d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])] + \operatorname{Sin} [d (a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n])] \right) \end{aligned}$$

**Problem 324: Result more than twice size of optimal antiderivative.**

$$\int x^m \operatorname{Csc} [a + b \operatorname{Log}[c x^n]]^{3/2} dx$$

Optimal (type 5, 130 leaves, 3 steps):

$$\frac{1}{2 + 2 m + 3 i b n} 2 x^{1+m} \left( 1 - e^{2 i a} (c x^n)^{2 i b} \right)^{3/2} \operatorname{Csc} [a + b \operatorname{Log}[c x^n]]^{3/2} \operatorname{Hypergeometric2F1} \left[ \frac{3}{2}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -\frac{2 i + 2 i m - 7 b n}{4 b n}, e^{2 i a} (c x^n)^{2 i b} \right]$$

Result (type 5, 466 leaves):

$$\begin{aligned} & \left( x^{1+m-i b n} \left( (4 + 8 m + 4 m^2 + b^2 n^2) x^{2 i b n} \sqrt{2 - 2 e^{2 i a} (c x^n)^{2 i b}} \right. \right. \\ & \left. \sqrt{\frac{i e^{i a} (c x^n)^{i b}}{-1 + e^{2 i a} (c x^n)^{2 i b}}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m - 3 b n}{4 b n}, -\frac{2 i + 2 i m - 7 b n}{4 b n}, e^{2 i a} (c x^n)^{2 i b} \right] + \right. \\ & \left. (-2 i - 2 i m + 3 b n) \left( (-2 i - 2 i m + b n) \sqrt{2 - 2 e^{2 i a} (c x^n)^{2 i b}} \sqrt{\frac{i e^{i a} (c x^n)^{i b}}{-1 + e^{2 i a} (c x^n)^{2 i b}}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, -\frac{2 i + 2 i m + b n}{4 b n}, \right. \right. \right. \\ & \left. \left. \left. -\frac{2 i + 2 i m - 3 b n}{4 b n}, e^{2 i a} (c x^n)^{2 i b} \right] - 2 x^{i b n} \sqrt{\operatorname{Csc} [a + b \operatorname{Log}[c x^n]]} (b n \operatorname{Cos} [b n \operatorname{Log}[x]] - 2 (1+m) \operatorname{Sin} [b n \operatorname{Log}[x]]) \right) \right) \Bigg) / \\ & (b n (-2 i - 2 i m + 3 b n) (b n \operatorname{Cos} [a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]] + 2 (1+m) \operatorname{Sin} [a - b n \operatorname{Log}[x] + b \operatorname{Log}[c x^n]])) \end{aligned}$$



### Problem 326: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{\sqrt{\text{Csc}[a + b \text{Log}[c x^n]]}} dx$$

Optimal (type 5, 129 leaves, 3 steps):

$$\frac{2 x^{1+m} \text{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{2i+2im+bn}{4bn}, -\frac{2i+2im-3bn}{4bn}, e^{2ia} (c x^n)^{2ib}\right]}{(2+2m-i)bn \sqrt{1-e^{2ia} (c x^n)^{2ib}} \sqrt{\text{Csc}[a + b \text{Log}[c x^n]]}}$$

Result (type 5, 637 leaves):

$$\left( 2 \sqrt{2} b e^{i(a+b(-n \text{Log}[x] + \text{Log}[c x^n]))} n x^{1+m-i} b n \sqrt{1-e^{2i(a+b(-n \text{Log}[x] + \text{Log}[c x^n]))} x^{2ib} n} \sqrt{\frac{i e^{i(a+b(-n \text{Log}[x] + \text{Log}[c x^n]))} x^{ib} n}{-1+e^{2i(a+b(-n \text{Log}[x] + \text{Log}[c x^n]))} x^{2ib} n}} \right. \\ \left. \left( (2+2m-i)bn x^{2ib} n \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2i+2im-3bn}{4bn}, -\frac{2i+2im-7bn}{4bn}, e^{2i(a+b(-n \text{Log}[x] + \text{Log}[c x^n]))} x^{2ib} n\right] - \right. \right. \\ \left. \left. (2+2m+3i)bn \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{2i+2im+bn}{4bn}, -\frac{2i+2im-3bn}{4bn}, e^{2i(a+b(-n \text{Log}[x] + \text{Log}[c x^n]))} x^{2ib} n\right] \right) \right) / \\ \left( (2+2m-i)bn (2+2m+3i)bn (-2-2m+i)bn + e^{2i(a+b(-n \text{Log}[x] + \text{Log}[c x^n]))} (2+2m+i)bn \right) + \\ \sqrt{\text{Csc}[a + b n \text{Log}[x] + b(-n \text{Log}[x] + \text{Log}[c x^n])]} \left( (2 x^{1+m} \text{Cos}[bn \text{Log}[x]] \text{Sin}[a + b(-n \text{Log}[x] + \text{Log}[c x^n])] )^2 / \right. \\ \left. (bn \text{Cos}[a + b(-n \text{Log}[x] + \text{Log}[c x^n])] + 2 \text{Sin}[a + b(-n \text{Log}[x] + \text{Log}[c x^n])] + 2m \text{Sin}[a + b(-n \text{Log}[x] + \text{Log}[c x^n])]) \right) + \\ \left. (x^{1+m} \text{Sin}[bn \text{Log}[x]] \text{Sin}[2(a + b(-n \text{Log}[x] + \text{Log}[c x^n]))]) / \right. \\ \left. (bn \text{Cos}[a + b(-n \text{Log}[x] + \text{Log}[c x^n])] + 2 \text{Sin}[a + b(-n \text{Log}[x] + \text{Log}[c x^n])] + 2m \text{Sin}[a + b(-n \text{Log}[x] + \text{Log}[c x^n])]) \right) \right)$$

### Test results for the 142 problems in "4.7.6 f^(a+b x+c x^2) trig(d+e x+f x^2)^n.m"

#### Problem 1: Unable to integrate problem.

$$\int F^{c(a+bx)} \text{Sin}[d+ex]^n dx$$

Optimal (type 5, 107 leaves, 2 steps):

$$-\frac{1}{ie^{n-bc \text{Log}[F]}} (1-e^{2i(d+ex)})^{-n} F^{c(a+bx)} \text{Hypergeometric2F1}\left[-n, -\frac{en+ibc \text{Log}[F]}{2e}, \frac{1}{2} \left(2-n-\frac{ibc \text{Log}[F]}{e}\right), e^{2i(d+ex)}\right] \text{Sin}[d+ex]^n$$

Result (type 8, 20 leaves):

$$\int F^{c(a+bx)} \sin[d+ex]^n dx$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int F^{c(a+bx)} \operatorname{Csc}[d+ex]^3 dx$$

Optimal (type 5, 137 leaves, 2 steps):

$$\frac{F^{c(a+bx)} \operatorname{Cot}[d+ex] \operatorname{Csc}[d+ex]}{2e} - \frac{bc F^{c(a+bx)} \operatorname{Csc}[d+ex] \operatorname{Log}[F]}{2e^2} - \frac{e^{i(d+ex)} F^{c(a+bx)} \operatorname{Hypergeometric2F1}\left[1, \frac{e-i bc \operatorname{Log}[F]}{2e}, \frac{1}{2} \left(3 - \frac{i bc \operatorname{Log}[F]}{e}\right), e^{2i(d+ex)}\right] (e + i bc \operatorname{Log}[F])}{e^2}$$

Result (type 5, 450 leaves):

$$\begin{aligned} & -\frac{F^{a+bcx} \operatorname{Csc}\left[\frac{d}{2} + \frac{ex}{2}\right]^2}{8e} - \frac{bc F^{a+bcx} \operatorname{Csc}[d] \operatorname{Log}[F]}{2e^2} + \frac{F^{c(a+bx)} \operatorname{Csc}[d] (e^2 + b^2 c^2 \operatorname{Log}[F]^2)}{2bc e^2 \operatorname{Log}[F]} + \frac{F^{a+bcx} \operatorname{Sec}\left[\frac{d}{2} + \frac{ex}{2}\right]^2}{8e} - \left(i F^{c(a+bx)} (e^2 + b^2 c^2 \operatorname{Log}[F]^2)\right. \\ & \left. \left(1 + \operatorname{Hypergeometric2F1}\left[1, -\frac{i bc \operatorname{Log}[F]}{e}, 1 - \frac{i bc \operatorname{Log}[F]}{e}, \operatorname{Cos}[d+ex] + i \operatorname{Sin}[d+ex]\right] (-1 + \operatorname{Cos}[d] + i \operatorname{Sin}[d])\right)\right) / \\ & \left(2bc e^2 \operatorname{Log}[F] (-1 + \operatorname{Cos}[d] + i \operatorname{Sin}[d])\right) - \left(i F^{c(a+bx)} (e^2 + b^2 c^2 \operatorname{Log}[F]^2)\right. \\ & \left. \left(1 - \operatorname{Hypergeometric2F1}\left[1, -\frac{i bc \operatorname{Log}[F]}{e}, 1 - \frac{i bc \operatorname{Log}[F]}{e}, -\operatorname{Cos}[d+ex] - i \operatorname{Sin}[d+ex]\right] (1 + \operatorname{Cos}[d] + i \operatorname{Sin}[d])\right)\right) / \\ & \left(2bc e^2 \operatorname{Log}[F] (1 + \operatorname{Cos}[d] + i \operatorname{Sin}[d])\right) + \frac{bc F^{a+bcx} \operatorname{Csc}\left[\frac{d}{2}\right] \operatorname{Csc}\left[\frac{d}{2} + \frac{ex}{2}\right] \operatorname{Log}[F] \operatorname{Sin}\left[\frac{ex}{2}\right]}{4e^2} - \frac{bc F^{a+bcx} \operatorname{Log}[F] \operatorname{Sec}\left[\frac{d}{2}\right] \operatorname{Sec}\left[\frac{d}{2} + \frac{ex}{2}\right] \operatorname{Sin}\left[\frac{ex}{2}\right]}{4e^2} \end{aligned}$$

Problem 10: Unable to integrate problem.

$$\int F^{c(a+bx)} \operatorname{Cos}[d+ex]^n dx$$

Optimal (type 5, 107 leaves, 2 steps):

$$-\frac{1}{i en - bc \operatorname{Log}[F]} \left(1 + e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \operatorname{Cos}[d+ex]^n \operatorname{Hypergeometric2F1}\left[-n, -\frac{en + i bc \operatorname{Log}[F]}{2e}, \frac{1}{2} \left(2 - n - \frac{i bc \operatorname{Log}[F]}{e}\right), -e^{2i(d+ex)}\right]$$

Result (type 8, 20 leaves):

$$\int F^{c(a+bx)} \operatorname{Cos}[d+ex]^n dx$$

### Problem 14: Unable to integrate problem.

$$\int F^{c(a+bx)} \operatorname{Sec}[d+ex] dx$$

Optimal (type 5, 84 leaves, 1 step):

$$\frac{2 e^{i(d+ex)} F^{c(a+bx)} \operatorname{Hypergeometric2F1}\left[1, \frac{e-ibc \operatorname{Log}[F]}{2e}, \frac{1}{2} \left(3 - \frac{ibc \operatorname{Log}[F]}{e}\right), -e^{2i(d+ex)}\right]}{ie + bc \operatorname{Log}[F]}$$

Result (type 8, 18 leaves):

$$\int F^{c(a+bx)} \operatorname{Sec}[d+ex] dx$$

### Problem 16: Unable to integrate problem.

$$\int F^{c(a+bx)} \operatorname{Sec}[d+ex]^3 dx$$

Optimal (type 5, 141 leaves, 2 steps):

$$-\frac{e^{i(d+ex)} F^{c(a+bx)} \operatorname{Hypergeometric2F1}\left[1, \frac{e-ibc \operatorname{Log}[F]}{2e}, \frac{1}{2} \left(3 - \frac{ibc \operatorname{Log}[F]}{e}\right), -e^{2i(d+ex)}\right] (ie - bc \operatorname{Log}[F])}{e^2} - \frac{bc F^{c(a+bx)} \operatorname{Log}[F] \operatorname{Sec}[d+ex]}{2e^2} + \frac{F^{c(a+bx)} \operatorname{Sec}[d+ex] \operatorname{Tan}[d+ex]}{2e}$$

Result (type 8, 20 leaves):

$$\int F^{c(a+bx)} \operatorname{Sec}[d+ex]^3 dx$$

### Problem 21: Result more than twice size of optimal antiderivative.

$$\int e^{c(a+bx)} \operatorname{Tan}[d+ex] dx$$

Optimal (type 5, 78 leaves, 4 steps):

$$-\frac{ie^{c(a+bx)}}{bc} + \frac{2ie^{c(a+bx)} \operatorname{Hypergeometric2F1}\left[1, -\frac{ibc}{2e}, 1 - \frac{ibc}{2e}, -e^{2i(d+ex)}\right]}{bc}$$

Result (type 5, 166 leaves):

$$\frac{1}{bc(i bc - 2e)(1 + e^{2id})} e^{c(a+bx)} \left( 2bc e^{2i(d+ex)} \text{Hypergeometric2F1}\left[1, 1 - \frac{ibc}{2e}, 2 - \frac{ibc}{2e}, -e^{2i(d+ex)}\right] - (bc + 2ie) \left( 1 - e^{2id} + 2e^{2id} \text{Hypergeometric2F1}\left[1, -\frac{ibc}{2e}, 1 - \frac{ibc}{2e}, -e^{2i(d+ex)}\right] \right) \right)$$

**Problem 22: Result more than twice size of optimal antiderivative.**

$$\int e^{c(a+bx)} \text{Cot}[d+ex] dx$$

Optimal (type 5, 76 leaves, 4 steps):

$$\frac{ie^{c(a+bx)}}{bc} - \frac{2ie^{c(a+bx)} \text{Hypergeometric2F1}\left[1, -\frac{ibc}{2e}, 1 - \frac{ibc}{2e}, e^{2i(d+ex)}\right]}{bc}$$

Result (type 5, 163 leaves):

$$\left( e^{c(a+bx)} \left( 2ibc e^{2i(d+ex)} \text{Hypergeometric2F1}\left[1, 1 - \frac{ibc}{2e}, 2 - \frac{ibc}{2e}, e^{2i(d+ex)}\right] + i(bc + 2ie) \left( 1 + e^{2id} - 2e^{2id} \text{Hypergeometric2F1}\left[1, -\frac{ibc}{2e}, 1 - \frac{ibc}{2e}, e^{2i(d+ex)}\right] \right) \right) \right) / (bc(bc + 2ie)(-1 + e^{2id}))$$

**Problem 26: Unable to integrate problem.**

$$\int F^{c(a+bx)} \text{Sec}[d+ex]^n dx$$

Optimal (type 5, 100 leaves, 2 steps):

$$\frac{1}{ie^{n+bc \text{Log}[F]}} (1 + e^{2i(d+ex)})^n F^{ac+bcx} \text{Hypergeometric2F1}\left[n, \frac{en - ibc \text{Log}[F]}{2e}, \frac{1}{2} \left( 2+n - \frac{ibc \text{Log}[F]}{e} \right), -e^{2i(d+ex)}\right] \text{Sec}[d+ex]^n$$

Result (type 8, 20 leaves):

$$\int F^{c(a+bx)} \text{Sec}[d+ex]^n dx$$

**Problem 27: Unable to integrate problem.**

$$\int F^{c(a+bx)} \text{Csc}[d+ex]^n dx$$

Optimal (type 5, 102 leaves, 2 steps):

$$-\frac{1}{i e n - b c \operatorname{Log}[F]} \left(1 - e^{-2 i (d+e x)}\right)^n F^{a+b c x} \operatorname{Csc}[d+e x]^n \operatorname{Hypergeometric2F1}\left[n, \frac{e n + i b c \operatorname{Log}[F]}{2 e}, \frac{1}{2} \left(2+n + \frac{i b c \operatorname{Log}[F]}{e}\right), e^{-2 i (d+e x)}\right]$$

Result (type 8, 20 leaves):

$$\int F^{c(a+b x)} \operatorname{Csc}[d+e x]^n dx$$

**Problem 63: Result more than twice size of optimal antiderivative.**

$$\int e^x \operatorname{Csc}[e^x] \operatorname{Sec}[e^x] dx$$

Optimal (type 3, 5 leaves, 3 steps):

$$\operatorname{Log}[\operatorname{Tan}[e^x]]$$

Result (type 3, 21 leaves):

$$2 \left( -\frac{1}{2} \operatorname{Log}[\operatorname{Cos}[e^x]] + \frac{1}{2} \operatorname{Log}[\operatorname{Sin}[e^x]] \right)$$

**Problem 70: Result more than twice size of optimal antiderivative.**

$$\int e^x \operatorname{Sec}[e^x] dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\operatorname{ArcTanh}[\operatorname{Sin}[e^x]]$$

Result (type 3, 41 leaves):

$$-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{e^x}{2}\right] - \operatorname{Sin}\left[\frac{e^x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Cos}\left[\frac{e^x}{2}\right] + \operatorname{Sin}\left[\frac{e^x}{2}\right]\right]$$

**Problem 93: Result more than twice size of optimal antiderivative.**

$$\int f^{a+c x^2} \operatorname{Sin}[d+e x+f x^2]^3 dx$$

Optimal (type 4, 377 leaves, 14 steps):

$$\frac{3 i e^{-i d - \frac{e^2}{4 i f - 4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{i e + 2 x (i f - c \operatorname{Log}[f])}{2 \sqrt{i f - c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f - c \operatorname{Log}[f]}} - \frac{i e^{-3 i d - \frac{9 e^2}{4 (3 i f - c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{3 i e + 2 x (3 i f - c \operatorname{Log}[f])}{2 \sqrt{3 i f - c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f - c \operatorname{Log}[f]}} -$$

$$\frac{3 i e^{i d + \frac{e^2}{4 i f + 4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{i e + 2 x (i f + c \operatorname{Log}[f])}{2 \sqrt{i f + c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f + c \operatorname{Log}[f]}} + \frac{i e^{3 i d + \frac{9 e^2}{4 (3 i f + c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{3 i e + 2 x (3 i f + c \operatorname{Log}[f])}{2 \sqrt{3 i f + c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f + c \operatorname{Log}[f]}}$$

Result (type 4, 3003 leaves):

$$\left( f^a \sqrt{\pi} \left( -27 (-1)^{3/4} e^{-\frac{i e^2}{4 (f - i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e + 2 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{f - i c \operatorname{Log}[f]}}\right] \sqrt{f - i c \operatorname{Log}[f]} + \right.$$

$$27 (-1)^{1/4} c e^{-\frac{i e^2}{4 (f - i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e + 2 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f - i c \operatorname{Log}[f]} -$$

$$3 (-1)^{3/4} c^2 e^{-\frac{i e^2}{4 (f - i c \operatorname{Log}[f])}} f \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e + 2 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f - i c \operatorname{Log}[f]} +$$

$$3 (-1)^{1/4} c^3 e^{-\frac{i e^2}{4 (f - i c \operatorname{Log}[f])}} \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e + 2 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f - i c \operatorname{Log}[f]} +$$

$$3 (-1)^{3/4} e^{-\frac{9 i e^2}{4 (3 f - i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e + 6 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f - i c \operatorname{Log}[f]}}\right] \sqrt{3 f - i c \operatorname{Log}[f]} -$$

$$(-1)^{1/4} c e^{-\frac{9 i e^2}{4 (3 f - i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e + 6 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f - i c \operatorname{Log}[f]} +$$

$$3 (-1)^{3/4} c^2 e^{-\frac{9 i e^2}{4 (3 f - i c \operatorname{Log}[f])}} f \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e + 6 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f - i c \operatorname{Log}[f]} -$$

$$(-1)^{1/4} c^3 e^{-\frac{9 i e^2}{4 (3 f - i c \operatorname{Log}[f])}} \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e + 6 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f - i c \operatorname{Log}[f]} +$$

$$27 (-1)^{1/4} e^{\frac{i e^2}{4 (f + i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e + 2 f x + 2 i c x \operatorname{Log}[f])}{2 \sqrt{f + i c \operatorname{Log}[f]}}\right] \sqrt{f + i c \operatorname{Log}[f]} -$$

$$27 (-1)^{3/4} c e^{\frac{i e^2}{4 (f + i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e + 2 f x + 2 i c x \operatorname{Log}[f])}{2 \sqrt{f + i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f + i c \operatorname{Log}[f]} +$$

$$3 (-1)^{1/4} c^2 e^{\frac{i e^2}{4 (f + i c \operatorname{Log}[f])}} f \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e + 2 f x + 2 i c x \operatorname{Log}[f])}{2 \sqrt{f + i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f + i c \operatorname{Log}[f]} -$$

$$\begin{aligned}
& 3 (-1)^{3/4} c^3 e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f+i c \operatorname{Log}[f]} - \\
& 3 (-1)^{1/4} e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \sqrt{3 f+i c \operatorname{Log}[f]} + \\
& (-1)^{3/4} c e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f+i c \operatorname{Log}[f]} - \\
& 3 (-1)^{1/4} c^2 e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f+i c \operatorname{Log}[f]} + \\
& (-1)^{3/4} c^3 e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f+i c \operatorname{Log}[f]} + \\
& 27 (-1)^{1/4} e^{-\frac{i e^2}{4(f-i c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e+2 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \sqrt{f-i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 27 (-1)^{3/4} c e^{-\frac{i e^2}{4(f-i c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e+2 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f-i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i e^2}{4(f-i c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e+2 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f-i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{3/4} c^3 e^{-\frac{i e^2}{4(f-i c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e+2 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f-i c \operatorname{Log}[f]} \operatorname{Sin}[d] - \\
& 27 (-1)^{3/4} e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \sqrt{f+i c \operatorname{Log}[f]} \operatorname{Sin}[d] - \\
& 27 (-1)^{1/4} c e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f+i c \operatorname{Log}[f]} \operatorname{Sin}[d] - \\
& 3 (-1)^{3/4} c^2 e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f+i c \operatorname{Log}[f]} \operatorname{Sin}[d] - \\
& 3 (-1)^{1/4} c^3 e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f+i c \operatorname{Log}[f]} \operatorname{Sin}[d] - \\
& 3 (-1)^{1/4} e^{-\frac{9 i e^2}{4(3 f-i c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e+6 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f-i c \operatorname{Log}[f]}}\right] \sqrt{3 f-i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] - \\
& (-1)^{3/4} c e^{-\frac{9 i e^2}{4(3 f-i c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e+6 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f-i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] -
\end{aligned}$$

$$\begin{aligned}
& 3 (-1)^{1/4} c^2 e^{-\frac{9 i e^2}{4(3 f-i c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3 e+6 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f-i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] - \\
& (-1)^{3/4} c^3 e^{-\frac{9 i e^2}{4(3 f-i c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3 e+6 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f-i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& 3 (-1)^{3/4} e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \sqrt{3 f+i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& (-1)^{1/4} c e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f+i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& 3 (-1)^{3/4} c^2 e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f+i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& (-1)^{1/4} c^3 e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f+i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] \Big) \Big) / \\
& (16 (i f-c \operatorname{Log}[f]) (f-i c \operatorname{Log}[f]) (3 f-i c \operatorname{Log}[f]) (3 f+i c \operatorname{Log}[f]))
\end{aligned}$$

**Problem 99: Result more than twice size of optimal antiderivative.**

$$\int f^{a+b x+c x^2} \operatorname{Sin}[d+f x^2]^3 dx$$

Optimal (type 4, 386 leaves, 14 steps):

$$\begin{aligned}
& \frac{3 i e^{-i d+\frac{b^2 \operatorname{Log}[f]^2}{4 i f-4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{b \operatorname{Log}[f]-2 x(i f-c \operatorname{Log}[f])}{2 \sqrt{i f-c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f-c \operatorname{Log}[f]}} + \frac{i e^{-3 i d+\frac{b^2 \operatorname{Log}[f]^2}{12 i f-4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{b \operatorname{Log}[f]-2 x(3 i f-c \operatorname{Log}[f])}{2 \sqrt{3 i f-c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f-c \operatorname{Log}[f]}} - \\
& \frac{3 i e^{i d-\frac{b^2 \operatorname{Log}[f]^2}{4 i f+4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{b \operatorname{Log}[f]+2 x(i f+c \operatorname{Log}[f])}{2 \sqrt{i f+c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f+c \operatorname{Log}[f]}} + \frac{i e^{3 i d-\frac{b^2 \operatorname{Log}[f]^2}{4(3 i f+c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{b \operatorname{Log}[f]+2 x(3 i f+c \operatorname{Log}[f])}{2 \sqrt{3 i f+c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f+c \operatorname{Log}[f]}}
\end{aligned}$$

Result (type 4, 3291 leaves):

$$\begin{aligned}
& \left( f^a \sqrt{\pi} \left( -27 (-1)^{3/4} e^{\frac{i b^2 \operatorname{Log}[f]^2}{4(f-i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(2 f x-i b \operatorname{Log}[f]-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \sqrt{f-i c \operatorname{Log}[f]} + \right. \right. \\
& \left. \left. 27 (-1)^{1/4} c e^{\frac{i b^2 \operatorname{Log}[f]^2}{4(f-i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(2 f x-i b \operatorname{Log}[f]-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f-i c \operatorname{Log}[f]} - \right. \right.
\end{aligned}$$



$$\begin{aligned}
& 3 (-1)^{3/4} c^2 e^{\frac{i b^2 \text{Log}[f]^2}{4 (f-i c \text{Log}[f])}} f \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f - i c \text{Log}[f]} + \\
& 3 (-1)^{1/4} c^3 e^{\frac{i b^2 \text{Log}[f]^2}{4 (f-i c \text{Log}[f])}} \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f - i c \text{Log}[f]} + \\
& 3 (-1)^{3/4} e^{\frac{i b^2 \text{Log}[f]^2}{4 (3 f-i c \text{Log}[f])}} f^3 \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \sqrt{3 f - i c \text{Log}[f]} - \\
& (-1)^{1/4} c e^{\frac{i b^2 \text{Log}[f]^2}{4 (3 f-i c \text{Log}[f])}} f^2 \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{3 f - i c \text{Log}[f]} + \\
& 3 (-1)^{3/4} c^2 e^{\frac{i b^2 \text{Log}[f]^2}{4 (3 f-i c \text{Log}[f])}} f \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3 f - i c \text{Log}[f]} - \\
& (-1)^{1/4} c^3 e^{\frac{i b^2 \text{Log}[f]^2}{4 (3 f-i c \text{Log}[f])}} \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3 f - i c \text{Log}[f]} + \\
& 27 (-1)^{1/4} e^{-\frac{i b^2 \text{Log}[f]^2}{4 (f+i c \text{Log}[f])}} f^3 \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \sqrt{f + i c \text{Log}[f]} - \\
& 27 (-1)^{3/4} c e^{-\frac{i b^2 \text{Log}[f]^2}{4 (f+i c \text{Log}[f])}} f^2 \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{f + i c \text{Log}[f]} + \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i b^2 \text{Log}[f]^2}{4 (f+i c \text{Log}[f])}} f \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f + i c \text{Log}[f]} - \\
& 3 (-1)^{3/4} c^3 e^{-\frac{i b^2 \text{Log}[f]^2}{4 (f+i c \text{Log}[f])}} \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f + i c \text{Log}[f]} - \\
& 3 (-1)^{1/4} e^{-\frac{i b^2 \text{Log}[f]^2}{4 (3 f+i c \text{Log}[f])}} f^3 \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \sqrt{3 f + i c \text{Log}[f]} + \\
& (-1)^{3/4} c e^{-\frac{i b^2 \text{Log}[f]^2}{4 (3 f+i c \text{Log}[f])}} f^2 \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{3 f + i c \text{Log}[f]} - \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i b^2 \text{Log}[f]^2}{4 (3 f+i c \text{Log}[f])}} f \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3 f + i c \text{Log}[f]} + \\
& (-1)^{3/4} c^3 e^{-\frac{i b^2 \text{Log}[f]^2}{4 (3 f+i c \text{Log}[f])}} \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3 f + i c \text{Log}[f]} + \\
& 27 (-1)^{1/4} e^{\frac{i b^2 \text{Log}[f]^2}{4 (f-i c \text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \sqrt{f - i c \text{Log}[f]} \text{Sin}[d] +
\end{aligned}$$

$$\begin{aligned}
& 27 (-1)^{3/4} c e^{\frac{i b^2 \text{Log}[f]^2}{4(f-i c \text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{f - i c \text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{1/4} c^2 e^{\frac{i b^2 \text{Log}[f]^2}{4(f-i c \text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f - i c \text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{3/4} c^3 e^{\frac{i b^2 \text{Log}[f]^2}{4(f-i c \text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f - i c \text{Log}[f]} \text{Sin}[d] - \\
& 27 (-1)^{3/4} e^{-\frac{i b^2 \text{Log}[f]^2}{4(f+i c \text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \sqrt{f + i c \text{Log}[f]} \text{Sin}[d] - \\
& 27 (-1)^{1/4} c e^{-\frac{i b^2 \text{Log}[f]^2}{4(f+i c \text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{f + i c \text{Log}[f]} \text{Sin}[d] - \\
& 3 (-1)^{3/4} c^2 e^{-\frac{i b^2 \text{Log}[f]^2}{4(f+i c \text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f + i c \text{Log}[f]} \text{Sin}[d] - \\
& 3 (-1)^{1/4} c^3 e^{-\frac{i b^2 \text{Log}[f]^2}{4(f+i c \text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f + i c \text{Log}[f]} \text{Sin}[d] - \\
& 3 (-1)^{1/4} e^{\frac{i b^2 \text{Log}[f]^2}{4(3f-i c \text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \sqrt{3 f - i c \text{Log}[f]} \text{Sin}[3 d] - \\
& (-1)^{3/4} c e^{\frac{i b^2 \text{Log}[f]^2}{4(3f-i c \text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{3 f - i c \text{Log}[f]} \text{Sin}[3 d] - \\
& 3 (-1)^{1/4} c^2 e^{\frac{i b^2 \text{Log}[f]^2}{4(3f-i c \text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3 f - i c \text{Log}[f]} \text{Sin}[3 d] - \\
& (-1)^{3/4} c^3 e^{\frac{i b^2 \text{Log}[f]^2}{4(3f-i c \text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3 f - i c \text{Log}[f]} \text{Sin}[3 d] + \\
& 3 (-1)^{3/4} e^{-\frac{i b^2 \text{Log}[f]^2}{4(3f+i c \text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \sqrt{3 f + i c \text{Log}[f]} \text{Sin}[3 d] + \\
& (-1)^{1/4} c e^{-\frac{i b^2 \text{Log}[f]^2}{4(3f+i c \text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{3 f + i c \text{Log}[f]} \text{Sin}[3 d] + \\
& 3 (-1)^{3/4} c^2 e^{-\frac{i b^2 \text{Log}[f]^2}{4(3f+i c \text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3 f + i c \text{Log}[f]} \text{Sin}[3 d] + \\
& (-1)^{1/4} c^3 e^{-\frac{i b^2 \text{Log}[f]^2}{4(3f+i c \text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3 f + i c \text{Log}[f]} \text{Sin}[3 d] \Big) \Big) /
\end{aligned}$$

$$(16 (i f - c \text{Log}[f]) (f - i c \text{Log}[f]) (3 f - i c \text{Log}[f]) (3 f + i c \text{Log}[f]))$$

## Problem 101: Result more than twice size of optimal antiderivative.

$$\int f^{a+bx+cx^2} \sin[d+ex+fx^2]^2 dx$$

Optimal (type 4, 268 leaves, 10 steps):

$$\frac{f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{Erfi}\left[\frac{(b+2cx)\sqrt{\operatorname{Log}[f]}}{2\sqrt{c}}\right]}{4\sqrt{c}\sqrt{\operatorname{Log}[f]}} - \frac{e^{-2id} \frac{(2e+ib\operatorname{Log}[f])^2}{8if-4c\operatorname{Log}[f]} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{2ie-b\operatorname{Log}[f]+2x(2if-c\operatorname{Log}[f])}{2\sqrt{2if-c\operatorname{Log}[f]}}\right]}{8\sqrt{2if-c\operatorname{Log}[f]}} - \frac{e^{2id} \frac{(2e-ib\operatorname{Log}[f])^2}{8if+4c\operatorname{Log}[f]} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{2ie+b\operatorname{Log}[f]+2x(2if+c\operatorname{Log}[f])}{2\sqrt{2if+c\operatorname{Log}[f]}}\right]}{8\sqrt{2if+c\operatorname{Log}[f]}}$$

Result (type 4, 1120 leaves):

$$\frac{1}{8c\operatorname{Log}[f](2f-ic\operatorname{Log}[f])(2f+ic\operatorname{Log}[f])} f^a \sqrt{\pi} \left( 8\sqrt{c} f^{2-\frac{b^2}{4c}} \operatorname{Erfi}\left[\frac{(b+2cx)\sqrt{\operatorname{Log}[f]}}{2\sqrt{c}}\right] \sqrt{\operatorname{Log}[f]} + 2c^{5/2} f^{-\frac{b^2}{4c}} \operatorname{Erfi}\left[\frac{(b+2cx)\sqrt{\operatorname{Log}[f]}}{2\sqrt{c}}\right] \operatorname{Log}[f]^{5/2} + \right. \\ 2(-1)^{1/4} c e^{\frac{i(-4e^2+4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f-ic\operatorname{Log}[f])}} f \operatorname{Cos}[2d] \operatorname{Erf}\left[\frac{(-1)^{3/4}(2e+4fx-ib\operatorname{Log}[f]-2icx\operatorname{Log}[f])}{2\sqrt{2f-ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{2f-ic\operatorname{Log}[f]} + \\ (-1)^{3/4} c^2 e^{\frac{i(-4e^2+4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f-ic\operatorname{Log}[f])}} \operatorname{Cos}[2d] \operatorname{Erf}\left[\frac{(-1)^{3/4}(2e+4fx-ib\operatorname{Log}[f]-2icx\operatorname{Log}[f])}{2\sqrt{2f-ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{2f-ic\operatorname{Log}[f]} + \\ 2(-1)^{3/4} c e^{\frac{-i(-4e^2-4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f+ic\operatorname{Log}[f])}} f \operatorname{Cos}[2d] \operatorname{Erf}\left[\frac{(-1)^{1/4}(2e+4fx+ib\operatorname{Log}[f]+2icx\operatorname{Log}[f])}{2\sqrt{2f+ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{2f+ic\operatorname{Log}[f]} + \\ (-1)^{1/4} c^2 e^{\frac{-i(-4e^2-4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f+ic\operatorname{Log}[f])}} \operatorname{Cos}[2d] \operatorname{Erf}\left[\frac{(-1)^{1/4}(2e+4fx+ib\operatorname{Log}[f]+2icx\operatorname{Log}[f])}{2\sqrt{2f+ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{2f+ic\operatorname{Log}[f]} + \\ 2(-1)^{3/4} c e^{\frac{i(-4e^2+4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f-ic\operatorname{Log}[f])}} f \operatorname{Erf}\left[\frac{(-1)^{3/4}(2e+4fx-ib\operatorname{Log}[f]-2icx\operatorname{Log}[f])}{2\sqrt{2f-ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{2f-ic\operatorname{Log}[f]} \operatorname{Sin}[2d] - \\ (-1)^{1/4} c^2 e^{\frac{i(-4e^2+4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f-ic\operatorname{Log}[f])}} \operatorname{Erf}\left[\frac{(-1)^{3/4}(2e+4fx-ib\operatorname{Log}[f]-2icx\operatorname{Log}[f])}{2\sqrt{2f-ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{2f-ic\operatorname{Log}[f]} \operatorname{Sin}[2d] + \\ 2(-1)^{1/4} c e^{\frac{-i(-4e^2-4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f+ic\operatorname{Log}[f])}} f \operatorname{Erf}\left[\frac{(-1)^{1/4}(2e+4fx+ib\operatorname{Log}[f]+2icx\operatorname{Log}[f])}{2\sqrt{2f+ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{2f+ic\operatorname{Log}[f]} \operatorname{Sin}[2d] - \\ (-1)^{3/4} c^2 e^{\frac{-i(-4e^2-4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f+ic\operatorname{Log}[f])}} \operatorname{Erf}\left[\frac{(-1)^{1/4}(2e+4fx+ib\operatorname{Log}[f]+2icx\operatorname{Log}[f])}{2\sqrt{2f+ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{2f+ic\operatorname{Log}[f]} \operatorname{Sin}[2d] \left. \right)$$

## Problem 102: Result more than twice size of optimal antiderivative.

$$\int f^{a+bx+cx^2} \sin[d+ex+fx^2]^3 dx$$

Optimal (type 4, 430 leaves, 14 steps):

$$\frac{3 i e^{-i d - \frac{(e+ib \operatorname{Log}[f])^2}{4 i f - 4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{i e - b \operatorname{Log}[f] + 2 x (i f - c \operatorname{Log}[f])}{2 \sqrt{i f - c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f - c \operatorname{Log}[f]}} - \frac{i e^{-3 i d - \frac{(3e+ib \operatorname{Log}[f])^2}{4 (3 i f - c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{3 i e - b \operatorname{Log}[f] + 2 x (3 i f - c \operatorname{Log}[f])}{2 \sqrt{3 i f - c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f - c \operatorname{Log}[f]}}$$

$$+ \frac{3 i e^{i d + \frac{(e-ib \operatorname{Log}[f])^2}{4 i f + 4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{i e + b \operatorname{Log}[f] + 2 x (i f + c \operatorname{Log}[f])}{2 \sqrt{i f + c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f + c \operatorname{Log}[f]}} + \frac{i e^{3 i d + \frac{(3e+ib \operatorname{Log}[f])^2}{4 (3 i f + c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{3 i e + b \operatorname{Log}[f] + 2 x (3 i f + c \operatorname{Log}[f])}{2 \sqrt{3 i f + c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f + c \operatorname{Log}[f]}}$$

Result (type 4, 3835 leaves):

$$\left( f^a \sqrt{\pi} \left( -27 (-1)^{3/4} e^{\frac{i(-e^2+2ib \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(f-ic \operatorname{Log}[f])}} f^3 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{f-ic \operatorname{Log}[f]}}\right] \sqrt{f-ic \operatorname{Log}[f]} + \right.$$

$$27 (-1)^{1/4} c e^{\frac{i(-e^2+2ib \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(f-ic \operatorname{Log}[f])}} f^2 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f-ic \operatorname{Log}[f]} -$$

$$3 (-1)^{3/4} c^2 e^{\frac{i(-e^2+2ib \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(f-ic \operatorname{Log}[f])}} f \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f-ic \operatorname{Log}[f]} +$$

$$3 (-1)^{1/4} c^3 e^{\frac{i(-e^2+2ib \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(f-ic \operatorname{Log}[f])}} \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f-ic \operatorname{Log}[f]} +$$

$$3 (-1)^{3/4} e^{\frac{i(-9e^2+6ib \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(3f-ic \operatorname{Log}[f])}} f^3 \operatorname{Cos}[3d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{3f-ic \operatorname{Log}[f]}}\right] \sqrt{3f-ic \operatorname{Log}[f]} -$$

$$(-1)^{1/4} c e^{\frac{i(-9e^2+6ib \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(3f-ic \operatorname{Log}[f])}} f^2 \operatorname{Cos}[3d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{3f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3f-ic \operatorname{Log}[f]} +$$

$$3 (-1)^{3/4} c^2 e^{\frac{i(-9e^2+6ib \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(3f-ic \operatorname{Log}[f])}} f \operatorname{Cos}[3d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{3f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3f-ic \operatorname{Log}[f]} -$$

$$(-1)^{1/4} c^3 e^{\frac{i(-9e^2+6ib \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(3f-ic \operatorname{Log}[f])}} \operatorname{Cos}[3d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{3f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3f-ic \operatorname{Log}[f]} +$$

$$\begin{aligned}
& 27 (-1)^{1/4} e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f^3 \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \sqrt{f+ic\text{Log}[f]} - \\
& 27 (-1)^{3/4} c e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f^2 \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{f+ic\text{Log}[f]} + \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f+ic\text{Log}[f]} - \\
& 3 (-1)^{3/4} c^3 e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f+ic\text{Log}[f]} - \\
& 3 (-1)^{1/4} e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f^3 \text{Cos}[3d] \text{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \sqrt{3f+ic\text{Log}[f]} + \\
& (-1)^{3/4} c e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f^2 \text{Cos}[3d] \text{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{3f+ic\text{Log}[f]} - \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f \text{Cos}[3d] \text{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3f+ic\text{Log}[f]} + \\
& (-1)^{3/4} c^3 e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} \text{Cos}[3d] \text{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3f+ic\text{Log}[f]} + \\
& 27 (-1)^{1/4} e^{\frac{i(-e^2+2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f-ic\text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{f-ic\text{Log}[f]}}\right] \sqrt{f-ic\text{Log}[f]} \text{Sin}[d] + \\
& 27 (-1)^{3/4} c e^{\frac{i(-e^2+2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f-ic\text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{f-ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{f-ic\text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{1/4} c^2 e^{\frac{i(-e^2+2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f-ic\text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{f-ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f-ic\text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{3/4} c^3 e^{\frac{i(-e^2+2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f-ic\text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{f-ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f-ic\text{Log}[f]} \text{Sin}[d] - \\
& 27 (-1)^{3/4} e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \sqrt{f+ic\text{Log}[f]} \text{Sin}[d] - \\
& 27 (-1)^{1/4} c e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{f+ic\text{Log}[f]} \text{Sin}[d] -
\end{aligned}$$

$$\begin{aligned}
& 3 (-1)^{3/4} c^2 e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f+ic\text{Log}[f]} \sin[d] - \\
& 3 (-1)^{1/4} c^3 e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f+ic\text{Log}[f]} \sin[d] - \\
& 3 (-1)^{1/4} e^{\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f-ic\text{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{3f-ic\text{Log}[f]}}\right] \sqrt{3f-ic\text{Log}[f]} \sin[3d] - \\
& (-1)^{3/4} c e^{\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f-ic\text{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{3f-ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{3f-ic\text{Log}[f]} \sin[3d] - \\
& 3 (-1)^{1/4} c^2 e^{\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f-ic\text{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{3f-ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3f-ic\text{Log}[f]} \sin[3d] - \\
& (-1)^{3/4} c^3 e^{\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f-ic\text{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{3f-ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3f-ic\text{Log}[f]} \sin[3d] + \\
& 3 (-1)^{3/4} e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \sqrt{3f+ic\text{Log}[f]} \sin[3d] + \\
& (-1)^{1/4} c e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{3f+ic\text{Log}[f]} \sin[3d] + \\
& 3 (-1)^{3/4} c^2 e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3f+ic\text{Log}[f]} \sin[3d] + \\
& (-1)^{1/4} c^3 e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3f+ic\text{Log}[f]} \sin[3d] \Big) \Big) / \\
& (16 (if - c \text{Log}[f]) (f - ic \text{Log}[f]) (3f - ic \text{Log}[f]) (3f + ic \text{Log}[f]))
\end{aligned}$$

**Problem 124: Result more than twice size of optimal antiderivative.**

$$\int f^{a+cx^2} \cos[d+ex+fx^2]^3 dx$$

Optimal (type 4, 369 leaves, 14 steps):

$$\frac{3 e^{-i d - \frac{e^2}{4 i f - 4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{i e + 2 x (i f - c \operatorname{Log}[f])}{2 \sqrt{i f - c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f - c \operatorname{Log}[f]}} + \frac{e^{-3 i d - \frac{9 e^2}{4 (3 i f - c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{3 i e + 2 x (3 i f - c \operatorname{Log}[f])}{2 \sqrt{3 i f - c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f - c \operatorname{Log}[f]}} +$$

$$\frac{3 e^{i d + \frac{e^2}{4 i f + 4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{i e + 2 x (i f + c \operatorname{Log}[f])}{2 \sqrt{i f + c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f + c \operatorname{Log}[f]}} + \frac{e^{3 i d + \frac{9 e^2}{4 (3 i f + c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{3 i e + 2 x (3 i f + c \operatorname{Log}[f])}{2 \sqrt{3 i f + c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f + c \operatorname{Log}[f]}}$$

Result (type 4, 2997 leaves):

$$\left( f^a \sqrt{\pi} \left( -27 (-1)^{3/4} e^{-\frac{i e^2}{4 (f - i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e + 2 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{f - i c \operatorname{Log}[f]}}\right] \sqrt{f - i c \operatorname{Log}[f]} + \right.$$

$$27 (-1)^{1/4} c e^{-\frac{i e^2}{4 (f - i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e + 2 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f - i c \operatorname{Log}[f]} -$$

$$3 (-1)^{3/4} c^2 e^{-\frac{i e^2}{4 (f - i c \operatorname{Log}[f])}} f \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e + 2 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f - i c \operatorname{Log}[f]} +$$

$$3 (-1)^{1/4} c^3 e^{-\frac{i e^2}{4 (f - i c \operatorname{Log}[f])}} \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e + 2 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f - i c \operatorname{Log}[f]} -$$

$$3 (-1)^{3/4} e^{-\frac{9 i e^2}{4 (3 f - i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e + 6 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f - i c \operatorname{Log}[f]}}\right] \sqrt{3 f - i c \operatorname{Log}[f]} +$$

$$(-1)^{1/4} c e^{-\frac{9 i e^2}{4 (3 f - i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e + 6 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f - i c \operatorname{Log}[f]} -$$

$$3 (-1)^{3/4} c^2 e^{-\frac{9 i e^2}{4 (3 f - i c \operatorname{Log}[f])}} f \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e + 6 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f - i c \operatorname{Log}[f]} +$$

$$(-1)^{1/4} c^3 e^{-\frac{9 i e^2}{4 (3 f - i c \operatorname{Log}[f])}} \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e + 6 f x - 2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f - i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f - i c \operatorname{Log}[f]} -$$

$$27 (-1)^{1/4} e^{\frac{i e^2}{4 (f + i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e + 2 f x + 2 i c x \operatorname{Log}[f])}{2 \sqrt{f + i c \operatorname{Log}[f]}}\right] \sqrt{f + i c \operatorname{Log}[f]} +$$

$$27 (-1)^{3/4} c e^{\frac{i e^2}{4 (f + i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e + 2 f x + 2 i c x \operatorname{Log}[f])}{2 \sqrt{f + i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f + i c \operatorname{Log}[f]} -$$

$$3 (-1)^{1/4} c^2 e^{\frac{i e^2}{4 (f + i c \operatorname{Log}[f])}} f \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e + 2 f x + 2 i c x \operatorname{Log}[f])}{2 \sqrt{f + i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f + i c \operatorname{Log}[f]} +$$

$$\begin{aligned}
& 3 (-1)^{3/4} c^3 e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f+i c \operatorname{Log}[f]} - \\
& 3 (-1)^{1/4} e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \sqrt{3 f+i c \operatorname{Log}[f]} + \\
& (-1)^{3/4} c e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f+i c \operatorname{Log}[f]} - \\
& 3 (-1)^{1/4} c^2 e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f+i c \operatorname{Log}[f]} + \\
& (-1)^{3/4} c^3 e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} \operatorname{Cos}[3 d] \operatorname{Erfi}\left[\frac{(-1)^{3/4} (3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f+i c \operatorname{Log}[f]} + \\
& 27 (-1)^{1/4} e^{-\frac{i e^2}{4(f-i c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e+2 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \sqrt{f-i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 27 (-1)^{3/4} c e^{-\frac{i e^2}{4(f-i c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e+2 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f-i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i e^2}{4(f-i c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e+2 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f-i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{3/4} c^3 e^{-\frac{i e^2}{4(f-i c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{1/4} (e+2 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f-i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 27 (-1)^{3/4} e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \sqrt{f+i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 27 (-1)^{1/4} c e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f+i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{3/4} c^2 e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f+i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{1/4} c^3 e^{\frac{i e^2}{4(f+i c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4} (e+2 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f+i c \operatorname{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{1/4} e^{-\frac{9 i e^2}{4(3 f-i c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e+6 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f-i c \operatorname{Log}[f]}}\right] \sqrt{3 f-i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& (-1)^{3/4} c e^{-\frac{9 i e^2}{4(3 f-i c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{1/4} (3 e+6 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f-i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] +
\end{aligned}$$



$$\begin{aligned}
& 3 (-1)^{1/4} c^2 e^{-\frac{9 i e^2}{4(3 f-i c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3 e+6 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f-i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& (-1)^{3/4} c^3 e^{-\frac{9 i e^2}{4(3 f-i c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3 e+6 f x-2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f-i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& 3 (-1)^{3/4} e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \sqrt{3 f+i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& (-1)^{1/4} c e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3 f+i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& 3 (-1)^{3/4} c^2 e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3 f+i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] + \\
& (-1)^{1/4} c^3 e^{\frac{9 i e^2}{4(3 f+i c \operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3 e+6 f x+2 i c x \operatorname{Log}[f])}{2 \sqrt{3 f+i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3 f+i c \operatorname{Log}[f]} \operatorname{Sin}[3 d] \Big) \Big) / \\
& (16 (f-i c \operatorname{Log}[f]) (3 f-i c \operatorname{Log}[f]) (f+i c \operatorname{Log}[f]) (3 f+i c \operatorname{Log}[f]))
\end{aligned}$$

**Problem 130: Result more than twice size of optimal antiderivative.**

$$\int f^{a+b x+c x^2} \operatorname{Cos}[d+f x^2]^3 dx$$

Optimal (type 4, 378 leaves, 14 steps):

$$\begin{aligned}
& \frac{3 e^{-i d+\frac{b^2 \operatorname{Log}[f]^2}{4 i f-4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{b \operatorname{Log}[f]-2 x(i f-c \operatorname{Log}[f])}{2 \sqrt{i f-c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f-c \operatorname{Log}[f]}} - \frac{e^{-3 i d+\frac{b^2 \operatorname{Log}[f]^2}{12 i f-4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{b \operatorname{Log}[f]-2 x(3 i f-c \operatorname{Log}[f])}{2 \sqrt{3 i f-c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f-c \operatorname{Log}[f]}} + \\
& \frac{3 e^{i d-\frac{b^2 \operatorname{Log}[f]^2}{4 i f+4 c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{b \operatorname{Log}[f]+2 x(i f+c \operatorname{Log}[f])}{2 \sqrt{i f+c \operatorname{Log}[f]}}\right]}{16 \sqrt{i f+c \operatorname{Log}[f]}} + \frac{e^{3 i d-\frac{b^2 \operatorname{Log}[f]^2}{4(3 i f+c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{b \operatorname{Log}[f]+2 x(3 i f+c \operatorname{Log}[f])}{2 \sqrt{3 i f+c \operatorname{Log}[f]}}\right]}{16 \sqrt{3 i f+c \operatorname{Log}[f]}}
\end{aligned}$$

Result (type 4, 3285 leaves):

$$\begin{aligned}
& \left( f^a \sqrt{\pi} \left( -27 (-1)^{3/4} e^{\frac{i b^2 \operatorname{Log}[f]^2}{4(f-i c \operatorname{Log}[f])}} f^3 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(2 f x-i b \operatorname{Log}[f]-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \sqrt{f-i c \operatorname{Log}[f]} + \right. \right. \\
& \left. \left. 27 (-1)^{1/4} c e^{\frac{i b^2 \operatorname{Log}[f]^2}{4(f-i c \operatorname{Log}[f])}} f^2 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(2 f x-i b \operatorname{Log}[f]-2 i c x \operatorname{Log}[f])}{2 \sqrt{f-i c \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f-i c \operatorname{Log}[f]} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 3 (-1)^{3/4} c^2 e^{\frac{i b^2 \text{Log}[f]^2}{4 (f-i c \text{Log}[f])}} f \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f - i c \text{Log}[f]} + \\
& 3 (-1)^{1/4} c^3 e^{\frac{i b^2 \text{Log}[f]^2}{4 (f-i c \text{Log}[f])}} \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f - i c \text{Log}[f]} - \\
& 3 (-1)^{3/4} e^{\frac{i b^2 \text{Log}[f]^2}{4 (3 f-i c \text{Log}[f])}} f^3 \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \sqrt{3 f - i c \text{Log}[f]} + \\
& (-1)^{1/4} c e^{\frac{i b^2 \text{Log}[f]^2}{4 (3 f-i c \text{Log}[f])}} f^2 \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{3 f - i c \text{Log}[f]} - \\
& 3 (-1)^{3/4} c^2 e^{\frac{i b^2 \text{Log}[f]^2}{4 (3 f-i c \text{Log}[f])}} f \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3 f - i c \text{Log}[f]} + \\
& (-1)^{1/4} c^3 e^{\frac{i b^2 \text{Log}[f]^2}{4 (3 f-i c \text{Log}[f])}} \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3 f - i c \text{Log}[f]} - \\
& 27 (-1)^{1/4} e^{-\frac{i b^2 \text{Log}[f]^2}{4 (f+i c \text{Log}[f])}} f^3 \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \sqrt{f + i c \text{Log}[f]} + \\
& 27 (-1)^{3/4} c e^{-\frac{i b^2 \text{Log}[f]^2}{4 (f+i c \text{Log}[f])}} f^2 \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{f + i c \text{Log}[f]} - \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i b^2 \text{Log}[f]^2}{4 (f+i c \text{Log}[f])}} f \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f + i c \text{Log}[f]} + \\
& 3 (-1)^{3/4} c^3 e^{-\frac{i b^2 \text{Log}[f]^2}{4 (f+i c \text{Log}[f])}} \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f + i c \text{Log}[f]} - \\
& 3 (-1)^{1/4} e^{-\frac{i b^2 \text{Log}[f]^2}{4 (3 f+i c \text{Log}[f])}} f^3 \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \sqrt{3 f + i c \text{Log}[f]} + \\
& (-1)^{3/4} c e^{-\frac{i b^2 \text{Log}[f]^2}{4 (3 f+i c \text{Log}[f])}} f^2 \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{3 f + i c \text{Log}[f]} - \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i b^2 \text{Log}[f]^2}{4 (3 f+i c \text{Log}[f])}} f \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3 f + i c \text{Log}[f]} + \\
& (-1)^{3/4} c^3 e^{-\frac{i b^2 \text{Log}[f]^2}{4 (3 f+i c \text{Log}[f])}} \text{Cos}[3 d] \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3 f + i c \text{Log}[f]} + \\
& 27 (-1)^{1/4} e^{\frac{i b^2 \text{Log}[f]^2}{4 (f-i c \text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \sqrt{f - i c \text{Log}[f]} \text{Sin}[d] +
\end{aligned}$$

$$\begin{aligned}
& 27 (-1)^{3/4} c e^{\frac{i b^2 \text{Log}[f]^2}{4(f-i c \text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{f - i c \text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{1/4} c^2 e^{\frac{i b^2 \text{Log}[f]^2}{4(f-i c \text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f - i c \text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{3/4} c^3 e^{\frac{i b^2 \text{Log}[f]^2}{4(f-i c \text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{1/4} (2 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{f - i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f - i c \text{Log}[f]} \text{Sin}[d] + \\
& 27 (-1)^{3/4} e^{-\frac{i b^2 \text{Log}[f]^2}{4(f+i c \text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \sqrt{f + i c \text{Log}[f]} \text{Sin}[d] + \\
& 27 (-1)^{1/4} c e^{-\frac{i b^2 \text{Log}[f]^2}{4(f+i c \text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{f + i c \text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{3/4} c^2 e^{-\frac{i b^2 \text{Log}[f]^2}{4(f+i c \text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f + i c \text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{1/4} c^3 e^{-\frac{i b^2 \text{Log}[f]^2}{4(f+i c \text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{3/4} (2 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{f + i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f + i c \text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{1/4} e^{\frac{i b^2 \text{Log}[f]^2}{4(3f-i c \text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \sqrt{3 f - i c \text{Log}[f]} \text{Sin}[3 d] + \\
& (-1)^{3/4} c e^{\frac{i b^2 \text{Log}[f]^2}{4(3f-i c \text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{3 f - i c \text{Log}[f]} \text{Sin}[3 d] + \\
& 3 (-1)^{1/4} c^2 e^{\frac{i b^2 \text{Log}[f]^2}{4(3f-i c \text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3 f - i c \text{Log}[f]} \text{Sin}[3 d] + \\
& (-1)^{3/4} c^3 e^{\frac{i b^2 \text{Log}[f]^2}{4(3f-i c \text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{1/4} (6 f x - i b \text{Log}[f] - 2 i c x \text{Log}[f])}{2 \sqrt{3 f - i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3 f - i c \text{Log}[f]} \text{Sin}[3 d] + \\
& 3 (-1)^{3/4} e^{-\frac{i b^2 \text{Log}[f]^2}{4(3f+i c \text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \sqrt{3 f + i c \text{Log}[f]} \text{Sin}[3 d] + \\
& (-1)^{1/4} c e^{-\frac{i b^2 \text{Log}[f]^2}{4(3f+i c \text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f] \sqrt{3 f + i c \text{Log}[f]} \text{Sin}[3 d] + \\
& 3 (-1)^{3/4} c^2 e^{-\frac{i b^2 \text{Log}[f]^2}{4(3f+i c \text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3 f + i c \text{Log}[f]} \text{Sin}[3 d] + \\
& (-1)^{1/4} c^3 e^{-\frac{i b^2 \text{Log}[f]^2}{4(3f+i c \text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{3/4} (6 f x + i b \text{Log}[f] + 2 i c x \text{Log}[f])}{2 \sqrt{3 f + i c \text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3 f + i c \text{Log}[f]} \text{Sin}[3 d] \Big) \Big) /
\end{aligned}$$

$$(16 (f - i c \text{Log}[f]) (3 f - i c \text{Log}[f]) (f + i c \text{Log}[f]) (3 f + i c \text{Log}[f]))$$

### Problem 132: Result more than twice size of optimal antiderivative.

$$\int f^{a+bx+cx^2} \cos[d+ex+fx^2]^2 dx$$

Optimal (type 4, 268 leaves, 10 steps):

$$\frac{f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{Erfi}\left[\frac{(b+2cx)\sqrt{\operatorname{Log}[f]}}{2\sqrt{c}}\right]}{4\sqrt{c}\sqrt{\operatorname{Log}[f]}} + \frac{e^{-2id-\frac{(2e+ib\operatorname{Log}[f])^2}{8if-4c\operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{2ie-b\operatorname{Log}[f]+2x(2if-c\operatorname{Log}[f])}{2\sqrt{2if-c\operatorname{Log}[f]}}\right]}{8\sqrt{2if-c\operatorname{Log}[f]}} + \frac{e^{2id+\frac{(2e-ib\operatorname{Log}[f])^2}{8if+4c\operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{2ie+b\operatorname{Log}[f]+2x(2if+c\operatorname{Log}[f])}{2\sqrt{2if+c\operatorname{Log}[f]}}\right]}{8\sqrt{2if+c\operatorname{Log}[f]}}$$

Result (type 4, 1118 leaves):

$$\frac{1}{8c\operatorname{Log}[f](2f-ic\operatorname{Log}[f])(2f+ic\operatorname{Log}[f])} f^a \sqrt{\pi} \left( 8\sqrt{c} f^{2-\frac{b^2}{4c}} \operatorname{Erfi}\left[\frac{(b+2cx)\sqrt{\operatorname{Log}[f]}}{2\sqrt{c}}\right] \sqrt{\operatorname{Log}[f]} + 2c^{5/2} f^{-\frac{b^2}{4c}} \operatorname{Erfi}\left[\frac{(b+2cx)\sqrt{\operatorname{Log}[f]}}{2\sqrt{c}}\right] \operatorname{Log}[f]^{5/2} - \right. \\ 2(-1)^{3/4} c e^{\frac{i(-4e^2+4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f-ic\operatorname{Log}[f])}} f \operatorname{Cos}[2d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(2e+4fx-ib\operatorname{Log}[f]-2icx\operatorname{Log}[f])}{2\sqrt{2f-ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{2f-ic\operatorname{Log}[f]} + \\ (-1)^{1/4} c^2 e^{\frac{i(-4e^2+4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f-ic\operatorname{Log}[f])}} \operatorname{Cos}[2d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(2e+4fx-ib\operatorname{Log}[f]-2icx\operatorname{Log}[f])}{2\sqrt{2f-ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{2f-ic\operatorname{Log}[f]} - \\ 2(-1)^{1/4} c e^{\frac{-i(-4e^2-4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f+ic\operatorname{Log}[f])}} f \operatorname{Cos}[2d] \operatorname{Erfi}\left[\frac{(-1)^{3/4}(2e+4fx+ib\operatorname{Log}[f]+2icx\operatorname{Log}[f])}{2\sqrt{2f+ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{2f+ic\operatorname{Log}[f]} + \\ (-1)^{3/4} c^2 e^{\frac{-i(-4e^2-4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f+ic\operatorname{Log}[f])}} \operatorname{Cos}[2d] \operatorname{Erfi}\left[\frac{(-1)^{3/4}(2e+4fx+ib\operatorname{Log}[f]+2icx\operatorname{Log}[f])}{2\sqrt{2f+ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{2f+ic\operatorname{Log}[f]} + \\ 2(-1)^{1/4} c e^{\frac{i(-4e^2+4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f-ic\operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{1/4}(2e+4fx-ib\operatorname{Log}[f]-2icx\operatorname{Log}[f])}{2\sqrt{2f-ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{2f-ic\operatorname{Log}[f]} \operatorname{Sin}[2d] + \\ (-1)^{3/4} c^2 e^{\frac{i(-4e^2+4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f-ic\operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{1/4}(2e+4fx-ib\operatorname{Log}[f]-2icx\operatorname{Log}[f])}{2\sqrt{2f-ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{2f-ic\operatorname{Log}[f]} \operatorname{Sin}[2d] + \\ 2(-1)^{3/4} c e^{\frac{-i(-4e^2-4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f+ic\operatorname{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4}(2e+4fx+ib\operatorname{Log}[f]+2icx\operatorname{Log}[f])}{2\sqrt{2f+ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{2f+ic\operatorname{Log}[f]} \operatorname{Sin}[2d] + \\ \left. (-1)^{1/4} c^2 e^{\frac{-i(-4e^2-4ib\operatorname{Log}[f]+b^2\operatorname{Log}[f]^2)}{4(2f+ic\operatorname{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4}(2e+4fx+ib\operatorname{Log}[f]+2icx\operatorname{Log}[f])}{2\sqrt{2f+ic\operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{2f+ic\operatorname{Log}[f]} \operatorname{Sin}[2d] \right)$$

### Problem 133: Result more than twice size of optimal antiderivative.

$$\int f^{a+bx+cx^2} \operatorname{Cos}[d+ex+fx^2]^3 dx$$

Optimal (type 4, 422 leaves, 14 steps):

$$\frac{3 e^{-i d - \frac{(e+ib \operatorname{Log}[f])^2}{4if-4c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{ie-b \operatorname{Log}[f]+2x(if-c \operatorname{Log}[f])}{2\sqrt{if-c \operatorname{Log}[f]}}\right]}{16 \sqrt{if-c \operatorname{Log}[f]}}} + \frac{e^{-3id - \frac{(3e+ib \operatorname{Log}[f])^2}{4(3if-c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erf}\left[\frac{3ie-b \operatorname{Log}[f]+2x(3if-c \operatorname{Log}[f])}{2\sqrt{3if-c \operatorname{Log}[f]}}\right]}{16 \sqrt{3if-c \operatorname{Log}[f]}}} +$$

$$\frac{3 e^{id + \frac{(e-ib \operatorname{Log}[f])^2}{4if+4c \operatorname{Log}[f]}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{ie+b \operatorname{Log}[f]+2x(if+c \operatorname{Log}[f])}{2\sqrt{if+c \operatorname{Log}[f]}}\right]}{16 \sqrt{if+c \operatorname{Log}[f]}}} + \frac{e^{3id - \frac{(3e-ib \operatorname{Log}[f])^2}{4(3if+c \operatorname{Log}[f])}} f^a \sqrt{\pi} \operatorname{Erfi}\left[\frac{3ie+b \operatorname{Log}[f]+2x(3if+c \operatorname{Log}[f])}{2\sqrt{3if+c \operatorname{Log}[f]}}\right]}{16 \sqrt{3if+c \operatorname{Log}[f]}}}$$

Result (type 4, 3829 leaves):

$$\left( f^a \sqrt{\pi} \left( -27 (-1)^{3/4} e^{\frac{i(-e^2+2ibe \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(f-ic \operatorname{Log}[f])}} f^3 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{f-ic \operatorname{Log}[f]}}\right] \sqrt{f-ic \operatorname{Log}[f]} + \right.$$

$$27 (-1)^{1/4} c e^{\frac{i(-e^2+2ibe \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(f-ic \operatorname{Log}[f])}} f^2 \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{f-ic \operatorname{Log}[f]} -$$

$$3 (-1)^{3/4} c^2 e^{\frac{i(-e^2+2ibe \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(f-ic \operatorname{Log}[f])}} f \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{f-ic \operatorname{Log}[f]} +$$

$$3 (-1)^{1/4} c^3 e^{\frac{i(-e^2+2ibe \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(f-ic \operatorname{Log}[f])}} \operatorname{Cos}[d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{f-ic \operatorname{Log}[f]} -$$

$$3 (-1)^{3/4} e^{\frac{i(-9e^2+6ibe \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(3f-ic \operatorname{Log}[f])}} f^3 \operatorname{Cos}[3d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{3f-ic \operatorname{Log}[f]}}\right] \sqrt{3f-ic \operatorname{Log}[f]} +$$

$$(-1)^{1/4} c e^{\frac{i(-9e^2+6ibe \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(3f-ic \operatorname{Log}[f])}} f^2 \operatorname{Cos}[3d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{3f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f] \sqrt{3f-ic \operatorname{Log}[f]} -$$

$$3 (-1)^{3/4} c^2 e^{\frac{i(-9e^2+6ibe \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(3f-ic \operatorname{Log}[f])}} f \operatorname{Cos}[3d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{3f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^2 \sqrt{3f-ic \operatorname{Log}[f]} +$$

$$(-1)^{1/4} c^3 e^{\frac{i(-9e^2+6ibe \operatorname{Log}[f]+b^2 \operatorname{Log}[f]^2)}{4(3f-ic \operatorname{Log}[f])}} \operatorname{Cos}[3d] \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib \operatorname{Log}[f]-2icx \operatorname{Log}[f])}{2\sqrt{3f-ic \operatorname{Log}[f]}}\right] \operatorname{Log}[f]^3 \sqrt{3f-ic \operatorname{Log}[f]} -$$

$$\begin{aligned}
& 27 (-1)^{1/4} e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f^3 \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \sqrt{f+ic\text{Log}[f]} + \\
& 27 (-1)^{3/4} c e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f^2 \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{f+ic\text{Log}[f]} - \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f+ic\text{Log}[f]} + \\
& 3 (-1)^{3/4} c^3 e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} \text{Cos}[d] \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f+ic\text{Log}[f]} - \\
& 3 (-1)^{1/4} e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f^3 \text{Cos}[3d] \text{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \sqrt{3f+ic\text{Log}[f]} + \\
& (-1)^{3/4} c e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f^2 \text{Cos}[3d] \text{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{3f+ic\text{Log}[f]} - \\
& 3 (-1)^{1/4} c^2 e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f \text{Cos}[3d] \text{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3f+ic\text{Log}[f]} + \\
& (-1)^{3/4} c^3 e^{-\frac{i(-9e^2-6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} \text{Cos}[3d] \text{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3f+ic\text{Log}[f]} + \\
& 27 (-1)^{1/4} e^{\frac{i(-e^2+2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f-ic\text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{f-ic\text{Log}[f]}}\right] \sqrt{f-ic\text{Log}[f]} \text{Sin}[d] + \\
& 27 (-1)^{3/4} c e^{\frac{i(-e^2+2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f-ic\text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{f-ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{f-ic\text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{1/4} c^2 e^{\frac{i(-e^2+2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f-ic\text{Log}[f])}} f \text{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{f-ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f-ic\text{Log}[f]} \text{Sin}[d] + \\
& 3 (-1)^{3/4} c^3 e^{\frac{i(-e^2+2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f-ic\text{Log}[f])}} \text{Erfi}\left[\frac{(-1)^{1/4}(e+2fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{f-ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f-ic\text{Log}[f]} \text{Sin}[d] + \\
& 27 (-1)^{3/4} e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f^3 \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \sqrt{f+ic\text{Log}[f]} \text{Sin}[d] + \\
& 27 (-1)^{1/4} c e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f^2 \text{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{f+ic\text{Log}[f]} \text{Sin}[d] +
\end{aligned}$$

$$\begin{aligned}
& 3 (-1)^{3/4} c^2 e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{f+ic\text{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{1/4} c^3 e^{-\frac{i(-e^2-2ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(f+ic\text{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4}(e+2fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{f+ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{f+ic\text{Log}[f]} \operatorname{Sin}[d] + \\
& 3 (-1)^{1/4} e^{\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f-ic\text{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{3f-ic\text{Log}[f]}}\right] \sqrt{3f-ic\text{Log}[f]} \operatorname{Sin}[3d] + \\
& (-1)^{3/4} c e^{\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f-ic\text{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{3f-ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{3f-ic\text{Log}[f]} \operatorname{Sin}[3d] + \\
& 3 (-1)^{1/4} c^2 e^{\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f-ic\text{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{3f-ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3f-ic\text{Log}[f]} \operatorname{Sin}[3d] + \\
& (-1)^{3/4} c^3 e^{\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f-ic\text{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{1/4}(3e+6fx-ib\text{Log}[f]-2icx\text{Log}[f])}{2\sqrt{3f-ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3f-ic\text{Log}[f]} \operatorname{Sin}[3d] + \\
& 3 (-1)^{3/4} e^{-\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f^3 \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \sqrt{3f+ic\text{Log}[f]} \operatorname{Sin}[3d] + \\
& (-1)^{1/4} c e^{-\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f^2 \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f] \sqrt{3f+ic\text{Log}[f]} \operatorname{Sin}[3d] + \\
& 3 (-1)^{3/4} c^2 e^{-\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} f \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f]^2 \sqrt{3f+ic\text{Log}[f]} \operatorname{Sin}[3d] + \\
& (-1)^{1/4} c^3 e^{-\frac{i(-9e^2+6ib\text{Log}[f]+b^2\text{Log}[f]^2)}{4(3f+ic\text{Log}[f])}} \operatorname{Erfi}\left[\frac{(-1)^{3/4}(3e+6fx+ib\text{Log}[f]+2icx\text{Log}[f])}{2\sqrt{3f+ic\text{Log}[f]}}\right] \text{Log}[f]^3 \sqrt{3f+ic\text{Log}[f]} \operatorname{Sin}[3d] \Big) \Big) / \\
& (16 (f-ic\text{Log}[f]) (3f-ic\text{Log}[f]) (f+ic\text{Log}[f]) (3f+ic\text{Log}[f]))
\end{aligned}$$

**Problem 141: Result more than twice size of optimal antiderivative.**

$$\int \frac{F^{c(a+bx)}}{f+f\cos[d+ex]} dx$$

Optimal (type 5, 79 leaves, 2 steps):

$$\frac{2 e^{i(d+ex)} F^{c(a+bx)} \operatorname{Hypergeometric2F1}\left[2, 1 - \frac{ibc\text{Log}[F]}{e}, 2 - \frac{ibc\text{Log}[F]}{e}, -e^{i(d+ex)}\right]}{f(i e + bc \text{Log}[F])}$$

Result (type 5, 248 leaves):

$$\frac{1}{e f (1 + \cos [d + e x]) (e - i b c \log [F])}$$

$$2 F^{-\frac{b c d}{e}} \cos \left[ \frac{1}{2} (d + e x) \right] \left( b c e^{\frac{(d+e x)(i e - b c \log [F])}{e}} F^{a c} \cos \left[ \frac{1}{2} (d + e x) \right] \text{Hypergeometric2F1} \left[ 1, 1 - \frac{i b c \log [F]}{e}, 2 - \frac{i b c \log [F]}{e}, -e^{i (d+e x)} \right] \log [F] - \right.$$

$$i F^{c (a+b \frac{d+e x}{e})} \cos \left[ \frac{1}{2} (d + e x) \right] \text{Hypergeometric2F1} \left[ 1, -\frac{i b c \log [F]}{e}, 1 - \frac{i b c \log [F]}{e}, -e^{i (d+e x)} \right] (e - i b c \log [F]) +$$

$$\left. F^{c (a+b \frac{d+e x}{e})} (e - i b c \log [F]) \sin \left[ \frac{1}{2} (d + e x) \right] \right)$$

**Problem 142: Result more than twice size of optimal antiderivative.**

$$\int \frac{F^{c (a+b x)}}{(f + f \cos [d + e x])^2} dx$$

Optimal (type 5, 169 leaves, 3 steps):

$$\frac{2 e^{i (d+e x)} F^{c (a+b x)} \text{Hypergeometric2F1} \left[ 2, 1 - \frac{i b c \log [F]}{e}, 2 - \frac{i b c \log [F]}{e}, -e^{i (d+e x)} \right] (i e - b c \log [F])}{3 e^2 f^2} -$$

$$\frac{b c F^{c (a+b x)} \log [F] \sec \left[ \frac{d}{2} + \frac{e x}{2} \right]^2}{6 e^2 f^2} + \frac{F^{c (a+b x)} \sec \left[ \frac{d}{2} + \frac{e x}{2} \right]^2 \tan \left[ \frac{d}{2} + \frac{e x}{2} \right]}{6 e f^2}$$

Result (type 5, 749 leaves):



$$\begin{aligned}
& - \frac{2 b c F^{\frac{c(-bd+ae)}{e} + \frac{2bc}{e} \left(\frac{d+ex}{2}\right)} \operatorname{Cos}\left[\frac{d}{2} + \frac{ex}{2}\right]^2 \operatorname{Log}[F]}{3 e^2 (f + f \operatorname{Cos}[d + ex])^2} + \frac{1}{3 e^4 (f + f \operatorname{Cos}[d + ex])^2} 8 i b c F^{\frac{c(-bd+ae)}{e}} \operatorname{Cos}\left[\frac{d}{2} + \frac{ex}{2}\right]^4 \operatorname{Log}[F] \\
& \left( -i e + b c \operatorname{Log}[F] \right) \left( i e + b c \operatorname{Log}[F] \right) \left[ - \frac{e F^{a c - \frac{bcd}{e} - \frac{c(-bd+ae)}{e} + \frac{2bc}{e} \left(\frac{d+ex}{2}\right)} \operatorname{Hypergeometric2F1}\left[1, -\frac{i b c \operatorname{Log}[F]}{e}, 1 - \frac{i b c \operatorname{Log}[F]}{e}, -e^{2 i \left(\frac{d+ex}{2}\right)}\right]}{2 b c \operatorname{Log}[F]} \right] - \\
& \frac{1}{2 (e - i b c \operatorname{Log}[F])} i e e^{\left(\frac{d+ex}{2}\right)} \left( 2 i + \frac{\left( a c - \frac{bcd}{e} - \frac{c(-bd+ae)}{e} + \frac{2bc}{e} \left(\frac{d+ex}{2}\right) \right) \operatorname{Log}[F]}{\frac{d+ex}{2}} \right) \left( e^{2 i \left(\frac{d+ex}{2}\right)} \right) \\
& \left[ 1 - \frac{i \left( a c - \frac{bcd}{e} - \frac{c(-bd+ae)}{e} + \frac{2bc}{e} \left(\frac{d+ex}{2}\right) \right) \operatorname{Log}[F]}{2 \left(\frac{d+ex}{2}\right)} \right] + \frac{1}{2} i \left( 2 i + \frac{\left( a c - \frac{bcd}{e} - \frac{c(-bd+ae)}{e} + \frac{2bc}{e} \left(\frac{d+ex}{2}\right) \right) \operatorname{Log}[F]}{\frac{d+ex}{2}} \right) \\
& \left. \operatorname{Hypergeometric2F1}\left[1, \frac{e - i b c \operatorname{Log}[F]}{e}, 1 + \frac{e - i b c \operatorname{Log}[F]}{e}, -e^{2 i \left(\frac{d+ex}{2}\right)}\right] \right] + \\
& \frac{2 F^{\frac{c(-bd+ae)}{e} + \frac{2bc}{e} \left(\frac{d+ex}{2}\right)} \operatorname{Cos}\left[\frac{d}{2} + \frac{ex}{2}\right] \operatorname{Sin}\left[\frac{d}{2} + \frac{ex}{2}\right]}{3 e (f + f \operatorname{Cos}[d + ex])^2} + \frac{4 F^{\frac{c(-bd+ae)}{e} + \frac{2bc}{e} \left(\frac{d+ex}{2}\right)} \operatorname{Cos}\left[\frac{d}{2} + \frac{ex}{2}\right]^3 (e^2 + b^2 c^2 \operatorname{Log}[F]^2) \operatorname{Sin}\left[\frac{d}{2} + \frac{ex}{2}\right]}{3 e^3 (f + f \operatorname{Cos}[d + ex])^2}
\end{aligned}$$

## Test results for the 950 problems in "4.7.7 Trig functions.m"

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sin}[a + b x]}{c + d x^2} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\text{CosIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} + bx\right] \text{Sin}\left[a - \frac{b\sqrt{-c}}{\sqrt{d}}\right]}{2\sqrt{-c}\sqrt{d}} + \frac{\text{CosIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right] \text{Sin}\left[a + \frac{b\sqrt{-c}}{\sqrt{d}}\right]}{2\sqrt{-c}\sqrt{d}} \\
& - \frac{\text{Cos}\left[a + \frac{b\sqrt{-c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right]}{2\sqrt{-c}\sqrt{d}} - \frac{\text{Cos}\left[a - \frac{b\sqrt{-c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} + bx\right]}{2\sqrt{-c}\sqrt{d}}
\end{aligned}$$

Result (type 4, 172 leaves):

$$\begin{aligned}
& \frac{1}{2\sqrt{c}\sqrt{d}} i \left( \text{CosIntegral}\left[b\left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] \text{Sin}\left[a - \frac{ib\sqrt{c}}{\sqrt{d}}\right] - \text{CosIntegral}\left[b\left(-\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] \text{Sin}\left[a + \frac{ib\sqrt{c}}{\sqrt{d}}\right] + \right. \\
& \left. \text{Cos}\left[a - \frac{ib\sqrt{c}}{\sqrt{d}}\right] \text{SinIntegral}\left[b\left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] + \text{Cos}\left[a + \frac{ib\sqrt{c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{ib\sqrt{c}}{\sqrt{d}} - bx\right] \right)
\end{aligned}$$

**Problem 34: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{b - \frac{a}{x^2}} \text{Sin}[x]}{\sqrt{a - bx^2}} dx$$

Optimal (type 4, 28 leaves, 3 steps):

$$\frac{\sqrt{b - \frac{a}{x^2}} x \text{SinIntegral}[x]}{\sqrt{a - bx^2}}$$

Result (type 4, 46 leaves):

$$\frac{i \sqrt{b - \frac{a}{x^2}} x \left( \text{ExpIntegralEi}[-ix] - \text{ExpIntegralEi}[ix] \right)}{2\sqrt{a - bx^2}}$$

**Problem 35: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{x(1 + \text{Sin}[\text{Log}[x]])} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$-\frac{\text{Cos}[\text{Log}[x]]}{1 + \text{Sin}[\text{Log}[x]]}$$

Result (type 3, 26 leaves):

$$\frac{2 \operatorname{Sin}\left[\frac{\operatorname{Log}[x]}{2}\right]}{\operatorname{Cos}\left[\frac{\operatorname{Log}[x]}{2}\right] + \operatorname{Sin}\left[\frac{\operatorname{Log}[x]}{2}\right]}$$

Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{Sin}\left[\frac{a + b x}{c + d x}\right] dx$$

Optimal (type 4, 100 leaves, 5 steps):

$$\frac{(b c - a d) \operatorname{Cos}\left[\frac{b}{d}\right] \operatorname{CosIntegral}\left[\frac{b c - a d}{d (c + d x)}\right]}{d^2} + \frac{(c + d x) \operatorname{Sin}\left[\frac{a + b x}{c + d x}\right]}{d} + \frac{(b c - a d) \operatorname{Sin}\left[\frac{b}{d}\right] \operatorname{SinIntegral}\left[\frac{b c - a d}{d (c + d x)}\right]}{d^2}$$

Result (type 4, 918 leaves):

$$\frac{(b c^2 - a c d) \left( \frac{i e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(-e^{-\frac{2ia}{c+dx} + e^{\frac{2ibc}{d(c+dx)}}}\right)}{4(b c - a d)} - \frac{i e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(e^{-\frac{2ia}{c+dx} + e^{\frac{2ibc}{d(c+dx)}}}\right)}{4(b c - a d)} \right)}{2 d} -$$

$$\frac{(-b c^2 + a c d) \left( \frac{i e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(-e^{-\frac{2ia}{c+dx} + e^{\frac{2ibc}{d(c+dx)}}}\right)}{4(b c - a d)} - \frac{i e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(e^{-\frac{2ia}{c+dx} + e^{\frac{2ibc}{d(c+dx)}}}\right)}{4(b c - a d)} \right)}{2 d} -$$

$$\frac{i (b c^2 - a c d) \left( \frac{e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(-e^{-\frac{2ia}{c+dx} + e^{\frac{2ibc}{d(c+dx)}}}\right)}{4(b c - a d)} - \frac{e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(e^{-\frac{2ia}{c+dx} + e^{\frac{2ibc}{d(c+dx)}}}\right)}{4(b c - a d)} \right)}{2 d} -$$

$$\frac{i (-b c^2 + a c d) \left( \frac{e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(-e^{-\frac{2ia}{c+dx} + e^{\frac{2ibc}{d(c+dx)}}}\right)}{4(b c - a d)} - \frac{e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(e^{-\frac{2ia}{c+dx} + e^{\frac{2ibc}{d(c+dx)}}}\right)}{4(b c - a d)} \right)}{2 d} + x \operatorname{Cos}\left[\frac{-b c + a d}{d (c + d x)}\right] \operatorname{Sin}\left[\frac{b}{d}\right] +$$

$$x \operatorname{Cos}\left[\frac{b}{d}\right] \operatorname{Sin}\left[\frac{-b c + a d}{d (c + d x)}\right] - \frac{(-b c + a d) \left( \operatorname{Cos}\left[\frac{b}{d}\right] \operatorname{CosIntegral}\left[\frac{-b c + a d}{d (c + d x)}\right] - \operatorname{Sin}\left[\frac{b}{d}\right] \operatorname{SinIntegral}\left[\frac{-b c + a d}{d (c + d x)}\right] \right)}{d^2}$$

**Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin\left[\frac{a+bx}{c+dx}\right]^2 dx$$

Optimal (type 4, 107 leaves, 6 steps):

$$\frac{(bc-a)d \operatorname{CosIntegral}\left[\frac{2(bc-ad)}{d(c+dx)}\right] \operatorname{Sin}\left[\frac{2b}{d}\right]}{d^2} + \frac{(c+dx) \operatorname{Sin}\left[\frac{a+bx}{c+dx}\right]^2}{d} - \frac{(bc-a)d \operatorname{Cos}\left[\frac{2b}{d}\right] \operatorname{SinIntegral}\left[\frac{2(bc-ad)}{d(c+dx)}\right]}{d^2}$$

Result (type 4, 401 leaves):

$$\frac{(-bc^2 + acd) \left( \frac{e^{-\frac{2i(2bc-ad-bdx)}{d(c+dx)}} \left(-1 + e^{\frac{4ib}{d}}\right) \left(-\frac{4ia}{e^{c+dx} + e^{d(c+dx)}} - \frac{4ibc}{e^{c+dx} + e^{d(c+dx)}}\right)}{8(bc-ad)} - \frac{e^{-\frac{2i(2bc-ad-bdx)}{d(c+dx)}} \left(1 + e^{\frac{4ib}{d}}\right) \left(\frac{4ia}{e^{c+dx} + e^{d(c+dx)}} - \frac{4ibc}{e^{c+dx} + e^{d(c+dx)}}\right)}{8(bc-ad)} \right)}{d}$$

$$\frac{1}{2} x \operatorname{Cos}\left[\frac{2b}{d}\right] \operatorname{Cos}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] + \frac{1}{2} x \operatorname{Sin}\left[\frac{2b}{d}\right] \operatorname{Sin}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] + \frac{1}{2d^2} \left( d^2 x + 2bc \operatorname{CosIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \operatorname{Sin}\left[\frac{2b}{d}\right] - 2ad \operatorname{CosIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \operatorname{Sin}\left[\frac{2b}{d}\right] + 2bc \operatorname{Cos}\left[\frac{2b}{d}\right] \operatorname{SinIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] - 2ad \operatorname{Cos}\left[\frac{2b}{d}\right] \operatorname{SinIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \right)$$

**Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin\left[\frac{a+bx}{c+dx}\right]^3 dx$$

Optimal (type 4, 194 leaves, 9 steps):

$$\frac{3(bc-a)d \operatorname{Cos}\left[\frac{b}{d}\right] \operatorname{CosIntegral}\left[\frac{bc-ad}{d(c+dx)}\right]}{4d^2} - \frac{3(bc-a)d \operatorname{Cos}\left[\frac{3b}{d}\right] \operatorname{CosIntegral}\left[\frac{3(bc-ad)}{d(c+dx)}\right]}{4d^2} + \frac{(c+dx) \operatorname{Sin}\left[\frac{a+bx}{c+dx}\right]^3}{d} + \frac{3(bc-a)d \operatorname{Sin}\left[\frac{b}{d}\right] \operatorname{SinIntegral}\left[\frac{bc-ad}{d(c+dx)}\right]}{4d^2} - \frac{3(bc-a)d \operatorname{Sin}\left[\frac{3b}{d}\right] \operatorname{SinIntegral}\left[\frac{3(bc-ad)}{d(c+dx)}\right]}{4d^2}$$

Result (type 4, 657 leaves):

$$\begin{aligned}
& \frac{3(-bc^2 + acd) \left( \frac{i e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(-e^{c+dx} + e^{\frac{2ia}{d(c+dx)}}\right) - i e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(e^{c+dx} + e^{\frac{2ia}{d(c+dx)}}\right)}{4(bc-a)d} \right)}{4d} + \\
& \frac{3(-bc^2 + acd) \left( \frac{i e^{-\frac{3i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{6ib}{d}}\right) \left(-e^{c+dx} + e^{\frac{6ia}{d(c+dx)}}\right) - i e^{-\frac{3i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{6ib}{d}}\right) \left(e^{c+dx} + e^{\frac{6ia}{d(c+dx)}}\right)}{12(bc-a)d} \right)}{4d} + \\
& \frac{3}{4} x \operatorname{Cos} \left[ \frac{-bc+ad}{d(c+dx)} \right] \operatorname{Sin} \left[ \frac{b}{d} \right] - \frac{1}{4} x \operatorname{Cos} \left[ \frac{3(-bc+ad)}{d(c+dx)} \right] \operatorname{Sin} \left[ \frac{3b}{d} \right] + \frac{3}{4} x \operatorname{Cos} \left[ \frac{b}{d} \right] \operatorname{Sin} \left[ \frac{-bc+ad}{d(c+dx)} \right] - \\
& \frac{1}{4} x \operatorname{Cos} \left[ \frac{3b}{d} \right] \operatorname{Sin} \left[ \frac{3(-bc+ad)}{d(c+dx)} \right] + \frac{1}{4d^2} 3(-bc+ad) \left( -\operatorname{Cos} \left[ \frac{b}{d} \right] \operatorname{CosIntegral} \left[ \frac{-bc+ad}{d(c+dx)} \right] + \right. \\
& \left. \operatorname{Cos} \left[ \frac{3b}{d} \right] \operatorname{CosIntegral} \left[ \frac{3(-bc+ad)}{d(c+dx)} \right] + \operatorname{Sin} \left[ \frac{b}{d} \right] \operatorname{SinIntegral} \left[ \frac{-bc+ad}{d(c+dx)} \right] - \operatorname{Sin} \left[ \frac{3b}{d} \right] \operatorname{SinIntegral} \left[ \frac{3(-bc+ad)}{d(c+dx)} \right] \right)
\end{aligned}$$

**Problem 46: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[a + bx]}{c + dx^2} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\begin{aligned}
& \frac{\operatorname{Cos} \left[ a + \frac{b\sqrt{-c}}{\sqrt{d}} \right] \operatorname{CosIntegral} \left[ \frac{b\sqrt{-c}}{\sqrt{d}} - bx \right]}{2\sqrt{-c}\sqrt{d}} - \frac{\operatorname{Cos} \left[ a - \frac{b\sqrt{-c}}{\sqrt{d}} \right] \operatorname{CosIntegral} \left[ \frac{b\sqrt{-c}}{\sqrt{d}} + bx \right]}{2\sqrt{-c}\sqrt{d}} + \\
& \frac{\operatorname{Sin} \left[ a + \frac{b\sqrt{-c}}{\sqrt{d}} \right] \operatorname{SinIntegral} \left[ \frac{b\sqrt{-c}}{\sqrt{d}} - bx \right]}{2\sqrt{-c}\sqrt{d}} + \frac{\operatorname{Sin} \left[ a - \frac{b\sqrt{-c}}{\sqrt{d}} \right] \operatorname{SinIntegral} \left[ \frac{b\sqrt{-c}}{\sqrt{d}} + bx \right]}{2\sqrt{-c}\sqrt{d}}
\end{aligned}$$

Result (type 4, 172 leaves):

$$\begin{aligned}
& -\frac{1}{2\sqrt{c}\sqrt{d}} i \left( \operatorname{Cos} \left[ a + \frac{ib\sqrt{c}}{\sqrt{d}} \right] \operatorname{CosIntegral} \left[ b \left( -\frac{i\sqrt{c}}{\sqrt{d}} + x \right) \right] - \operatorname{Cos} \left[ a - \frac{ib\sqrt{c}}{\sqrt{d}} \right] \operatorname{CosIntegral} \left[ b \left( \frac{i\sqrt{c}}{\sqrt{d}} + x \right) \right] \right) + \\
& \operatorname{Sin} \left[ a - \frac{ib\sqrt{c}}{\sqrt{d}} \right] \operatorname{SinIntegral} \left[ b \left( \frac{i\sqrt{c}}{\sqrt{d}} + x \right) \right] + \operatorname{Sin} \left[ a + \frac{ib\sqrt{c}}{\sqrt{d}} \right] \operatorname{SinIntegral} \left[ \frac{ib\sqrt{c}}{\sqrt{d}} - bx \right]
\end{aligned}$$

**Problem 51: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos \left[ \frac{a + b x}{c + d x} \right] dx$$

Optimal (type 4, 101 leaves, 5 steps):

$$\frac{(c + d x) \cos \left[ \frac{a + b x}{c + d x} \right]}{d} - \frac{(b c - a d) \operatorname{CosIntegral} \left[ \frac{b c - a d}{d (c + d x)} \right] \operatorname{Sin} \left[ \frac{b}{d} \right]}{d^2} + \frac{(b c - a d) \cos \left[ \frac{b}{d} \right] \operatorname{SinIntegral} \left[ \frac{b c - a d}{d (c + d x)} \right]}{d^2}$$

Result (type 4, 317 leaves):

$$\frac{(-b c^2 + a c d) \left( \frac{e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left( -1 + e^{\frac{2ib}{d}} \right) \left( -\frac{2ia}{e^{c+dx} + e^{d(c+dx)}} \right)}{4(b c - a d)} - \frac{e^{-\frac{i(2bc-ad+bdx)}{d(c+dx)}} \left( 1 + e^{\frac{2ib}{d}} \right) \left( \frac{2ia}{e^{c+dx} + e^{d(c+dx)}} \right)}{4(b c - a d)} \right)}{d} + x \cos \left[ \frac{b}{d} \right] \cos \left[ \frac{-b c + a d}{d (c + d x)} \right] -$$

$$x \sin \left[ \frac{b}{d} \right] \sin \left[ \frac{-b c + a d}{d (c + d x)} \right] + \frac{(-b c + a d) \left( \operatorname{CosIntegral} \left[ \frac{-b c + a d}{d (c + d x)} \right] \operatorname{Sin} \left[ \frac{b}{d} \right] + \cos \left[ \frac{b}{d} \right] \operatorname{SinIntegral} \left[ \frac{-b c + a d}{d (c + d x)} \right] \right)}{d^2}$$

**Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos \left[ \frac{a + b x}{c + d x} \right]^2 dx$$

Optimal (type 4, 107 leaves, 6 steps):

$$\frac{(c + d x) \cos \left[ \frac{a + b x}{c + d x} \right]^2}{d} - \frac{(b c - a d) \operatorname{CosIntegral} \left[ \frac{2(b c - a d)}{d (c + d x)} \right] \operatorname{Sin} \left[ \frac{2b}{d} \right]}{d^2} + \frac{(b c - a d) \cos \left[ \frac{2b}{d} \right] \operatorname{SinIntegral} \left[ \frac{2(b c - a d)}{d (c + d x)} \right]}{d^2}$$

Result (type 4, 400 leaves):

$$\frac{(-b c^2 + a c d) \left( \frac{e^{-\frac{2i(2bc-ad+bdx)}{d(c+dx)}} \left( -1 + e^{\frac{4ib}{d}} \right) \left( -\frac{4ia}{e^{c+dx} + e^{d(c+dx)}} \right)}{8(b c - a d)} - \frac{e^{-\frac{2i(2bc-ad+bdx)}{d(c+dx)}} \left( 1 + e^{\frac{4ib}{d}} \right) \left( \frac{4ia}{e^{c+dx} + e^{d(c+dx)}} \right)}{8(b c - a d)} \right)}{d} +$$

$$\frac{1}{2} x \cos \left[ \frac{2b}{d} \right] \cos \left[ \frac{2(-b c + a d)}{d (c + d x)} \right] - \frac{1}{2} x \sin \left[ \frac{2b}{d} \right] \sin \left[ \frac{2(-b c + a d)}{d (c + d x)} \right] + \frac{1}{2 d^2} \left( d^2 x - 2 b c \operatorname{CosIntegral} \left[ \frac{2(-b c + a d)}{d (c + d x)} \right] \operatorname{Sin} \left[ \frac{2b}{d} \right] + \right.$$

$$\left. 2 a d \operatorname{CosIntegral} \left[ \frac{2(-b c + a d)}{d (c + d x)} \right] \operatorname{Sin} \left[ \frac{2b}{d} \right] - 2 b c \cos \left[ \frac{2b}{d} \right] \operatorname{SinIntegral} \left[ \frac{2(-b c + a d)}{d (c + d x)} \right] + 2 a d \cos \left[ \frac{2b}{d} \right] \operatorname{SinIntegral} \left[ \frac{2(-b c + a d)}{d (c + d x)} \right] \right)$$

### Problem 63: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + b \operatorname{Sec}[c + d x]}}{1 + \operatorname{Cos}[c + d x]} dx$$

Optimal (type 4, 92 leaves, 2 steps):

$$\frac{\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}[c + d x]}{1 + \operatorname{Sec}[c + d x]}\right], \frac{a - b}{a + b}\right] \sqrt{\frac{1}{1 + \operatorname{Sec}[c + d x]}} \sqrt{a + b \operatorname{Sec}[c + d x]}}{d \sqrt{\frac{a + b \operatorname{Sec}[c + d x]}{(a + b)(1 + \operatorname{Sec}[c + d x])}}}$$

Result (type 4, 1979 leaves):

$$\begin{aligned} & \frac{\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \sqrt{a + b \operatorname{Sec}[c + d x]} \left(-2 \operatorname{Sin}[c + d x] + 2 \operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right)}{d(1 + \operatorname{Cos}[c + d x])} + \left(\operatorname{Cos}\left[\frac{c}{2} + \frac{d x}{2}\right]^2 \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]\right)^5 \\ & \left(\frac{b}{\sqrt{b + a \operatorname{Cos}[c + d x]} \sqrt{\operatorname{Sec}[c + d x]}} + \frac{a \sqrt{\operatorname{Sec}[c + d x]}}{\sqrt{b + a \operatorname{Cos}[c + d x]}} + \frac{b \sqrt{\operatorname{Sec}[c + d x]}}{\sqrt{b + a \operatorname{Cos}[c + d x]}} + \frac{a \operatorname{Cos}[2(c + d x)] \sqrt{\operatorname{Sec}[c + d x]}}{\sqrt{b + a \operatorname{Cos}[c + d x]}}\right) \\ & \sqrt{1 + \operatorname{Sec}[c + d x]} \sqrt{a + b \operatorname{Sec}[c + d x]} \left(2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \sqrt{\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] + \right. \\ & \left. \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{(a + b)(1 + \operatorname{Cos}[c + d x])}} \left(-\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right]\right)\right) \Bigg/ \left(4 d \sqrt{\frac{1}{1 + \operatorname{Cos}[c + d x]}} \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{(a + b)(1 + \operatorname{Cos}[c + d x])}} \sqrt{\operatorname{Sec}[c + d x]}\right) \\ & - \left(\left(\left(a \operatorname{Sec}\left[\frac{1}{2}(c + d x)\right]\right)^5 \sqrt{1 + \operatorname{Sec}[c + d x]} \operatorname{Sin}[c + d x] \left(2 \operatorname{Cos}\left[\frac{1}{2}(c + d x)\right] \sqrt{\frac{\operatorname{Cos}[c + d x]}{1 + \operatorname{Cos}[c + d x]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + d x)\right]\right], \frac{a - b}{a + b}\right] + \right. \right. \right. \\ & \left. \left. \sqrt{\frac{b + a \operatorname{Cos}[c + d x]}{(a + b)(1 + \operatorname{Cos}[c + d x])}} \left(-\operatorname{Sin}\left[\frac{1}{2}(c + d x)\right] + \operatorname{Sin}\left[\frac{3}{2}(c + d x)\right]\right)\right)\right) \Bigg/ \end{aligned}$$

$$\begin{aligned}
& \left( 8 \left( \frac{1}{1 + \cos[c + dx]} \right)^{3/2} \sqrt{b + a \cos[c + dx]} \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \right) - \left( 3 \sqrt{b + a \cos[c + dx]} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 \right. \\
& \left. \sqrt{1 + \operatorname{Sec}[c + dx]} \sin[c + dx] \left( 2 \cos\left[\frac{1}{2}(c + dx)\right] \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] + \right. \right. \\
& \left. \left. \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \left( -\sin\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{3}{2}(c + dx)\right] \right) \right) \right) / \left( 8 \sqrt{\frac{1}{1 + \cos[c + dx]}} \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \right) - \\
& \left( \sqrt{b + a \cos[c + dx]} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 \sqrt{1 + \operatorname{Sec}[c + dx]} \left( -\frac{a \sin[c + dx]}{(a + b)(1 + \cos[c + dx])} + \frac{(b + a \cos[c + dx]) \sin[c + dx]}{(a + b)(1 + \cos[c + dx])^2} \right) \right. \\
& \left. \left( 2 \cos\left[\frac{1}{2}(c + dx)\right] \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] + \right. \right. \\
& \left. \left. \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \left( -\sin\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{3}{2}(c + dx)\right] \right) \right) \right) / \left( 8 \left( \frac{1}{1 + \cos[c + dx]} \right)^{3/2} \left( \frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])} \right)^{3/2} \right) + \\
& \left( 5 \sqrt{b + a \cos[c + dx]} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 \sqrt{1 + \operatorname{Sec}[c + dx]} \left( 2 \cos\left[\frac{1}{2}(c + dx)\right] \sqrt{\frac{\cos[c + dx]}{1 + \cos[c + dx]}} \operatorname{EllipticE}\left[ \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c + dx)\right]\right], \frac{a - b}{a + b}\right] + \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \left( -\sin\left[\frac{1}{2}(c + dx)\right] + \sin\left[\frac{3}{2}(c + dx)\right] \right) \right) \operatorname{Tan}\left[\frac{1}{2}(c + dx)\right] \right) / \\
& \left( 8 \left( \frac{1}{1 + \cos[c + dx]} \right)^{3/2} \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \right) + \frac{1}{4 \left( \frac{1}{1 + \cos[c + dx]} \right)^{3/2} \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}}} \\
& \sqrt{b + a \cos[c + dx]} \operatorname{Sec}\left[\frac{1}{2}(c + dx)\right]^5 \sqrt{1 + \operatorname{Sec}[c + dx]} \left( \sqrt{\frac{b + a \cos[c + dx]}{(a + b)(1 + \cos[c + dx])}} \left( -\frac{1}{2} \cos\left[\frac{1}{2}(c + dx)\right] + \frac{3}{2} \cos\left[\frac{3}{2}(c + dx)\right] \right) \right) -
\end{aligned}$$



$$\begin{aligned}
& \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \sin \left[\frac{1}{2}(c+d x)\right] + \\
& \frac{\cos \left[\frac{1}{2}(c+d x)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] \left(\frac{\cos [c+d x] \sin [c+d x]}{(1+\cos [c+d x])^2} - \frac{\sin [c+d x]}{1+\cos [c+d x]}\right)}{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}}} + \\
& \frac{\left(-\frac{a \sin [c+d x]}{(a+b)(1+\cos [c+d x])} + \frac{(b+a \cos [c+d x]) \sin [c+d x]}{(a+b)(1+\cos [c+d x])^2}\right) \left(-\sin \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{3}{2}(c+d x)\right]\right)}{2 \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}}} + \\
& \left. \frac{\sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right] \sqrt{1-\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}{a+b}}}{\sqrt{1-\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]^2}}\right) + \\
& \left(\sqrt{b+a \cos [c+d x]} \operatorname{Sec}\left[\frac{1}{2}(c+d x)\right]^5 \operatorname{Sec}[c+d x] \left(2 \cos \left[\frac{1}{2}(c+d x)\right] \sqrt{\frac{\cos [c+d x]}{1+\cos [c+d x]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Tan}\left[\frac{1}{2}(c+d x)\right]\right], \frac{a-b}{a+b}\right] + \right.\right. \\
& \left.\left.\sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \left(-\sin \left[\frac{1}{2}(c+d x)\right] + \sin \left[\frac{3}{2}(c+d x)\right]\right)\right) \operatorname{Tan}[c+d x]\right) / \\
& \left.\left.\left.\left(8\left(\frac{1}{1+\cos [c+d x]}\right)^{3 / 2} \sqrt{\frac{b+a \cos [c+d x]}{(a+b)(1+\cos [c+d x])}} \sqrt{1+\operatorname{Sec}[c+d x]}\right)\right)\right)\right)
\end{aligned}$$

**Problem 64:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sec [a+b x] \sec [2 a+2 b x] d x$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{b} + \frac{\sqrt{2} \text{ArcTanh}[\sqrt{2} \text{Sin}[a + b x]]}{b}$$

Result (type 3, 331 leaves):

$$\frac{1}{4b} \left( \frac{(2 + 2i) \left( (-1 - i) + \sqrt{2} \right) \text{ArcTan} \left[ \frac{\cos \left[ \frac{1}{2} (a + b x) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{2} (a + b x) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{2} (a + b x) \right] - \sin \left[ \frac{1}{2} (a + b x) \right]} \right]}{(-1 + i) + \sqrt{2}} - 2i \sqrt{2} \text{ArcTan} \left[ \frac{\cos \left[ \frac{1}{2} (a + b x) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{2} (a + b x) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{2} (a + b x) \right] - \sin \left[ \frac{1}{2} (a + b x) \right]} \right] + \right. \\ \left. 4 \text{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] - \sin \left[ \frac{1}{2} (a + b x) \right] \right] - 4 \text{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] + \sin \left[ \frac{1}{2} (a + b x) \right] \right] + 2 \sqrt{2} \text{Log} \left[ \sqrt{2} + 2 \text{Sin}[a + b x] \right] - \right. \\ \left. \sqrt{2} \text{Log} \left[ 2 - \sqrt{2} \cos[a + b x] - \sqrt{2} \sin[a + b x] \right] + \frac{(1 - i) \left( (-1 - i) + \sqrt{2} \right) \text{Log} \left[ 2 + \sqrt{2} \cos[a + b x] - \sqrt{2} \sin[a + b x] \right]}{(-1 + i) + \sqrt{2}} \right)$$

**Problem 65: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Sec}[a + b x] \text{Sec}[2(a + b x)] dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{b} + \frac{\sqrt{2} \text{ArcTanh}[\sqrt{2} \text{Sin}[a + b x]]}{b}$$

Result (type 3, 331 leaves):

$$\frac{1}{4b} \left( \frac{(2 + 2i) \left( (-1 - i) + \sqrt{2} \right) \text{ArcTan} \left[ \frac{\cos \left[ \frac{1}{2} (a + b x) \right] - (-1 + \sqrt{2}) \sin \left[ \frac{1}{2} (a + b x) \right]}{(1 + \sqrt{2}) \cos \left[ \frac{1}{2} (a + b x) \right] - \sin \left[ \frac{1}{2} (a + b x) \right]} \right]}{(-1 + i) + \sqrt{2}} - 2i \sqrt{2} \text{ArcTan} \left[ \frac{\cos \left[ \frac{1}{2} (a + b x) \right] - (1 + \sqrt{2}) \sin \left[ \frac{1}{2} (a + b x) \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{1}{2} (a + b x) \right] - \sin \left[ \frac{1}{2} (a + b x) \right]} \right] + \right. \\ \left. 4 \text{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] - \sin \left[ \frac{1}{2} (a + b x) \right] \right] - 4 \text{Log} \left[ \cos \left[ \frac{1}{2} (a + b x) \right] + \sin \left[ \frac{1}{2} (a + b x) \right] \right] + 2 \sqrt{2} \text{Log} \left[ \sqrt{2} + 2 \text{Sin}[a + b x] \right] - \right. \\ \left. \sqrt{2} \text{Log} \left[ 2 - \sqrt{2} \cos[a + b x] - \sqrt{2} \sin[a + b x] \right] + \frac{(1 - i) \left( (-1 - i) + \sqrt{2} \right) \text{Log} \left[ 2 + \sqrt{2} \cos[a + b x] - \sqrt{2} \sin[a + b x] \right]}{(-1 + i) + \sqrt{2}} \right)$$

**Problem 74: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin[x] \tan[2x] dx$$

Optimal (type 3, 20 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\sqrt{2} \sin[x]]}{\sqrt{2}} - \sin[x]$$

Result (type 3, 179 leaves):

$$-\frac{1}{4\sqrt{2}} \left( 2i \text{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] + 2i \text{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] - \right. \\ \left. 2 \text{Log}[\sqrt{2} + 2 \sin[x]] + \text{Log}[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + \text{Log}[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + 4\sqrt{2} \sin[x] \right)$$

**Problem 76: Result is not expressed in closed-form.**

$$\int \sin[x] \tan[4x] dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$\frac{1}{4} \sqrt{2 - \sqrt{2}} \text{ArcTanh} \left[ \frac{2 \sin[x]}{\sqrt{2 - \sqrt{2}}} \right] + \frac{1}{4} \sqrt{2 + \sqrt{2}} \text{ArcTanh} \left[ \frac{2 \sin[x]}{\sqrt{2 + \sqrt{2}}} \right] - \sin[x]$$

Result (type 7, 96 leaves):

$$\frac{1}{16} \text{RootSum}[1 + \#1^8 \&, \frac{1}{\#1^7} \left( 2 \text{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] - i \text{Log}[1 - 2 \cos[x] \#1 + \#1^2] + 2 \text{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^6 - i \text{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^6 \right) \&] - \sin[x]$$

**Problem 77: Result is not expressed in closed-form.**

$$\int \sin[x] \tan[5x] dx$$

Optimal (type 3, 112 leaves, 10 steps):

$$\frac{1}{5} \operatorname{ArcTanh}[\sin[x]] - \frac{1}{20} (1 - \sqrt{5}) \operatorname{Log}[1 - \sqrt{5} - 4 \sin[x]] - \frac{1}{20} (1 + \sqrt{5}) \operatorname{Log}[1 + \sqrt{5} - 4 \sin[x]] +$$

$$\frac{1}{20} (1 - \sqrt{5}) \operatorname{Log}[1 - \sqrt{5} + 4 \sin[x]] + \frac{1}{20} (1 + \sqrt{5}) \operatorname{Log}[1 + \sqrt{5} + 4 \sin[x]] - \sin[x]$$

Result (type 7, 248 leaves):

$$\frac{1}{20} \left( \operatorname{RootSum}[1 - \#1^2 + \#1^4 - \#1^6 + \#1^8 \&, \right.$$

$$\left( 6 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] - 3 i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] - 2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^2 + i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^2 - \right.$$

$$\left. 2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^4 + i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^4 + 6 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^6 - 3 i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^6 \right) /$$

$$\left( -\#1 + 2 \#1^3 - 3 \#1^5 + 4 \#1^7 \right) \& \left. - 4 \left( \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + 5 \sin[x] \right) \right)$$

Problem 78: Result is not expressed in closed-form.

$$\int \sin[x] \tan[6x] dx$$

Optimal (type 3, 89 leaves, 10 steps):

$$\frac{\operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{3 \sqrt{2}} + \frac{1}{6} \sqrt{2 - \sqrt{3}} \operatorname{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{2 - \sqrt{3}}}\right] + \frac{1}{6} \sqrt{2 + \sqrt{3}} \operatorname{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{2 + \sqrt{3}}}\right] - \sin[x]$$

Result (type 7, 366 leaves):

$$\frac{1}{24} \left( \operatorname{RootSum}[1 - \#1^4 + \#1^8 \&, \right.$$

$$\frac{1}{-\#1^3 + 2 \#1^7} \left( 4 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] - 2 i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] - 2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^2 + i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^2 - \right.$$

$$\left. 2 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^4 + i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^4 + 4 \operatorname{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^6 - 2 i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^6 \right) \& \left. - \right.$$

$$\sqrt{2} \left( 2 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + 2 i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] - 2 \operatorname{Log}[\sqrt{2} + 2 \sin[x]] + \right.$$

$$\left. \operatorname{Log}[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + \operatorname{Log}[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + 12 \sqrt{2} \sin[x] \right)$$

### Problem 80: Result more than twice size of optimal antiderivative.

$$\int \cot[2x] \sin[x] dx$$

Optimal (type 3, 10 leaves, 3 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\sin[x]] + \sin[x]$$

Result (type 3, 41 leaves):

$$\frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + \sin[x]$$

### Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cot[4x] \sin[x] dx$$

Optimal (type 3, 28 leaves, 6 steps):

$$-\frac{1}{4} \operatorname{ArcTanh}[\sin[x]] - \frac{\operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{2\sqrt{2}} + \sin[x]$$

Result (type 3, 223 leaves):

$$\frac{1}{8\sqrt{2}} \left( 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + 2\sqrt{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \right. \\ \left. 2\sqrt{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - 2 \operatorname{Log}\left[\sqrt{2} + 2 \sin[x]\right] + \operatorname{Log}\left[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]\right] + \operatorname{Log}\left[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]\right] + 8\sqrt{2} \sin[x] \right)$$

### Problem 83: Result more than twice size of optimal antiderivative.

$$\int \cot[5x] \sin[x] dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$-\frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \operatorname{ArcTanh}\left[2 \sqrt{\frac{2}{5 + \sqrt{5}}} \sin[x]\right] - \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{5} (5 + \sqrt{5})} \sin[x]\right] + \sin[x]$$

Result (type 3, 201 leaves):

$$\frac{(-1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{(-3 + \sqrt{5}) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{10 - 2\sqrt{5}}}\right] - (-1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{(5 + \sqrt{5}) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{10 - 2\sqrt{5}}}\right]}{\sqrt{50 - 10\sqrt{5}}} + \frac{(1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{(-5 + \sqrt{5}) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{2(5 + \sqrt{5})}}\right] - (1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{(3 + \sqrt{5}) \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{2(5 + \sqrt{5})}}\right]}{\sqrt{10(5 + \sqrt{5})}} + \operatorname{Sin}[x]$$

Problem 84: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cot}[6x] \operatorname{Sin}[x] dx$$

Optimal (type 3, 38 leaves, 7 steps):

$$-\frac{1}{6} \operatorname{ArcTanh}[\operatorname{Sin}[x]] - \frac{1}{6} \operatorname{ArcTanh}[2 \operatorname{Sin}[x]] - \frac{\operatorname{ArcTanh}\left[\frac{2 \operatorname{Sin}[x]}{\sqrt{3}}\right]}{2\sqrt{3}} + \operatorname{Sin}[x]$$

Result (type 3, 99 leaves):

$$\frac{1}{12} \left( -2\sqrt{3} \operatorname{ArcTanh}\left[\frac{\operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{3}}\right] - 2\sqrt{3} \operatorname{ArcTanh}\left[\sqrt{3} \operatorname{Tan}\left[\frac{x}{2}\right]\right] + 2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] - 2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \operatorname{Log}[1 - 2 \operatorname{Sin}[x]] - \operatorname{Log}[1 + 2 \operatorname{Sin}[x]] + 12 \operatorname{Sin}[x] \right)$$

Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \operatorname{Sec}[2x] \operatorname{Sin}[x] dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sqrt{2} \operatorname{Cos}[x]]}{\sqrt{2}}$$

Result (type 3, 174 leaves):

$$\frac{1}{4\sqrt{2}} \left( 2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] - 2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] + \right. \\ \left. 4 \operatorname{ArcTanh} \left[ \sqrt{2} + \tan\left[\frac{x}{2}\right] \right] - \operatorname{Log} \left[ 2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] + \operatorname{Log} \left[ 2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] \right)$$

**Problem 87: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[4x] \sin[x] dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh} \left[ \frac{2 \cos[x]}{\sqrt{2-\sqrt{2}}} \right]}{2\sqrt{2(2-\sqrt{2})}} + \frac{\operatorname{ArcTanh} \left[ \frac{2 \cos[x]}{\sqrt{2+\sqrt{2}}} \right]}{2\sqrt{2(2+\sqrt{2})}}$$

Result (type 3, 5090 leaves):

$$\left( \left( -2(-1)^{3/8} (1 + \sqrt{2}) x - \frac{2(-1)^{1/4} (-2 - (1-i)(-1)^{5/8} + (-1)^{5/8} \sqrt{2}) \operatorname{ArcTan} \left[ \frac{-\cos[x] + (1 + \sqrt{2}) \sin[x]}{2(-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x]} \right]}{(-1+i) + 2(-1)^{3/8} + \sqrt{2}} \right) - \right. \\ \left( 2(1-i)^{3/2} 2^{1/4} \left( (-3-i) + 2(-1)^{5/8} + (2+i)\sqrt{2} - (2+2i)(-1)^{3/8} \sqrt{2} + 2(-1)^{5/8} \sqrt{2} \right) \right. \\ \left. \operatorname{ArcTan} \left[ \frac{(1+i) + i\sqrt{2} + \left( (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) \tan\left[\frac{x}{2}\right]}{\sqrt{1-i} 2^{3/4}} \right] \right) / \left( (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) + \\ \left. 2(-1)^{3/8} \operatorname{Log} \left[ \sec\left[\frac{x}{2}\right]^2 \right] + \left( (-1)^{3/4} (-2 - (1-i)(-1)^{5/8} + (-1)^{5/8} \sqrt{2}) \operatorname{Log} \left[ -\sec\left[\frac{x}{2}\right]^4 \right] \right) \right) / \left( (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) \\ \left. \left( -2 + (1-i)\sqrt{2} + 2(-1)^{3/8} (-1 + \sqrt{2}) \cos[x] + \sqrt{2} \cos[2x] - 2(-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right) / \left( (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) \\ \left( - \left( \left( \frac{1}{2} + \frac{i}{2} \right) / \left( \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right) \left( -(-1-i)^{3/2} (1-i)^{1/4} (1+i)^{1/4} - (1+i) \cos[x] + i\sqrt{1-i} \sqrt{1+i} \cos[x] + \right. \right. \right. \right. \\ \left. \left. \left. (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) \right) - \sin[x] / \left( \sqrt{1-i} (1-i)^{1/4} (1+i)^{1/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right) \right) \right)$$

$$\begin{aligned}
& \left( -(-1-i)^{3/2} (1-i)^{1/4} (1+i)^{1/4} - (1+i) \cos[x] + i \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x] \right) - \\
& \left( i \sqrt{1-i} (1-i)^{1/4} (1+i)^{1/4} \sin[x] \right) / \left( 2 \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right) \right) \\
& \left. \left( -(-1-i)^{3/2} (1-i)^{1/4} (1+i)^{1/4} - (1+i) \cos[x] + i \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) / \\
& \left( -2 (-1)^{3/8} (1+\sqrt{2}) - \left( 2 (-1)^{1/4} (-2 - (1-i) (-1)^{5/8} + (-1)^{5/8} \sqrt{2}) \left( \frac{(1+\sqrt{2}) \cos[x] + \sin[x]}{2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x]} - \right. \right. \right. \\
& \left. \left. \left. \frac{(\cos[x] - \sin[x] + \sqrt{2} \sin[x]) (-\cos[x] + (1+\sqrt{2}) \sin[x])}{(2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x])^2} \right) \right) / \\
& \left( \left( (-1+i) + 2 (-1)^{3/8} + \sqrt{2} \right) \left( 1 + \frac{(-\cos[x] + (1+\sqrt{2}) \sin[x])^2}{(2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x])^2} \right) \right) + 2 (-1)^{3/8} \tan\left[\frac{x}{2}\right] - \\
& \left( (-1)^{3/4} (-2 - (1-i) (-1)^{5/8} + (-1)^{5/8} \sqrt{2}) \cos\left[\frac{x}{2}\right]^4 \right. \\
& \left. \left( -\sec\left[\frac{x}{2}\right]^4 (-2 (-1)^{3/8} \cos[x] + 2 \sqrt{2} \cos[2x] - 2 (-1)^{3/8} (-1+\sqrt{2}) \sin[x] - 2 \sqrt{2} \sin[2x]) - \right. \right. \\
& \left. \left. 2 \sec\left[\frac{x}{2}\right]^4 (-2 + (1-i) \sqrt{2} + 2 (-1)^{3/8} (-1+\sqrt{2}) \cos[x] + \sqrt{2} \cos[2x] - 2 (-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x]) \tan\left[\frac{x}{2}\right] \right) \right) / \\
& \left( \left( (-1+i) + 2 (-1)^{3/8} + \sqrt{2} \right) (-2 + (1-i) \sqrt{2} + 2 (-1)^{3/8} (-1+\sqrt{2}) \cos[x] + \sqrt{2} \cos[2x] - 2 (-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x]) \right) - \\
& \left. \frac{(1-i) \left( (-3-i) + 2 (-1)^{5/8} + (2+i) \sqrt{2} - (2+2i) (-1)^{3/8} \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2}{\sqrt{2} \left( 1 + \frac{(\frac{1+i}{4+\frac{1}{4}}) \left( (1+i) + i \sqrt{2} + ((-1+i) + 2 (-1)^{3/8} + \sqrt{2}) \tan\left[\frac{x}{2}\right])^2}{\sqrt{2}} \right)} \right) + \\
& \left( \left( (-2-2i) \left( (1-i) + \sqrt{2} + (-1)^{7/8} \sqrt{2} \right) x + (2+2i) (-1)^{3/8} (2 - (1-i) (-1)^{5/8} + (1-i) (-1)^{7/8} - \sqrt{2}) \right. \right. \\
& \left. \left. \text{ArcTan}\left[ \frac{\sin[x]}{-(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x]} \right] + \right. \right. \\
& \left. \left( 4-4i \right) \left( (1+3i) + (1-i) (-1)^{1/8} + (1+2i) (-1)^{3/8} - (2+2i) (-1)^{5/8} + (2+i) (-1)^{7/8} - (1+2i) \sqrt{2} \right) \right. \\
& \left. \left. \text{ArcTan}\left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{5/8} \left( i + (-1)^{3/4} + (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \tan\left[\frac{x}{2}\right] \right) \right] + 2 (-1)^{7/8} \sqrt{2} (-1 + (-1)^{1/4}) \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \left( (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^2 \right] + (-1)^{7/8} \left( (-2 - 2i) + 2(-1)^{5/8} - 2(-1)^{7/8} + (1+i) \sqrt{2} \right) \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^4 \right. \right. \\
& \quad \left. \left. (1 - 3(-1)^{1/4} + 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] - (-i + (-1)^{1/4}) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x]) \right] \right) \\
& \left( i / \left( \sqrt{1-i} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \left( \sqrt{1-i} (1-i)^{3/4} (1+i)^{1/4} + \sqrt{1-i} \cos[x] - \sqrt{1+i} \cos[x] + \right. \right. \right. \\
& \quad \left. \left. i \sqrt{1-i} \sin[x] + i \sqrt{1+i} \sin[x] \right) \right) + 1 / \left( \sqrt{1+i} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \right. \\
& \quad \left. \left( \sqrt{1-i} (1-i)^{3/4} (1+i)^{1/4} + \sqrt{1-i} \cos[x] - \sqrt{1+i} \cos[x] + i \sqrt{1-i} \sin[x] + i \sqrt{1+i} \sin[x] \right) \right) - \\
& \left( 2 \sin[x] \right) / \left( \sqrt{1-i} (1-i)^{1/4} (1+i)^{3/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \right. \\
& \quad \left. \left( \sqrt{1-i} (1-i)^{3/4} (1+i)^{1/4} + \sqrt{1-i} \cos[x] - \sqrt{1+i} \cos[x] + i \sqrt{1-i} \sin[x] + i \sqrt{1+i} \sin[x] \right) \right) \Bigg) / \\
& \left( (-2 - 2i) \left( (1-i) + \sqrt{2} + (-1)^{7/8} \sqrt{2} \right) + \left( (2 + 2i) (-1)^{3/8} \left( 2 - (1-i) (-1)^{5/8} + (1-i) (-1)^{7/8} - \sqrt{2} \right) \right. \right. \\
& \quad \left. \left( - \frac{\sin[x] \left( (-1)^{3/4} \cos[x] - \sin[x] + (-1)^{1/4} \sin[x] \right)}{\left( -(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x] \right)^2} + \frac{\cos[x]}{-(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x]} \right) \right) / \\
& \left( 1 + \frac{\sin[x]^2}{\left( -(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x] \right)^2} \right) + 2(-1)^{7/8} \sqrt{2} (-1 + (-1)^{1/4}) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \tan \left[ \frac{x}{2} \right] + \\
& \left( (-1)^{7/8} \left( (-2 - 2i) + 2(-1)^{5/8} - 2(-1)^{7/8} + (1+i) \sqrt{2} \right) \cos \left[ \frac{x}{2} \right]^4 \right. \\
& \quad \left( \operatorname{Sec} \left[ \frac{x}{2} \right]^4 \left( 2(-1)^{3/8} \sqrt{2} \cos[x] + 2 \cos[2x] + 2(-1)^{3/4} \cos[2x] - 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \sin[x] + 2(-i + (-1)^{1/4}) \sin[2x] \right) + \right. \\
& \quad \left. 2 \operatorname{Sec} \left[ \frac{x}{2} \right]^4 \left( 1 - 3(-1)^{1/4} + 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] - \right. \right. \\
& \quad \left. \left. (-i + (-1)^{1/4}) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \tan \left[ \frac{x}{2} \right] \right) \Bigg) / \\
& \left( 1 - 3(-1)^{1/4} + 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] - (-i + (-1)^{1/4}) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) + \\
& \left( 2(-1)^{5/8} \left( (1 + 3i) + (1-i) (-1)^{1/8} + (1 + 2i) (-1)^{3/8} - (2 + 2i) (-1)^{5/8} + (2 + i) (-1)^{7/8} - (1 + 2i) \sqrt{2} \right) \right. \\
& \quad \left. (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \operatorname{Sec} \left[ \frac{x}{2} \right]^2 \right) / \left( 1 - \frac{1}{2} (-1)^{3/4} \left( i + (-1)^{3/4} + (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \tan \left[ \frac{x}{2} \right] \right)^2 \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \left( \left( 2 (-1)^{1/8} (1 + (-1)^{1/4}) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) x - 2 (-1)^{3/8} (2 - \sqrt{2} - (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2}) \right. \right. \\
& \quad \left. \left. \text{ArcTan} \left[ \frac{\text{Sin}[x]}{(-1 + (-1)^{1/4}) \text{Cos}[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \text{Sin}[x])} \right] - \right. \right. \\
& \quad 4 \left( (3 - i) - 2 (-1)^{1/8} + 2 (-1)^{3/8} - (2 - i) \sqrt{2} + (-1)^{1/8} \sqrt{2} - (2 + i) (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2} \right) \\
& \quad \left. \left. \text{ArcTan} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{5/8} (i + (-1)^{3/4} + (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) \text{Tan} \left[ \frac{x}{2} \right]) \right] - \right. \right. \\
& \quad \left. \left. 2 (-1)^{7/8} (1 + (-1)^{3/4}) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) \text{Log} \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + (-1)^{7/8} (2 - \sqrt{2} - (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2}) \text{Log} \left[ -\text{Sec} \left[ \frac{x}{2} \right]^4 \right. \right. \right. \\
& \quad \left. \left. \left. (-1 + 3 (-1)^{1/4} - 2 (-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \text{Cos}[x] + (-i + (-1)^{1/4}) \text{Cos}[2x] + 2 (-1)^{3/8} \sqrt{2} \text{Sin}[x] + \text{Sin}[2x] + (-1)^{3/4} \text{Sin}[2x] \right] \right) \right) \\
& \left( 1 / \left( \sqrt{1-i} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right)^2 (-\sqrt{-1+i} (1-i)^{1/4} (1+i)^{3/4} + \sqrt{1-i} \text{Cos}[x] - \sqrt{1+i} \text{Cos}[x] - \right. \right. \\
& \quad \left. \left. i \sqrt{1-i} \text{Sin}[x] - i \sqrt{1+i} \text{Sin}[x]) \right) - i / \left( \sqrt{1+i} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \right. \right. \\
& \quad \left. \left. (-\sqrt{-1+i} (1-i)^{1/4} (1+i)^{3/4} + \sqrt{1-i} \text{Cos}[x] - \sqrt{1+i} \text{Cos}[x] - i \sqrt{1-i} \text{Sin}[x] - i \sqrt{1+i} \text{Sin}[x]) \right) \right) + \\
& \quad \left( 2 \text{Sin}[x] \right) / \left( \sqrt{-1+i} (1-i)^{3/4} (1+i)^{1/4} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \right. \\
& \quad \left. \left. (-\sqrt{-1+i} (1-i)^{1/4} (1+i)^{3/4} + \sqrt{1-i} \text{Cos}[x] - \sqrt{1+i} \text{Cos}[x] - i \sqrt{1-i} \text{Sin}[x] - i \sqrt{1+i} \text{Sin}[x]) \right) \right) \Big) / \\
& \left( 2 (-1)^{1/8} (1 + (-1)^{1/4}) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) - \left( 2 (-1)^{3/8} (2 - \sqrt{2} - (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2}) \right. \right. \\
& \quad \left. \left. \left( -\frac{\text{Sin}[x] \left( (-1)^{3/4} \text{Cos}[x] - (-1 + (-1)^{1/4}) \text{Sin}[x] \right)}{\left( (-1 + (-1)^{1/4}) \text{Cos}[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \text{Sin}[x]) \right)^2} + \frac{\text{Cos}[x]}{(-1 + (-1)^{1/4}) \text{Cos}[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \text{Sin}[x])} \right) \right) / \\
& \quad \left( 1 + \frac{\text{Sin}[x]^2}{\left( (-1 + (-1)^{1/4}) \text{Cos}[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \text{Sin}[x]) \right)^2} \right) - 2 (-1)^{7/8} (1 + (-1)^{3/4}) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) \text{Tan} \left[ \frac{x}{2} \right] - \\
& \quad \left( (-1)^{7/8} (2 - \sqrt{2} - (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2}) \text{Cos} \left[ \frac{x}{2} \right]^4 \right. \\
& \quad \left. \left( -\text{Sec} \left[ \frac{x}{2} \right]^4 (2 (-1)^{3/8} \sqrt{2} \text{Cos}[x] + 2 \text{Cos}[2x] + 2 (-1)^{3/4} \text{Cos}[2x] + 2 (-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \text{Sin}[x] - 2 (-i + (-1)^{1/4}) \text{Sin}[2x]) - \right. \right. \\
& \quad \left. \left. 2 \text{Sec} \left[ \frac{x}{2} \right]^4 (-1 + 3 (-1)^{1/4} - 2 (-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \text{Cos}[x] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( -i + (-1)^{1/4} \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \tan\left[\frac{x}{2}\right] \Bigg) \Bigg/ \\
& \left( -1 + 3(-1)^{1/4} - 2(-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \cos[x] + \left( -i + (-1)^{1/4} \right) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) - \\
& \left( (1+i)(-1)^{5/8} \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \left( (3-i) - 2(-1)^{1/8} + 2(-1)^{3/8} - (2-i) \sqrt{2} + (-1)^{1/8} \sqrt{2} - (2+i)(-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2} \right) \right. \\
& \left. \sec\left[\frac{x}{2}\right]^2 \right) \Bigg/ \left( 1 - \frac{1}{2}(-1)^{3/4} \left( i + (-1)^{3/4} + \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \tan\left[\frac{x}{2}\right] \right)^2 \right) \Bigg) + \\
& \left( \left( -4\sqrt{-1-i}(-1+\sqrt{2}) \operatorname{ArcTanh}\left[ \frac{-i \left( (1+i) + \sqrt{2} \right) + \left( (1+i) + 2(-1)^{5/8} - \sqrt{2} \right) \tan\left[\frac{x}{2}\right]}{\sqrt{-1-i} 2^{3/4}} \right] \right) + \right. \\
& \left. (-1)^{1/8} 2^{1/4} \left( 2 \operatorname{ArcTan}\left[ \frac{\cos[x] + (1+\sqrt{2}) \sin[x]}{2(-1)^{5/8} + (-1+\sqrt{2}) \cos[x] + \sin[x]} \right] - i \left( 2(1+\sqrt{2})x + 2 \log\left[\sec\left[\frac{x}{2}\right]^2\right] - \right. \right. \right. \\
& \left. \left. \log\left[\sec\left[\frac{x}{2}\right]^4 \left( 2 - (1+i)\sqrt{2} + 2(-1)^{5/8}(-1+\sqrt{2}) \cos[x] - \sqrt{2} \cos[2x] + 2(-1)^{5/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right] \right) \right) \Bigg) \Bigg) \\
& \left( \left( \frac{1}{2} + \frac{i}{2} \right) \Bigg/ \left( \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right) \left( -\frac{2(1-i)^{1/4} (1+i)^{1/4}}{\sqrt{-1+i}} - (1+i) \cos[x] + \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] - \right. \right. \right. \\
& \left. \left. i \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) - (2 \sin[x]) \Bigg/ \left( (-1+i)^{5/2} (1-i)^{1/4} (1+i)^{1/4} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right) \right. \\
& \left. \left( -\frac{2(1-i)^{1/4} (1+i)^{1/4}}{\sqrt{-1+i}} - (1+i) \cos[x] + \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] - i \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) \Bigg) + \\
& \left( i (1-i)^{1/4} (1+i)^{1/4} \sin[x] \right) \Bigg/ \left( (-1+i)^{3/2} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right) \right. \\
& \left. \left( -\frac{2(1-i)^{1/4} (1+i)^{1/4}}{\sqrt{-1+i}} - (1+i) \cos[x] + \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] - i \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) \Bigg) \Bigg) \Bigg/ \\
& \left( -\frac{2^{1/4} \left( (1+i) + 2(-1)^{5/8} - \sqrt{2} \right) (-1+\sqrt{2}) \sec\left[\frac{x}{2}\right]^2}{1 + \frac{\left(\frac{1-i}{4}\right) \left( -i \left( (1+i) + \sqrt{2} \right) + \left( (1+i) + 2(-1)^{5/8} - \sqrt{2} \right) \tan\left[\frac{x}{2}\right] \right)^2}{\sqrt{2}}} + (-1)^{1/8} 2^{1/4} \left( 2 \left( \frac{(1+\sqrt{2}) \cos[x] - \sin[x]}{2(-1)^{5/8} + (-1+\sqrt{2}) \cos[x] + \sin[x]} - \frac{(\cos[x] - (-1+\sqrt{2}) \sin[x]) (\cos[x] + (1+\sqrt{2}) \sin[x])}{(2(-1)^{5/8} + (-1+\sqrt{2}) \cos[x] + \sin[x])^2} \right) \right. \right. \right. \\
& \left. \left. \left. 1 + \frac{(\cos[x] + (1+\sqrt{2}) \sin[x])^2}{(2(-1)^{5/8} + (-1+\sqrt{2}) \cos[x] + \sin[x])^2} \right) \right) - \right. \\
& \left. i \left( 2(1+\sqrt{2}) + 2 \tan\left[\frac{x}{2}\right] - \left( \cos\left[\frac{x}{2}\right] \right)^4 \left( \sec\left[\frac{x}{2}\right]^4 \left( 2(-1)^{5/8} \cos[x] + 2\sqrt{2} \cos[2x] - 2(-1)^{5/8}(-1+\sqrt{2}) \sin[x] + 2\sqrt{2} \sin[2x] \right) + 2 \right. \right. \right.
\end{aligned}$$

$$\left. \left( \left. \left( \left. \left. \sec\left[\frac{x}{2}\right]^4 \left(2 - (1+i)\sqrt{2} + 2(-1)^{5/8}(-1+\sqrt{2})\cos[x] - \sqrt{2}\cos[2x] + 2(-1)^{5/8}\sin[x] + \sqrt{2}\sin[2x]\right)\tan\left[\frac{x}{2}\right]\right)\right)\right)\right)\right)$$

$$\left(2 - (1+i)\sqrt{2} + 2(-1)^{5/8}(-1+\sqrt{2})\cos[x] - \sqrt{2}\cos[2x] + 2(-1)^{5/8}\sin[x] + \sqrt{2}\sin[2x]\right)$$

**Problem 89:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sec[6x] \sin[x] dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$-\frac{\text{ArcTanh}\left[\sqrt{2}\cos[x]\right]}{3\sqrt{2}} + \frac{\text{ArcTanh}\left[\frac{2\cos[x]}{\sqrt{2-\sqrt{3}}}\right]}{6\sqrt{2-\sqrt{3}}} + \frac{\text{ArcTanh}\left[\frac{2\cos[x]}{\sqrt{2+\sqrt{3}}}\right]}{6\sqrt{2+\sqrt{3}}}$$

Result (type 3, 678 leaves):

$$\begin{aligned}
& \left( \frac{1}{6} + \frac{i}{6} \right) (-1)^{1/4} \operatorname{ArcTan} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \operatorname{Sec} \left[ \frac{x}{2} \right] \left( \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right) \right] - \left( \frac{1}{6} + \frac{i}{6} \right) (-1)^{3/4} \operatorname{ArcTanh} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \operatorname{Sec} \left[ \frac{x}{2} \right] \left( \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right) \right] + \\
& \frac{1}{12(2+\sqrt{2})} \left( 1 + \sqrt{2} \right) \left( x + 2\sqrt{3} \operatorname{ArcTanh} \left[ \frac{2 + (2+\sqrt{2}) \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{6}} \right] - \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^2 \right] + \operatorname{Log} \left[ -\operatorname{Sec} \left[ \frac{x}{2} \right]^2 (\sqrt{2} - 2 \cos [x] + 2 \sin [x]) \right] \right) - \\
& \frac{x - 2\sqrt{3} \operatorname{ArcTanh} \left[ \frac{\sqrt{2} + (-1+\sqrt{2}) \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] - \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^2 \right] + \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^2 (1 + \sqrt{2} \cos [x] - \sqrt{2} \sin [x]) \right]}{12\sqrt{2}} + \\
& \left( \left( 2(\sqrt{2} + \sqrt{3}) \operatorname{ArcTanh} \left[ \frac{2 + (2+\sqrt{6}) \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2}} \right] + (3 + \sqrt{6}) \left( x - \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^2 \right] + \operatorname{Log} \left[ -\operatorname{Sec} \left[ \frac{x}{2} \right]^2 (\sqrt{6} - 2 \cos [x] + 2 \sin [x]) \right] \right) \right) \right) \\
& \left( 1 + \sqrt{6} \sin [x] \right) \left( 3 + \sqrt{6} - (2 + \sqrt{6}) \cos [x] + (2 + \sqrt{6}) \sin [x] \right) \Big/ \\
& \left( 12 \left( (12 + 5\sqrt{6}) \cos [2x] + 2 \cos [x] (5 + 2\sqrt{6} + 5\sqrt{6} \sin [x]) - 2(12 + 5\sqrt{6} + 4(5 + 2\sqrt{6}) \sin [x] - 6 \sin [2x]) \right) \right) + \\
& \left( (-2(-2 + \sqrt{6}) \operatorname{ArcTanh} [\sqrt{2} + (\sqrt{2} - \sqrt{3}) \operatorname{Tan} \left[ \frac{x}{2} \right]] + (3\sqrt{2} - 2\sqrt{3}) \left( x - \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^2 \right] + \operatorname{Log} \left[ -\operatorname{Sec} \left[ \frac{x}{2} \right]^2 (\sqrt{3} + \sqrt{2} \cos [x] - \sqrt{2} \sin [x]) \right] \right) \right) \right) \\
& \left( \sqrt{2} - 2\sqrt{3} \sin [x] \right) \left( -3 + \sqrt{6} - (-2 + \sqrt{6}) \cos [x] + (-2 + \sqrt{6}) \sin [x] \right) \Big/ \\
& \left( 24 \left( (-12 + 5\sqrt{6}) \cos [2x] + 2 \cos [x] (-5 + 2\sqrt{6} + 5\sqrt{6} \sin [x]) - 2(-12 + 5\sqrt{6} + 4(-5 + 2\sqrt{6}) \sin [x] + 6 \sin [2x]) \right) \right)
\end{aligned}$$

**Problem 90: Result more than twice size of optimal antiderivative.**

$$\int \csc [2x] \sin [x] dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcTanh} [\sin [x]]$$

Result (type 3, 37 leaves):

$$\frac{1}{2} \left( -\operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] + \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \csc [4x] \sin [x] dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4} \operatorname{ArcTanh}[\sin[x]] + \frac{\operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{2\sqrt{2}}$$

Result (type 3, 218 leaves):

$$\frac{1}{8\sqrt{2}} \left( -2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] - 2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] + 2\sqrt{2} \operatorname{Log} \left[ \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right] \right] - \right. \\ \left. 2\sqrt{2} \operatorname{Log} \left[ \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right] + 2 \operatorname{Log} [\sqrt{2} + 2 \sin[x]] - \operatorname{Log} [2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] - \operatorname{Log} [2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] \right)$$

**Problem 94: Result more than twice size of optimal antiderivative.**

$$\int \csc[6x] \sin[x] dx$$

Optimal (type 3, 36 leaves, 7 steps):

$$\frac{1}{6} \operatorname{ArcTanh}[\sin[x]] + \frac{1}{6} \operatorname{ArcTanh}[2 \sin[x]] - \frac{\operatorname{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{3}}\right]}{2\sqrt{3}}$$

Result (type 3, 95 leaves):

$$\frac{1}{12} \left( -2\sqrt{3} \operatorname{ArcTanh} \left[ \frac{\tan\left[\frac{x}{2}\right]}{\sqrt{3}} \right] - 2\sqrt{3} \operatorname{ArcTanh} [\sqrt{3} \tan\left[\frac{x}{2}\right]] - \right. \\ \left. 2 \operatorname{Log} \left[ \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right] \right] + 2 \operatorname{Log} \left[ \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right] - \operatorname{Log} [1 - 2 \sin[x]] + \operatorname{Log} [1 + 2 \sin[x]] \right)$$

**Problem 105: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[x] \tan[2x] dx$$

Optimal (type 3, 20 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sqrt{2} \cos[x]]}{\sqrt{2}} - \cos[x]$$

Result (type 3, 183 leaves):

$$\frac{1}{4\sqrt{2}} \left( 2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] - 2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] + \right. \\ \left. 4 \operatorname{ArcTanh} \left[ \sqrt{2} + \tan\left[\frac{x}{2}\right] \right] - 4\sqrt{2} \cos[x] - \log \left[ 2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] + \log \left[ 2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] \right)$$

**Problem 106: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \tan[3x] dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh} \left[ \frac{2\cos[x]}{\sqrt{3}} \right]}{\sqrt{3}} - \cos[x]$$

Result (type 3, 48 leaves):

$$-\frac{\operatorname{ArcTanh} \left[ \frac{-2 + \tan\left[\frac{x}{2}\right]}{\sqrt{3}} \right]}{\sqrt{3}} + \frac{\operatorname{ArcTanh} \left[ \frac{2 + \tan\left[\frac{x}{2}\right]}{\sqrt{3}} \right]}{\sqrt{3}} - \cos[x]$$

**Problem 107: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[x] \tan[4x] dx$$

Optimal (type 3, 71 leaves, 6 steps):

$$\frac{1}{4} \sqrt{2 - \sqrt{2}} \operatorname{ArcTanh} \left[ \frac{2\cos[x]}{\sqrt{2 - \sqrt{2}}} \right] + \frac{1}{4} \sqrt{2 + \sqrt{2}} \operatorname{ArcTanh} \left[ \frac{2\cos[x]}{\sqrt{2 + \sqrt{2}}} \right] - \cos[x]$$

Result (type 3, 5854 leaves):

$$-\cos[x] + \left( (2 - 2i) (-1)^{3/8} x + \left( 2\sqrt{2} \left( (2 + 2i) - (1 + 3i) (-1)^{3/8} - (1 + i) \sqrt{2} + (1 + 2i) (-1)^{3/8} \sqrt{2} \right) \operatorname{ArcTan} \left[ \frac{\cos[x] + (1 + \sqrt{2}) \sin[x]}{2(-1)^{5/8} + (-1 + \sqrt{2}) \cos[x] + \sin[x]} \right] \right) /$$

$$\begin{aligned}
& \left( (-1 - i) - 2(-1)^{5/8} + \sqrt{2} \right) + \frac{4 \times 2^{3/4} \left( (-1 + i) + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \operatorname{ArcTanh} \left[ \frac{-i \left( (1+i) + \sqrt{2} \right) + \left( (1+i) + 2(-1)^{5/8} - \sqrt{2} \right) \tan \left[ \frac{x}{2} \right]}{\sqrt{-1-i} 2^{3/4}} \right]}{\sqrt{-1-i} \left( (1+i) + 2(-1)^{5/8} - \sqrt{2} \right)} - \\
& \left( (1-i) (-1)^{3/8} \sqrt{2} (-2 + \sqrt{2}) \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^2 \right] + \left( i \sqrt{2} \left( (2+2i) - (1+3i) (-1)^{3/8} - (1+i) \sqrt{2} + (1+2i) (-1)^{3/8} \sqrt{2} \right) \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^4 \right. \right. \right. \\
& \left. \left. \left. \left( 2 - (1+i) \sqrt{2} + 2(-1)^{5/8} (-1 + \sqrt{2}) \cos[x] - \sqrt{2} \cos[2x] + 2(-1)^{5/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right] \right) \right) / \left( (-1-i) - 2(-1)^{5/8} + \sqrt{2} \right) \Bigg) \\
& \left( \left( \frac{1}{2} + \frac{i}{2} \right) / \left( \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right) \left( -\frac{2(1-i)^{1/4} (1+i)^{1/4}}{\sqrt{-1+i}} - (1+i) \cos[x] + \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] - \right. \right. \right. \\
& \left. \left. \left. i \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) \right) + (2 \sin[x]) / \left( (-1+i)^{3/2} (1-i)^{3/4} (1+i)^{3/4} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right) \right. \\
& \left. \left( -\frac{2(1-i)^{1/4} (1+i)^{1/4}}{\sqrt{-1+i}} - (1+i) \cos[x] + \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] - i \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) + \\
& \left( 2i \sin[x] \right) / \left( (-1+i)^{3/2} (1-i)^{5/4} (1+i)^{1/4} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right) \right. \\
& \left. \left( -\frac{2(1-i)^{1/4} (1+i)^{1/4}}{\sqrt{-1+i}} - (1+i) \cos[x] + \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] - i \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) \Bigg) / \\
& \left( (2-2i) (-1)^{3/8} + \left( 2\sqrt{2} \left( (2+2i) - (1+3i) (-1)^{3/8} - (1+i) \sqrt{2} + (1+2i) (-1)^{3/8} \sqrt{2} \right) \right. \right. \\
& \left. \left. \left( \frac{(1+\sqrt{2}) \cos[x] - \sin[x]}{2(-1)^{5/8} + (-1+\sqrt{2}) \cos[x] + \sin[x]} - \frac{(\cos[x] - (-1+\sqrt{2}) \sin[x]) (\cos[x] + (1+\sqrt{2}) \sin[x])}{(2(-1)^{5/8} + (-1+\sqrt{2}) \cos[x] + \sin[x])^2} \right) \right) \right) / \\
& \left( \left( (-1-i) - 2(-1)^{5/8} + \sqrt{2} \right) \left( 1 + \frac{(\cos[x] + (1+\sqrt{2}) \sin[x])^2}{(2(-1)^{5/8} + (-1+\sqrt{2}) \cos[x] + \sin[x])^2} \right) \right) - (1-i) (-1)^{3/8} \sqrt{2} (-2 + \sqrt{2}) \tan \left[ \frac{x}{2} \right] + \\
& \left( i \sqrt{2} \left( (2+2i) - (1+3i) (-1)^{3/8} - (1+i) \sqrt{2} + (1+2i) (-1)^{3/8} \sqrt{2} \right) \cos \left[ \frac{x}{2} \right]^4 \right. \\
& \left. \left( \operatorname{Sec} \left[ \frac{x}{2} \right]^4 \left( 2(-1)^{5/8} \cos[x] + 2\sqrt{2} \cos[2x] - 2(-1)^{5/8} (-1 + \sqrt{2}) \sin[x] + 2\sqrt{2} \sin[2x] \right) + \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \left( \frac{2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 \left( 2 - (1+i)\sqrt{2} + 2(-1)^{5/8}(-1+\sqrt{2}) \cos[x] - \sqrt{2} \cos[2x] + 2(-1)^{5/8} \sin[x] + \sqrt{2} \sin[2x] \right) \operatorname{Tan}\left[\frac{x}{2}\right] \right)}{\left( (-1-i) - 2(-1)^{5/8} + \sqrt{2} \right) \left( 2 - (1+i)\sqrt{2} + 2(-1)^{5/8}(-1+\sqrt{2}) \cos[x] - \sqrt{2} \cos[2x] + 2(-1)^{5/8} \sin[x] + \sqrt{2} \sin[2x] \right)} \right) - \\
& \left. \frac{(1-i) \left( (-1+i) + \sqrt{2} + (1-i)(-1)^{5/8} \sqrt{2} \right) \operatorname{Sec}\left[\frac{x}{2}\right]^2}{1 + \frac{\left(\frac{1-i}{4}\right) \left( -i \left( (1+i) + \sqrt{2} \right) + \left( (1+i) + 2(-1)^{5/8} - \sqrt{2} \right) \operatorname{Tan}\left[\frac{x}{2}\right] \right)^2}{\sqrt{2}}} \right) + \\
& \left( \left( (-2+2i)(-1)^{5/8} x - \left( 2\sqrt{2} \left( (-2-2i) - (3+i)(-1)^{5/8} + (1+i)\sqrt{2} + (2+i)(-1)^{5/8} \sqrt{2} \right) \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTan}\left[ \frac{-\cos[x] + (1+\sqrt{2}) \sin[x]}{2(-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x]} \right] \right) \right) / \left( (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) + \\
& \frac{2(1-i)^{3/2} 2^{3/4} \left( (1+i) - \sqrt{2} + (1+i)(-1)^{3/8} \sqrt{2} \right) \operatorname{ArcTan}\left[ \frac{(1+i) + i\sqrt{2} + (-1+i) + 2(-1)^{3/8} + \sqrt{2}}{\sqrt{1-i}} 2^{3/4} \operatorname{Tan}\left[\frac{x}{2}\right] \right]}{(-1+i) + 2(-1)^{3/8} + \sqrt{2}} - \\
& \left. \frac{(1-i)(-1)^{5/8} \sqrt{2} (-2+\sqrt{2}) \operatorname{Log}\left[\operatorname{Sec}\left[\frac{x}{2}\right]^2\right] + \left( i\sqrt{2} \left( (-2-2i) - (3+i)(-1)^{5/8} + (1+i)\sqrt{2} + (2+i)(-1)^{5/8} \sqrt{2} \right) \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{x}{2}\right]^4 \right. \right. \right. \\
& \left. \left. \left. \left( -2 + (1-i)\sqrt{2} + 2(-1)^{3/8}(-1+\sqrt{2}) \cos[x] + \sqrt{2} \cos[2x] - 2(-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right] \right)}{(-1+i) + 2(-1)^{3/8} + \sqrt{2}} \right) \\
& \left( - \left( \left( \frac{1}{2} + \frac{i}{2} \right) / \left( \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right) \left( -(-1-i)^{3/2} (1-i)^{1/4} (1+i)^{1/4} - (1+i) \cos[x] + i\sqrt{1-i} \sqrt{1+i} \cos[x] + \right. \right. \right. \right. \\
& \left. \left. \left. (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) \right) + \left( \sqrt{-1-i} \sin[x] \right) / \left( (1-i)^{3/4} (1+i)^{3/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right) \right) \\
& \left( -(-1-i)^{3/2} (1-i)^{1/4} (1+i)^{1/4} - (1+i) \cos[x] + i\sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x] \right) - \\
& \left( i\sqrt{-1-i} (1-i)^{3/4} \sin[x] \right) / \left( 2(1+i)^{1/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right) \right) \\
& \left. \left( -(-1-i)^{3/2} (1-i)^{1/4} (1+i)^{1/4} - (1+i) \cos[x] + i\sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) \right) / \\
& \left( (-2+2i)(-1)^{5/8} - \left( 2\sqrt{2} \left( (-2-2i) - (3+i)(-1)^{5/8} + (1+i)\sqrt{2} + (2+i)(-1)^{5/8} \sqrt{2} \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{(1 + \sqrt{2}) \cos[x] + \sin[x]}{2(-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x]} - \frac{(\cos[x] - \sin[x] + \sqrt{2} \sin[x]) (-\cos[x] + (1 + \sqrt{2}) \sin[x])}{(2(-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x])^2} \right) \Bigg/ \\
& \left( \left( (-1 + i) + 2(-1)^{3/8} + \sqrt{2} \right) \left( 1 + \frac{(-\cos[x] + (1 + \sqrt{2}) \sin[x])^2}{(2(-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x])^2} \right) \right) - (1 - i) (-1)^{5/8} \sqrt{2} (-2 + \sqrt{2}) \tan\left[\frac{x}{2}\right] - \\
& \left( i \sqrt{2} \left( (-2 - 2i) - (3 + i) (-1)^{5/8} + (1 + i) \sqrt{2} + (2 + i) (-1)^{5/8} \sqrt{2} \right) \cos\left[\frac{x}{2}\right]^4 \right. \\
& \left. \left( -\sec\left[\frac{x}{2}\right]^4 \left( -2(-1)^{3/8} \cos[x] + 2\sqrt{2} \cos[2x] - 2(-1)^{3/8} (-1 + \sqrt{2}) \sin[x] - 2\sqrt{2} \sin[2x] \right) - \right. \right. \\
& \left. \left. 2 \sec\left[\frac{x}{2}\right]^4 \left( -2 + (1 - i) \sqrt{2} + 2(-1)^{3/8} (-1 + \sqrt{2}) \cos[x] + \sqrt{2} \cos[2x] - 2(-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x] \right) \tan\left[\frac{x}{2}\right] \right) \right) \Bigg/ \\
& \left( \left( (-1 + i) + 2(-1)^{3/8} + \sqrt{2} \right) \left( -2 + (1 - i) \sqrt{2} + 2(-1)^{3/8} (-1 + \sqrt{2}) \cos[x] + \sqrt{2} \cos[2x] - 2(-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right) + \\
& \left. \frac{(1 - i) \left( (1 + i) - \sqrt{2} + (1 + i) (-1)^{3/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2}{1 + \frac{\left(\frac{1+i}{4}\right) \left( (1+i) + i\sqrt{2} + (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) \tan\left[\frac{x}{2}\right]^2}{\sqrt{2}}} \right) + \\
& \left( \left( -2(-1)^{3/8} \left( 1 + (-1)^{1/4} \right) \left( -2 + \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) x - \right. \right. \\
& \left. \left. 2(-1)^{3/8} \left( (4 + 4i) - 2(-1)^{5/8} + 2(-1)^{7/8} - (3 + 3i) \sqrt{2} + 2(-1)^{5/8} \sqrt{2} - 2(-1)^{7/8} \sqrt{2} \right) \right. \right. \\
& \left. \left. \operatorname{ArcTan}\left[ \frac{\sin[x]}{-(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x]} \right] - \right. \right. \\
& \left. \left. (4 + 4i) \left( (-1 + i) + (1 + i) (-1)^{1/8} - (2 + i) (-1)^{3/8} + (1 + 2i) (-1)^{7/8} + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \right. \right. \\
& \left. \left. \operatorname{ArcTan}\left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{5/8} \left( i + (-1)^{3/4} + (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \tan\left[\frac{x}{2}\right] \right) \right] \right) + \right. \\
& \left. 2(-1)^{7/8} \left( (-3 - i) + (2 + i) \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \operatorname{Log}\left[\sec\left[\frac{x}{2}\right]^2\right] + \right. \\
& \left. (-1)^{7/8} \left( (4 + 4i) - 2(-1)^{5/8} + 2(-1)^{7/8} - (3 + 3i) \sqrt{2} + 2(-1)^{5/8} \sqrt{2} - 2(-1)^{7/8} \sqrt{2} \right) \operatorname{Log}\left[\sec\left[\frac{x}{2}\right]^4 \right. \right. \\
& \left. \left. \left( 1 - 3(-1)^{1/4} + 2(-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \cos[x] - \left( -i + (-1)^{1/4} \right) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \right] \right) \Bigg) \\
& \left( - \left( i \Bigg/ \left( \sqrt{1 - i} \left( (-1 + i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \left( -\sqrt{-1 - i} (1 - i)^{3/4} (1 + i)^{1/4} - \sqrt{1 - i} \cos[x] + \sqrt{1 + i} \cos[x] - \right. \right. \right. \right. \\
& \left. \left. \left. i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \right) \right) - 1 \Bigg/ \left( \sqrt{1 + i} \left( (-1 + i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \right. \\
& \left. \left. \left( -\sqrt{-1 - i} (1 - i)^{3/4} (1 + i)^{1/4} - \sqrt{1 - i} \cos[x] + \sqrt{1 + i} \cos[x] - i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 2 \operatorname{Sin}[x] \right) / \left( \sqrt{-1-i} (1-i)^{1/4} (1+i)^{3/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \left( -\sqrt{-1-i} (1-i)^{3/4} (1+i)^{1/4} - \sqrt{1-i} \operatorname{Cos}[x] + \right. \right. \\
& \quad \left. \left. \sqrt{1+i} \operatorname{Cos}[x] - i \sqrt{1-i} \operatorname{Sin}[x] - i \sqrt{1+i} \operatorname{Sin}[x] \right) \right) - \left( 2 i (1+i)^{3/4} \operatorname{Sin}[x] \right) / \left( \sqrt{-1-i} (1-i)^{3/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \right. \\
& \quad \left. \left( -\sqrt{-1-i} (1-i)^{3/4} (1+i)^{1/4} - \sqrt{1-i} \operatorname{Cos}[x] + \sqrt{1+i} \operatorname{Cos}[x] - i \sqrt{1-i} \operatorname{Sin}[x] - i \sqrt{1+i} \operatorname{Sin}[x] \right) \right) \Bigg) / \\
& \left( -2 (-1)^{3/8} \left( 1 + (-1)^{1/4} \right) \left( -2 + \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) - \right. \\
& \left( 2 (-1)^{3/8} \left( (4+4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - (3+3i) \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} - 2 (-1)^{7/8} \sqrt{2} \right) \right. \\
& \quad \left. \left( \frac{\operatorname{Sin}[x] \left( (-1)^{3/4} \operatorname{Cos}[x] - \operatorname{Sin}[x] + (-1)^{1/4} \operatorname{Sin}[x] \right)}{\left( -(-1)^{5/8} \sqrt{2} + \operatorname{Cos}[x] - (-1)^{1/4} \operatorname{Cos}[x] + (-1)^{3/4} \operatorname{Sin}[x] \right)^2} + \frac{\operatorname{Cos}[x]}{-(-1)^{5/8} \sqrt{2} + \operatorname{Cos}[x] - (-1)^{1/4} \operatorname{Cos}[x] + (-1)^{3/4} \operatorname{Sin}[x]} \right) \right) / \\
& \quad \left( 1 + \frac{\operatorname{Sin}[x]^2}{\left( -(-1)^{5/8} \sqrt{2} + \operatorname{Cos}[x] - (-1)^{1/4} \operatorname{Cos}[x] + (-1)^{3/4} \operatorname{Sin}[x] \right)^2} \right) + \\
& 2 (-1)^{7/8} \left( (-3-i) + (2+i) \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \operatorname{Tan}\left[\frac{x}{2}\right] + \\
& \left( (-1)^{7/8} \left( (4+4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - (3+3i) \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} - 2 (-1)^{7/8} \sqrt{2} \right) \operatorname{Cos}\left[\frac{x}{2}\right]^4 \right. \\
& \quad \left( \operatorname{Sec}\left[\frac{x}{2}\right]^4 \left( 2 (-1)^{3/8} \sqrt{2} \operatorname{Cos}[x] + 2 \operatorname{Cos}[2x] + 2 (-1)^{3/4} \operatorname{Cos}[2x] - 2 (-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \operatorname{Sin}[x] + 2 \left( -i + (-1)^{1/4} \right) \operatorname{Sin}[2x] \right) + \right. \\
& \quad \left. 2 \operatorname{Sec}\left[\frac{x}{2}\right]^4 \left( 1 - 3 (-1)^{1/4} + 2 (-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \operatorname{Cos}[x] - \right. \right. \\
& \quad \left. \left. \left( -i + (-1)^{1/4} \right) \operatorname{Cos}[2x] + 2 (-1)^{3/8} \sqrt{2} \operatorname{Sin}[x] + \operatorname{Sin}[2x] + (-1)^{3/4} \operatorname{Sin}[2x] \right) \operatorname{Tan}\left[\frac{x}{2}\right] \right) \Bigg) / \\
& \left( 1 - 3 (-1)^{1/4} + 2 (-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \operatorname{Cos}[x] - \left( -i + (-1)^{1/4} \right) \operatorname{Cos}[2x] + 2 (-1)^{3/8} \sqrt{2} \operatorname{Sin}[x] + \operatorname{Sin}[2x] + (-1)^{3/4} \operatorname{Sin}[2x] \right) + \\
& \left( 2 (-1)^{1/8} \left( -1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2} \right) \left( (-1+i) + (1+i) (-1)^{1/8} - (2+i) (-1)^{3/8} + (1+2i) (-1)^{7/8} + \sqrt{2} + (1-i) (-1)^{5/8} \sqrt{2} \right) \operatorname{Sec}\left[\frac{x}{2}\right]^2 \Bigg) / \\
& \left( 1 - \frac{1}{2} (-1)^{3/4} \left( i + (-1)^{3/4} + \left( -1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2} \right) \operatorname{Tan}\left[\frac{x}{2}\right] \right)^2 \right) \Bigg) + \\
& \left( \left( -2 (-1)^{3/8} \left( 1 + (-1)^{1/4} \right) \left( -2 + \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) x + 2 (-1)^{3/8} \left( (4+4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (3 + 3i) \sqrt{2} + 2(-1)^{5/8} \sqrt{2} - 2(-1)^{7/8} \sqrt{2} \Big) \operatorname{ArcTan} \left[ \frac{\operatorname{Sin}[x]}{(-1 + (-1)^{1/4}) \operatorname{Cos}[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \operatorname{Sin}[x])} \right] + \\
& (4 + 4i) \left( (-1 + i) + (1 + i) (-1)^{1/8} - (2 + i) (-1)^{3/8} + (1 + 2i) (-1)^{7/8} + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \\
& \operatorname{ArcTan} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{5/8} \left( i + (-1)^{3/4} + \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \operatorname{Tan} \left[ \frac{x}{2} \right] \right) \right] + \\
& 2(-1)^{7/8} \left( (3 + i) - (2 + i) \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \operatorname{Log} \left[ \operatorname{Sec} \left[ \frac{x}{2} \right]^2 \right] - \\
& (-1)^{7/8} \left( (4 + 4i) - 2(-1)^{5/8} + 2(-1)^{7/8} - (3 + 3i) \sqrt{2} + 2(-1)^{5/8} \sqrt{2} - 2(-1)^{7/8} \sqrt{2} \right) \operatorname{Log} \left[ -\operatorname{Sec} \left[ \frac{x}{2} \right]^4 \right. \\
& \left. \left. (-1 + 3(-1)^{1/4} - 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \operatorname{Cos}[x] + (-i + (-1)^{1/4}) \operatorname{Cos}[2x] + 2(-1)^{3/8} \sqrt{2} \operatorname{Sin}[x] + \operatorname{Sin}[2x] + (-1)^{3/4} \operatorname{Sin}[2x] \right) \right] \Big) \\
& \left( - \left( 1 / \left( \sqrt{1-i} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \left( \sqrt{-1+i} (1-i)^{1/4} (1+i)^{3/4} - \sqrt{1-i} \operatorname{Cos}[x] + \sqrt{1+i} \operatorname{Cos}[x] + \right. \right. \right. \right. \\
& \left. \left. \left. i \sqrt{1-i} \operatorname{Sin}[x] + i \sqrt{1+i} \operatorname{Sin}[x] \right) \right) \right) + i / \left( \sqrt{1+i} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \right. \\
& \left. \left( \sqrt{-1+i} (1-i)^{1/4} (1+i)^{3/4} - \sqrt{1-i} \operatorname{Cos}[x] + \sqrt{1+i} \operatorname{Cos}[x] + i \sqrt{1-i} \operatorname{Sin}[x] + i \sqrt{1+i} \operatorname{Sin}[x] \right) \right) - \\
& \left( 2i (1-i)^{3/4} \operatorname{Sin}[x] \right) / \left( \sqrt{-1+i} (1+i)^{3/4} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \left( \sqrt{-1+i} (1-i)^{1/4} (1+i)^{3/4} - \sqrt{1-i} \operatorname{Cos}[x] + \right. \right. \\
& \left. \left. \sqrt{1+i} \operatorname{Cos}[x] + i \sqrt{1-i} \operatorname{Sin}[x] + i \sqrt{1+i} \operatorname{Sin}[x] \right) \right) + (2 \operatorname{Sin}[x]) / \left( \sqrt{-1+i} (1-i)^{3/4} (1+i)^{1/4} \left( (-1-i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \right. \\
& \left. \left. \left( \sqrt{-1+i} (1-i)^{1/4} (1+i)^{3/4} - \sqrt{1-i} \operatorname{Cos}[x] + \sqrt{1+i} \operatorname{Cos}[x] + i \sqrt{1-i} \operatorname{Sin}[x] + i \sqrt{1+i} \operatorname{Sin}[x] \right) \right) \right) \Big) / \\
& \left( -2(-1)^{3/8} \left( 1 + (-1)^{1/4} \right) \left( -2 + \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) + \right. \\
& \left. \left( 2(-1)^{3/8} \left( (4 + 4i) - 2(-1)^{5/8} + 2(-1)^{7/8} - (3 + 3i) \sqrt{2} + 2(-1)^{5/8} \sqrt{2} - 2(-1)^{7/8} \sqrt{2} \right) \right. \right. \\
& \left. \left. \left( - \frac{\operatorname{Sin}[x] \left( (-1)^{3/4} \operatorname{Cos}[x] - (-1 + (-1)^{1/4}) \operatorname{Sin}[x] \right)}{\left( (-1 + (-1)^{1/4}) \operatorname{Cos}[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \operatorname{Sin}[x]) \right)^2} + \frac{\operatorname{Cos}[x]}{(-1 + (-1)^{1/4}) \operatorname{Cos}[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \operatorname{Sin}[x])} \right) \right) \right) / \\
& \left( 1 + \frac{\operatorname{Sin}[x]^2}{\left( (-1 + (-1)^{1/4}) \operatorname{Cos}[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \operatorname{Sin}[x]) \right)^2} \right) + \\
& 2(-1)^{7/8} \left( (3 + i) - (2 + i) \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \operatorname{Tan} \left[ \frac{x}{2} \right] +
\end{aligned}$$

$$\left( (-1)^{7/8} \left( (4 + 4i) - 2(-1)^{5/8} + 2(-1)^{7/8} - (3 + 3i) \sqrt{2} + 2(-1)^{5/8} \sqrt{2} - 2(-1)^{7/8} \sqrt{2} \right) \cos\left[\frac{x}{2}\right]^4 \right. \\ \left. \left( -\sec\left[\frac{x}{2}\right]^4 \left( 2(-1)^{3/8} \sqrt{2} \cos[x] + 2 \cos[2x] + 2(-1)^{3/4} \cos[2x] + 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \sin[x] - 2(-i + (-1)^{1/4}) \sin[2x] \right) - \right. \right. \\ \left. \left. 2 \sec\left[\frac{x}{2}\right]^4 \left( -1 + 3(-1)^{1/4} - 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] + \right. \right. \right. \\ \left. \left. \left. (-i + (-1)^{1/4}) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \tan\left[\frac{x}{2}\right] \right) \right) / \\ \left( -1 + 3(-1)^{1/4} - 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] + (-i + (-1)^{1/4}) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) - \\ \left( 2(-1)^{1/8} \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \left( (-1 + i) + (1 + i) (-1)^{1/8} - (2 + i) (-1)^{3/8} + (1 + 2i) (-1)^{7/8} + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2 \right) / \\ \left. \left( 1 - \frac{1}{2} (-1)^{3/4} \left( i + (-1)^{3/4} + \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \tan\left[\frac{x}{2}\right] \right)^2 \right) \right)$$

**Problem 108: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \tan[5x] dx$$

Optimal (type 3, 84 leaves, 6 steps):

$$\frac{1}{5} \sqrt{\frac{1}{2} (5 + \sqrt{5})} \operatorname{ArcTanh}\left[2 \sqrt{\frac{2}{5 + \sqrt{5}}} \cos[x]\right] + \frac{1}{5} \sqrt{\frac{1}{2} (5 - \sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{5} (5 + \sqrt{5})} \cos[x]\right] - \cos[x]$$

Result (type 3, 215 leaves):

$$\frac{(1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{4 - (-1 + \sqrt{5}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} (5 + \sqrt{5})}\right]}{\sqrt{10} (5 + \sqrt{5})} + \frac{(1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{4 + (-1 + \sqrt{5}) \tan\left[\frac{x}{2}\right]}{\sqrt{2} (5 + \sqrt{5})}\right]}{\sqrt{10} (5 + \sqrt{5})} + \\ \frac{(-1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{4 - (1 + \sqrt{5}) \tan\left[\frac{x}{2}\right]}{\sqrt{10 - 2\sqrt{5}}}\right]}{\sqrt{50 - 10\sqrt{5}}} + \frac{(-1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{4 + (1 + \sqrt{5}) \tan\left[\frac{x}{2}\right]}{\sqrt{10 - 2\sqrt{5}}}\right]}{\sqrt{50 - 10\sqrt{5}}} - \cos[x]$$

**Problem 109: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[x] \tan[6x] dx$$

Optimal (type 3, 89 leaves, 10 steps):

$$\frac{\text{ArcTanh}[\sqrt{2} \cos[x]]}{3\sqrt{2}} + \frac{1}{6} \sqrt{2-\sqrt{3}} \text{ArcTanh}\left[\frac{2\cos[x]}{\sqrt{2-\sqrt{3}}}\right] + \frac{1}{6} \sqrt{2+\sqrt{3}} \text{ArcTanh}\left[\frac{2\cos[x]}{\sqrt{2+\sqrt{3}}}\right] - \cos[x]$$

Result (type 3, 776 leaves):

$$\begin{aligned} & \left(-\frac{1}{6} - \frac{i}{6}\right) (-1)^{1/4} \text{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{1/4} \text{Sec}\left[\frac{x}{2}\right] \left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right)\right] + \left(\frac{1}{6} + \frac{i}{6}\right) (-1)^{3/4} \text{ArcTanh}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{3/4} \text{Sec}\left[\frac{x}{2}\right] \left(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right)\right] - \\ & \frac{x + 2\sqrt{3} \text{ArcTanh}\left[\frac{\sqrt{2} + (-1+\sqrt{2}) \tan\left[\frac{x}{2}\right]}{\sqrt{3}}\right] - \text{Log}\left[\text{Sec}\left[\frac{x}{2}\right]^2\right] + \text{Log}\left[\text{Sec}\left[\frac{x}{2}\right]^2 \left(1 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]\right)\right]}{12\sqrt{2}} + \\ & \left( (1 + \sqrt{2}) \left( x - 2\sqrt{3} \text{ArcTanh}\left[\frac{2 + (2 + \sqrt{2}) \tan\left[\frac{x}{2}\right]}{\sqrt{6}}\right] - \text{Log}\left[\text{Sec}\left[\frac{x}{2}\right]^2\right] + \text{Log}\left[-\text{Sec}\left[\frac{x}{2}\right]^2 \left(\sqrt{2} - 2\cos[x] + 2\sin[x]\right)\right] \right) \right. \\ & \quad \left. (2 + \sqrt{2} \sin[x]) \left(1 + \sqrt{2} - (2 + \sqrt{2}) \cos[x] + (2 + \sqrt{2}) \sin[x]\right) \right) / \\ & \left( 12 \left( -12 - 9\sqrt{2} + 4(3 + 2\sqrt{2}) \cos[x] + (4 + 3\sqrt{2}) \cos[2x] - 18 \sin[x] - 12\sqrt{2} \sin[x] + 4 \sin[2x] + 3\sqrt{2} \sin[2x] \right) - \right. \\ & \left( \left( 2(-2 + \sqrt{6}) \text{ArcTanh}\left[\sqrt{2} + (\sqrt{2} - \sqrt{3}) \tan\left[\frac{x}{2}\right]\right] + (3\sqrt{2} - 2\sqrt{3}) \left( x - \text{Log}\left[\text{Sec}\left[\frac{x}{2}\right]^2\right] + \text{Log}\left[-\text{Sec}\left[\frac{x}{2}\right]^2 \left(\sqrt{3} + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]\right)\right] \right) \right) \right. \\ & \quad \left. (\sqrt{2} - \sqrt{3} \sin[x]) \left(-3 + \sqrt{6} - (-2 + \sqrt{6}) \cos[x] + (-2 + \sqrt{6}) \sin[x]\right) \right) / \\ & \left( 12 \left( -36 + 15\sqrt{6} + (20 - 8\sqrt{6}) \cos[x] + (12 - 5\sqrt{6}) \cos[2x] - 50 \sin[x] + 20\sqrt{6} \sin[x] + 12 \sin[2x] - 5\sqrt{6} \sin[2x] \right) \right) + \\ & \left( \left( -2(\sqrt{2} + \sqrt{3}) \text{ArcTanh}\left[\frac{2 + (2 + \sqrt{6}) \tan\left[\frac{x}{2}\right]}{\sqrt{2}}\right] + (3 + \sqrt{6}) \left( x - \text{Log}\left[\text{Sec}\left[\frac{x}{2}\right]^2\right] + \text{Log}\left[-\text{Sec}\left[\frac{x}{2}\right]^2 \left(\sqrt{6} - 2\cos[x] + 2\sin[x]\right)\right] \right) \right) \right. \\ & \quad \left. (2 + \sqrt{6} \sin[x]) \left(3 + \sqrt{6} - (2 + \sqrt{6}) \cos[x] + (2 + \sqrt{6}) \sin[x]\right) \right) / \\ & \left( 12 \left( -36 - 15\sqrt{6} + 4(5 + 2\sqrt{6}) \cos[x] + (12 + 5\sqrt{6}) \cos[2x] - 50 \sin[x] - 20\sqrt{6} \sin[x] + 12 \sin[2x] + 5\sqrt{6} \sin[2x] \right) \right) \end{aligned}$$

**Problem 110: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \cot[2x] dx$$

Optimal (type 3, 10 leaves, 4 steps):

$$-\frac{1}{2} \text{ArcTanh}[\text{Cos}[x]] + \text{Cos}[x]$$

Result (type 3, 25 leaves):

$$\text{Cos}[x] - \frac{1}{2} \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \frac{1}{2} \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

**Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \text{Cos}[x] \text{Cot}[4x] dx$$

Optimal (type 3, 28 leaves, 6 steps):

$$-\frac{1}{4} \text{ArcTanh}[\text{Cos}[x]] - \frac{\text{ArcTanh}\left[\frac{\sqrt{2} \text{Cos}[x]}{2}\right]}{2\sqrt{2}} + \text{Cos}[x]$$

Result (type 3, 73 leaves):

$$\frac{1}{4} \left( (-1 - i) (-1)^{3/4} \text{ArcTanh}\left[\frac{-1 + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right] - (1 - i) (-1)^{1/4} \text{ArcTanh}\left[\frac{1 + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{2}}\right] + 4 \text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \right)$$

**Problem 114: Result more than twice size of optimal antiderivative.**

$$\int \text{Cos}[x] \text{Cot}[6x] dx$$

Optimal (type 3, 38 leaves, 7 steps):

$$-\frac{1}{6} \text{ArcTanh}[\text{Cos}[x]] - \frac{1}{6} \text{ArcTanh}[2 \text{Cos}[x]] - \frac{\text{ArcTanh}\left[\frac{2 \text{Cos}[x]}{\sqrt{3}}\right]}{2\sqrt{3}} + \text{Cos}[x]$$

Result (type 3, 87 leaves):

$$\frac{1}{12} \left( 2\sqrt{3} \text{ArcTanh}\left[\frac{-2 + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{3}}\right] - 2\sqrt{3} \text{ArcTanh}\left[\frac{2 + \text{Tan}\left[\frac{x}{2}\right]}{\sqrt{3}}\right] + 12 \text{Cos}[x] - 2 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}[1 - 2 \text{Cos}[x]] - \text{Log}[1 + 2 \text{Cos}[x]] + 2 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] \right)$$

**Problem 116: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[x] \operatorname{Sec}[2x] dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{\sqrt{2}}$$

Result (type 3, 174 leaves):

$$\frac{1}{4\sqrt{2}} \left( -2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] - 2i \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] + \right. \\ \left. 2 \operatorname{Log}[\sqrt{2} + 2 \sin[x]] - \operatorname{Log}[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] - \operatorname{Log}[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] \right)$$

**Problem 118: Result is not expressed in closed-form.**

$$\int \cos[x] \operatorname{Sec}[4x] dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{2\sin[x]}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2(2-\sqrt{2})}} - \frac{\operatorname{ArcTanh}\left[\frac{2\sin[x]}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2(2+\sqrt{2})}}$$

Result (type 7, 91 leaves):

$$\frac{1}{16} \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{1}{\#1^5} \left( 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] - i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] + 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^2 - i \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^2 \right) \& \right]$$

**Problem 120: Result is not expressed in closed-form.**

$$\int \cos[x] \operatorname{Sec}[6x] dx$$

Optimal (type 3, 85 leaves, 7 steps):



$$-\frac{\text{ArcTanh}[\sqrt{2} \sin[x]]}{3\sqrt{2}} + \frac{\text{ArcTanh}\left[\frac{2\sin[x]}{\sqrt{2-\sqrt{3}}}\right]}{6\sqrt{2-\sqrt{3}}} + \frac{\text{ArcTanh}\left[\frac{2\sin[x]}{\sqrt{2+\sqrt{3}}}\right]}{6\sqrt{2+\sqrt{3}}}$$

Result (type 7, 356 leaves):

$$\frac{1}{24} \left( \sqrt{2} \left( 2 \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] + 2 \operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]} \right] - \right. \right. \\ \left. \left. 2 \operatorname{Log}[\sqrt{2} + 2 \sin[x]] + \operatorname{Log}[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + \operatorname{Log}[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] \right) + \operatorname{RootSum}[1 - \#1^4 + \#1^8 \&, \right. \\ \left. \frac{1}{-\#1^3 + 2 \#1^7} \left( 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] - \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] + 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^2 - \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^2 + \right. \right. \\ \left. \left. 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^4 - \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^4 + 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^6 - \operatorname{Log}[1 - 2 \cos[x] \#1 + \#1^2] \#1^6 \right) \& \right)$$

**Problem 121: Result more than twice size of optimal antiderivative.**

$$\int \cos[2x] \sec[x] dx$$

Optimal (type 3, 10 leaves, 3 steps):

$$-\text{ArcTanh}[\sin[x]] + 2 \sin[x]$$

Result (type 3, 37 leaves):

$$\operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + 2 \sin[x]$$

**Problem 123: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \csc[2x] dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$-\frac{1}{2} \text{ArcTanh}[\cos[x]]$$

Result (type 3, 21 leaves):

$$\frac{1}{2} \left( -\text{Log} \left[ \text{Cos} \left[ \frac{x}{2} \right] \right] + \text{Log} \left[ \text{Sin} \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 125:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \text{Cos}[x] \text{Csc}[4x] dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4} \text{ArcTanh}[\text{Cos}[x]] + \frac{\text{ArcTanh}[\sqrt{2} \text{Cos}[x]]}{2\sqrt{2}}$$

Result (type 3, 66 leaves):

$$\frac{1}{4} \left( (1+i) (-1)^{3/4} \text{ArcTanh} \left[ \frac{-1 + \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2}} \right] + \sqrt{2} \text{ArcTanh} \left[ \frac{1 + \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2}} \right] - \text{Log} \left[ \text{Cos} \left[ \frac{x}{2} \right] \right] + \text{Log} \left[ \text{Sin} \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 127:** Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[x] \text{Csc}[6x] dx$$

Optimal (type 3, 36 leaves, 7 steps):

$$-\frac{1}{6} \text{ArcTanh}[\text{Cos}[x]] - \frac{1}{6} \text{ArcTanh}[2 \text{Cos}[x]] + \frac{\text{ArcTanh} \left[ \frac{2 \text{Cos}[x]}{\sqrt{3}} \right]}{2\sqrt{3}}$$

Result (type 3, 83 leaves):

$$\frac{1}{12} \left( -2\sqrt{3} \text{ArcTanh} \left[ \frac{-2 + \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] + 2\sqrt{3} \text{ArcTanh} \left[ \frac{2 + \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] - 2 \text{Log} \left[ \text{Cos} \left[ \frac{x}{2} \right] \right] + \text{Log} [1 - 2 \text{Cos}[x]] - \text{Log} [1 + 2 \text{Cos}[x]] + 2 \text{Log} \left[ \text{Sin} \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 174:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a - a \text{Sin}[e + f x]} (c + c \text{Sin}[e + f x])^{3/2}}{x} dx$$

Optimal (type 4, 186 leaves, 11 steps):

$$\begin{aligned}
& c \operatorname{CosIntegral}[f x] \operatorname{Sec}[e+f x] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} + \\
& \frac{1}{2} c \operatorname{CosIntegral}[2 f x] \operatorname{Sec}[e+f x] \operatorname{Sin}[2 e] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} - \\
& c \operatorname{Sec}[e+f x] \operatorname{Sin}[e] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} \operatorname{SinIntegral}[f x] + \\
& \frac{1}{2} c \operatorname{Cos}[2 e] \operatorname{Sec}[e+f x] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} \operatorname{SinIntegral}[2 f x]
\end{aligned}$$

Result (type 4, 150 leaves):

$$\begin{aligned}
& \frac{1}{2 \sqrt{2} \left(1 + e^{2 i (e+f x)}\right)} c e^{-i (e-f x)} \sqrt{-i c e^{-i (e+f x)} \left(i + e^{i (e+f x)}\right)^2} \\
& \left(2 e^{i e} \operatorname{ExpIntegralEi}[-i f x] + 2 e^{3 i e} \operatorname{ExpIntegralEi}[i f x] + i \left(\operatorname{ExpIntegralEi}[-2 i f x] - e^{4 i e} \operatorname{ExpIntegralEi}[2 i f x]\right)\right) \sqrt{a-a \operatorname{Sin}[e+f x]}
\end{aligned}$$

Problem 175: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a-a \operatorname{Sin}[e+f x]} (c+c \operatorname{Sin}[e+f x])^{3/2}}{x^2} dx$$

Optimal (type 4, 273 leaves, 13 steps):

$$\begin{aligned}
& -\frac{c \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]}}{x} + c f \operatorname{Cos}[2 e] \operatorname{CosIntegral}[2 f x] \operatorname{Sec}[e+f x] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} - \\
& \frac{c f \operatorname{CosIntegral}[f x] \operatorname{Sec}[e+f x] \operatorname{Sin}[e] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} -}{2 x} - \\
& \frac{c \operatorname{Sec}[e+f x] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} \operatorname{Sin}[2 e+2 f x]}{2 x} - \\
& c f \operatorname{Cos}[e] \operatorname{Sec}[e+f x] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} \operatorname{SinIntegral}[f x] - \\
& c f \operatorname{Sec}[e+f x] \operatorname{Sin}[2 e] \sqrt{a-a \operatorname{Sin}[e+f x]} \sqrt{c+c \operatorname{Sin}[e+f x]} \operatorname{SinIntegral}[2 f x]
\end{aligned}$$

Result (type 4, 231 leaves):

$$\begin{aligned}
& \frac{1}{2 \sqrt{2} \left(1 + e^{2 i (e+f x)}\right) x} \\
& c e^{-i (e+f x)} \sqrt{-i c e^{-i (e+f x)} \left(i + e^{i (e+f x)}\right)^2} \left(-i - 2 e^{i (e+f x)} - 2 e^{3 i (e+f x)} + i e^{4 i (e+f x)} - 2 i e^{i (e+2 f x)} f x \operatorname{ExpIntegralEi}[-i f x] + \right. \\
& \left. 2 i e^{3 i e+2 i f x} f x \operatorname{ExpIntegralEi}[i f x] + 2 e^{2 i f x} f x \operatorname{ExpIntegralEi}[-2 i f x] + 2 e^{2 i (2 e+f x)} f x \operatorname{ExpIntegralEi}[2 i f x]\right) \sqrt{a-a \operatorname{Sin}[e+f x]}
\end{aligned}$$

Problem 176: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a-a \operatorname{Sin}[e+f x]} (c+c \operatorname{Sin}[e+f x])^{3/2}}{x^3} dx$$

Optimal (type 4, 385 leaves, 15 steps):

$$\frac{c \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}}{2 x^2} - \frac{c f \cos[2 e + 2 f x] \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}}{2 x} -$$

$$\frac{1}{2} c f^2 \cos[e] \operatorname{CosIntegral}[f x] \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} -$$

$$c f^2 \operatorname{CosIntegral}[2 f x] \operatorname{Sec}[e + f x] \sin[2 e] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} -$$

$$\frac{c \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \sin[2 e + 2 f x]}{4 x^2} +$$

$$\frac{1}{2} c f^2 \operatorname{Sec}[e + f x] \sin[e] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \operatorname{SinIntegral}[f x] -$$

$$c f^2 \cos[2 e] \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \operatorname{SinIntegral}[2 f x] + \frac{c f \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \tan[e + f x]}{2 x}$$

Result (type 4, 317 leaves):

$$\frac{1}{4 \sqrt{2} (-i + e^{i(e+fx)}) \sqrt{-i c e^{-i(e+fx)} (i + e^{i(e+fx)})^2 x^2}} c^2 e^{-2i(e+fx)} (i + e^{i(e+fx)})$$

$$\left( -1 + 2i e^{i(e+fx)} + 2i e^{3i(e+fx)} + e^{4i(e+fx)} + 2i f x + 2 e^{i(e+fx)} f x - 2 e^{3i(e+fx)} f x + 2i e^{4i(e+fx)} f x + 2i e^{i(e+2fx)} f^2 x^2 \operatorname{ExpIntegralEi}[-i f x] + \right.$$

$$\left. 2i e^{3i e+2i f x} f^2 x^2 \operatorname{ExpIntegralEi}[i f x] - 4 e^{2i f x} f^2 x^2 \operatorname{ExpIntegralEi}[-2i f x] + 4 e^{2i(2e+fx)} f^2 x^2 \operatorname{ExpIntegralEi}[2i f x] \right) \sqrt{a - a \sin[e + f x]}$$

**Problem 182: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 \sqrt{a - a \sin[e + f x]}}{(c + c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 280 leaves, 34 steps):

$$-\frac{2 a x}{c f^2 \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} + \frac{2 a \operatorname{ArcTanh}[\sin[e + f x]] \cos[e + f x]}{c f^3 \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} + \frac{2 a \cos[e + f x] \log[\cos[e + f x]]}{c f^3 \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} -$$

$$\frac{a x^2 \operatorname{Sec}[e + f x]}{c f \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} + \frac{2 a x \sin[e + f x]}{c f^2 \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} + \frac{a x^2 \tan[e + f x]}{c f \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}}$$

Result (type 3, 178 leaves):

$$-\left( \left( \left( \cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \sqrt{a - a \sin[e + f x]} \right. \right.$$

$$\left. \left( 2i f x + f^2 x^2 + 2 f x \cos[e + f x] - 2 \log[1 + e^{2i(e+fx)}] + 2i f x \sin[e + f x] - 2 \log[1 + e^{2i(e+fx)}] \sin[e + f x] + \right. \right.$$

$$\left. \left. 4i \operatorname{ArcTan}[e^{i(e+fx)}] (1 + \sin[e + f x]) \right) \right) / \left( f^3 \left( \cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) (c (1 + \sin[e + f x]))^{3/2} \right)$$

### Problem 185: Result more than twice size of optimal antiderivative.

$$\int (a + a \cos [x]) (A + B \sec [x]) dx$$

Optimal (type 3, 18 leaves, 5 steps):

$$a (A + B) x + a B \operatorname{ArcTanh}[\sin [x]] + a A \sin [x]$$

Result (type 3, 51 leaves):

$$a A x + a B x - a B \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + a B \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + a A \sin [x]$$

### Problem 189: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \sec [x]}{a + a \cos [x]} dx$$

Optimal (type 3, 25 leaves, 4 steps):

$$\frac{B \operatorname{ArcTanh}[\sin [x]]}{a} + \frac{(A - B) \sin [x]}{a + a \cos [x]}$$

Result (type 3, 71 leaves):

$$\frac{2 \cos\left[\frac{x}{2}\right] \left( B \cos\left[\frac{x}{2}\right] \left( \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) + (-A + B) \sin\left[\frac{x}{2}\right] \right)}{a (1 + \cos [x])}$$

### Problem 193: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + a \cos [x])^{5/2} (A + B \sec [x]) dx$$

Optimal (type 3, 98 leaves, 6 steps):

$$2 a^{5/2} B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [x]}{\sqrt{a + a \cos [x]}}\right] + \frac{2 a^3 (32 A + 35 B) \sin [x]}{15 \sqrt{a + a \cos [x]}} + \frac{2}{15} a^2 (8 A + 5 B) \sqrt{a + a \cos [x]} \sin [x] + \frac{2}{5} a A (a + a \cos [x])^{3/2} \sin [x]$$

Result (type 3, 283 leaves):

$$\frac{1}{60} a^2 \sqrt{a(1+\cos[x])} \operatorname{Sec}\left[\frac{x}{2}\right] \left( -30i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (-1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] - \right.$$

$$30i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(-1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] + 30\sqrt{2} B \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{x}{2}\right]\right] - 15\sqrt{2} B \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] - \right.$$

$$\left. 15\sqrt{2} B \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] + 300A \sin\left[\frac{x}{2}\right] + 300B \sin\left[\frac{x}{2}\right] + 50A \sin\left[\frac{3x}{2}\right] + 20B \sin\left[\frac{3x}{2}\right] + 6A \sin\left[\frac{5x}{2}\right] \right)$$

**Problem 194:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + a \cos[x])^{3/2} (A + B \sec[x]) dx$$

Optimal (type 3, 72 leaves, 5 steps):

$$2a^{3/2} B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{a+a\cos[x]}}\right] + \frac{2a^2(4A+3B)\sin[x]}{3\sqrt{a+a\cos[x]}} + \frac{2}{3} a A \sqrt{a+a\cos[x]} \sin[x]$$

Result (type 3, 263 leaves):

$$\frac{1}{12} a \sqrt{a(1+\cos[x])} \operatorname{Sec}\left[\frac{x}{2}\right] \left( -6i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (-1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] - 6i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(-1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] + 6\sqrt{2} B \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{x}{2}\right]\right] - \right.$$

$$\left. 3\sqrt{2} B \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] - 3\sqrt{2} B \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] + 36A \sin\left[\frac{x}{2}\right] + 24B \sin\left[\frac{x}{2}\right] + 4A \sin\left[\frac{3x}{2}\right] \right)$$

**Problem 195:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{a+a\cos[x]} (A + B \sec[x]) dx$$

Optimal (type 3, 44 leaves, 4 steps):

$$2\sqrt{a} B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{a+a\cos[x]}}\right] + \frac{2aA \sin[x]}{\sqrt{a+a\cos[x]}}$$

Result (type 3, 244 leaves):

$$\frac{1}{4} \sqrt{a(1+\cos[x])} \operatorname{Sec}\left[\frac{x}{2}\right] \left( -2i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (-1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] - 2i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(-1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] + \right. \\ \left. 2\sqrt{2} B \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{x}{2}\right]\right] - \sqrt{2} B \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] - \sqrt{2} B \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] + 8A\sin\left[\frac{x}{2}\right] \right)$$

**Problem 196:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[x]}{\sqrt{a + a \cos[x]}} dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{2B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{a+a \cos[x]}}\right]}{\sqrt{a}} + \frac{\sqrt{2}(A-B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{2}\sqrt{a+a \cos[x]}}\right]}{\sqrt{a}}$$

Result (type 3, 307 leaves):

$$\frac{1}{2\sqrt{a(1+\cos[x])}} \\ \cos\left[\frac{x}{2}\right] \left( -2i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (-1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] - 2i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(-1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] - 4A \operatorname{Log}\left[\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]\right] + \right. \\ \left. 4B \operatorname{Log}\left[\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]\right] + 4A \operatorname{Log}\left[\cos\left[\frac{x}{4}\right] + \sin\left[\frac{x}{4}\right]\right] - 4B \operatorname{Log}\left[\cos\left[\frac{x}{4}\right] + \sin\left[\frac{x}{4}\right]\right] + \right. \\ \left. 2\sqrt{2} B \operatorname{Log}\left[\sqrt{2} + 2\sin\left[\frac{x}{2}\right]\right] - \sqrt{2} B \operatorname{Log}\left[2 - \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] - \sqrt{2} B \operatorname{Log}\left[2 + \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] \right)$$

**Problem 197:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Sec}[x]}{(a + a \cos[x])^{3/2}} dx$$

Optimal (type 3, 92 leaves, 7 steps):

$$\frac{2B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{a+a \cos[x]}}\right]}{a^{3/2}} + \frac{(A-5B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{2}\sqrt{a+a \cos[x]}}\right]}{2\sqrt{2}a^{3/2}} + \frac{(A-B)\sin[x]}{2(a+a \cos[x])^{3/2}}$$

Result (type 3, 524 leaves):

$$\frac{1}{4 a \sqrt{a (1 + \cos [x])}} \operatorname{Sec} \left[ \frac{x}{2} \right]$$

$$\left( -4 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - (-1 + \sqrt{2}) \sin \left[ \frac{x}{4} \right]}{(1 + \sqrt{2}) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] \cos \left[ \frac{x}{2} \right]^2 - 4 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - (1 + \sqrt{2}) \sin \left[ \frac{x}{4} \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] \cos \left[ \frac{x}{2} \right]^2 - A \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right] + \right.$$

$$5 B \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right] - A \cos [x] \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right] + 5 B \cos [x] \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right] + A \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right] -$$

$$5 B \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right] + A \cos [x] \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right] - 5 B \cos [x] \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right] + 2 \sqrt{2} B \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{x}{2} \right] \right] +$$

$$2 \sqrt{2} B \cos [x] \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{x}{2} \right] \right] - \sqrt{2} B \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] - \sqrt{2} B \cos [x] \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] -$$

$$\left. \sqrt{2} B \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] - \sqrt{2} B \cos [x] \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] + 2 A \sin \left[ \frac{x}{2} \right] - 2 B \sin \left[ \frac{x}{2} \right] \right)$$

**Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Sec} [x]}{(a + a \cos [x])^{5/2}} dx$$

Optimal (type 3, 120 leaves, 8 steps):

$$\frac{2 B \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [x]}{\sqrt{a + a \cos [x]}} \right]}{a^{5/2}} + \frac{(3 A - 43 B) \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [x]}{\sqrt{2} \sqrt{a + a \cos [x]}} \right]}{16 \sqrt{2} a^{5/2}} + \frac{(A - B) \sin [x]}{4 (a + a \cos [x])^{5/2}} + \frac{(3 A - 11 B) \sin [x]}{16 a (a + a \cos [x])^{3/2}}$$

Result (type 3, 393 leaves):

$$\frac{1}{8 (a (1 + \cos [x]))^{5/2} (B + A \cos [x])} \cos \left[ \frac{x}{2} \right]^5 \cos [x] (A + B \operatorname{Sec} [x])$$

$$\left( -32 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - (-1 + \sqrt{2}) \sin \left[ \frac{x}{4} \right]}{(1 + \sqrt{2}) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] - 32 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - (1 + \sqrt{2}) \sin \left[ \frac{x}{4} \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] + 2 (-3 A + 43 B) \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right] + \right.$$

$$2 (3 A - 43 B) \operatorname{Log} \left[ \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right] + 32 \sqrt{2} B \operatorname{Log} \left[ \sqrt{2} + 2 \sin \left[ \frac{x}{2} \right] \right] - 16 \sqrt{2} B \operatorname{Log} \left[ 2 - \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] -$$

$$\left. 16 \sqrt{2} B \operatorname{Log} \left[ 2 + \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] + \frac{A - B}{\left( \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right)^4} + \frac{3 A - 11 B}{\left( \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right)^2} + \frac{-A + B}{\left( \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right)^4} + \frac{-3 A + 11 B}{\left( \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right)^2} \right)$$



**Problem 228: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a \cos [c+d x]+b \sin [c+d x])^3} d x$$

Optimal (type 3, 103 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{b \cos [c+d x]-a \sin [c+d x]}{\sqrt{a^2+b^2}}\right]}{2\left(a^2+b^2\right)^{3/2} d}-\frac{b \cos [c+d x]-a \sin [c+d x]}{2\left(a^2+b^2\right) d\left(a \cos [c+d x]+b \sin [c+d x]\right)^2}$$

Result (type 3, 132 leaves):

$$\left(\left(a^2+b^2\right)\left(-b \cos [c+d x]+a \sin [c+d x]\right)+2 \sqrt{a^2+b^2} \operatorname{ArcTanh}\left[\frac{-b+a \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]\left(a \cos [c+d x]+b \sin [c+d x]\right)^2\right) / \left(2(a-i b)^2(a+i b)^2 d\left(a \cos [c+d x]+b \sin [c+d x]\right)^2\right)$$

**Problem 232: Result unnecessarily involves higher level functions.**

$$\int (a \cos [c+d x]+b \sin [c+d x])^{7/2} d x$$

Optimal (type 4, 186 leaves, 4 steps):

$$-\frac{10\left(a^2+b^2\right)\left(b \cos [c+d x]-a \sin [c+d x]\right) \sqrt{a \cos [c+d x]+b \sin [c+d x]}}{21 d}-\frac{2\left(b \cos [c+d x]-a \sin [c+d x]\right)\left(a \cos [c+d x]+b \sin [c+d x]\right)^{5/2}}{7 d}+\frac{10\left(a^2+b^2\right)^2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x-\operatorname{ArcTan}[a, b]), 2\right] \sqrt{\frac{a \cos [c+d x]+b \sin [c+d x]}{\sqrt{a^2+b^2}}}}{21 d \sqrt{a \cos [c+d x]+b \sin [c+d x]}}$$

Result (type 5, 205 leaves):

$$\frac{1}{42 d} \left( \sqrt{a \cos [c+d x]+b \sin [c+d x]} \right. \\ \left. (-23 b\left(a^2+b^2\right) \cos [c+d x]+(-9 a^2 b+3 b^3) \cos [3(c+d x)]+2 a\left(13 a^2+7 b^2+3\left(a^2-3 b^2\right) \cos [2(c+d x)]\right) \sin [c+d x]} \right) + \\ \left( 20\left(a^2+b^2\right)^2 \sqrt{\cos [c+d x+\operatorname{ArcTan}\left[\frac{a}{b}\right]]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin [c+d x+\operatorname{ArcTan}\left[\frac{a}{b}\right]]^2\right] \tan [c+d x+\operatorname{ArcTan}\left[\frac{a}{b}\right]] \right) / \\ \left( \sqrt{\sqrt{1+\frac{a^2}{b^2}} b \sin [c+d x+\operatorname{ArcTan}\left[\frac{a}{b}\right]]} \right)$$

**Problem 233: Result unnecessarily involves higher level functions.**

$$\int (a \cos [c+d x]+b \sin [c+d x])^{5 / 2} d x$$

Optimal (type 4, 131 leaves, 3 steps):

$$\frac{2(b \cos [c+d x]-a \sin [c+d x])(a \cos [c+d x]+b \sin [c+d x])^{3 / 2}}{5 d} + \\ \frac{6\left(a^2+b^2\right) \operatorname{EllipticE}\left[\frac{1}{2}(c+d x-\operatorname{ArcTan}[a, b]), 2\right] \sqrt{a \cos [c+d x]+b \sin [c+d x]}}{5 d \sqrt{\frac{a \cos [c+d x]+b \sin [c+d x]}{\sqrt{a^2+b^2}}}}$$

Result (type 5, 256 leaves):

$$\frac{1}{5 b d} \left( \sqrt{a \cos [c+d x]+b \sin [c+d x]} \left( 6 a \left( a^2+b^2 \right)-2 a b^2 \cos [2(c+d x)]+b \left( a^2-b^2 \right) \sin [2(c+d x)] \right) - \right. \\ \left. \left( 3 \left( a^2+b^2 \right)^2 \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right] \left( b \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2\right] \sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right] + \right. \right. \right. \\ \left. \left. \left. \sqrt{\sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2} \left( 2 a \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]-b \sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right] \right) \right) \right) \right) / \\ \left( \left( a \sqrt{1+\frac{b^2}{a^2}} \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right] \right)^{3 / 2} \sqrt{\sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2} \right)$$

**Problem 234:** Result unnecessarily involves higher level functions.

$$\int (a \cos [c+d x]+b \sin [c+d x])^{3 / 2} d x$$

Optimal (type 4, 131 leaves, 3 steps):

$$-\frac{2(b \cos [c+d x]-a \sin [c+d x]) \sqrt{a \cos [c+d x]+b \sin [c+d x]}}{3 d} + \frac{2\left(a^2+b^2\right) \operatorname{EllipticF}\left[\frac{1}{2}(c+d x-\operatorname{ArcTan}[a, b]), 2\right] \sqrt{\frac{a \cos [c+d x]+b \sin [c+d x]}{\sqrt{a^2+b^2}}}}{3 d \sqrt{a \cos [c+d x]+b \sin [c+d x]}}$$

Result (type 5, 143 leaves):

$$\frac{1}{3 d} \left( -b \cos [c+d x]+a \sin [c+d x] \right) \sqrt{a \cos [c+d x]+b \sin [c+d x]} + \\ \left( \left( a^2+b^2 \right) \sqrt{\cos \left[ c+d x+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin \left[ c+d x+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right]^2\right] \tan \left[ c+d x+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right] \right) / \\ \left( \sqrt{\sqrt{1+\frac{a^2}{b^2}} b \sin \left[ c+d x+\operatorname{ArcTan}\left[\frac{a}{b}\right]\right]} \right)$$

**Problem 235:** Result unnecessarily involves higher level functions and more than twice size of optimal

## antiderivative.

$$\int \sqrt{a \cos [c + d x] + b \sin [c + d x]} dx$$

Optimal (type 4, 75 leaves, 2 steps):

$$\frac{2 \operatorname{EllipticE}\left[\frac{1}{2}(c + d x - \operatorname{ArcTan}[a, b]), 2\right] \sqrt{a \cos [c + d x] + b \sin [c + d x]}}{d \sqrt{\frac{a \cos [c + d x] + b \sin [c + d x]}{\sqrt{a^2 + b^2}}}}$$

Result (type 5, 268 leaves):

$$\left( \cos [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]] \left( -b (a^2 + b^2) \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]]^2\right] \sin [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]] + \right. \right. \\ \left. \sqrt{\sin [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]]^2} \left( -2 a (a^2 + b^2) \cos [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]] + \right. \right. \\ \left. \left. 2 a^2 \sqrt{1 + \frac{b^2}{a^2}} \sqrt{a \sqrt{1 + \frac{b^2}{a^2}} \cos [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]] \sqrt{a \cos [c + d x] + b \sin [c + d x]} + b (a^2 + b^2) \sin [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]]} \right) \right) \right) / \\ \left( b d \left( a \sqrt{1 + \frac{b^2}{a^2}} \cos [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]] \right)^{3/2} \sqrt{\sin [c + d x - \operatorname{ArcTan}\left[\frac{b}{a}\right]]^2} \right)$$

**Problem 236: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a \cos [c + d x] + b \sin [c + d x]}} dx$$

Optimal (type 4, 75 leaves, 2 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x - \operatorname{ArcTan}[a, b]), 2\right] \sqrt{\frac{a \cos [c + d x] + b \sin [c + d x]}{\sqrt{a^2 + b^2}}}}{d \sqrt{a \cos [c + d x] + b \sin [c + d x]}}$$

Result (type 5, 92 leaves):

$$\left( 2 \sqrt{\cos\left[c + dx + \operatorname{ArcTan}\left[\frac{a}{b}\right]\right]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin\left[c + dx + \operatorname{ArcTan}\left[\frac{a}{b}\right]\right]^2\right] \tan\left[c + dx + \operatorname{ArcTan}\left[\frac{a}{b}\right]\right] \right) /$$

$$\left( d \sqrt{\sqrt{1 + \frac{a^2}{b^2}} b \sin\left[c + dx + \operatorname{ArcTan}\left[\frac{a}{b}\right]\right]} \right)$$

**Problem 237:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a \cos[c + dx] + b \sin[c + dx])^{3/2}} dx$$

Optimal (type 4, 138 leaves, 3 steps):

$$-\frac{2(b \cos[c + dx] - a \sin[c + dx])}{(a^2 + b^2) d \sqrt{a \cos[c + dx] + b \sin[c + dx]}} - \frac{2 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx - \operatorname{ArcTan}[a, b]), 2\right] \sqrt{a \cos[c + dx] + b \sin[c + dx]}}{(a^2 + b^2) d \sqrt{\frac{a \cos[c + dx] + b \sin[c + dx]}{\sqrt{a^2 + b^2}}}}$$

Result (type 5, 322 leaves):

$$\frac{\sqrt{a \cos [c+d x]+b \sin [c+d x]}\left(-\frac{2}{a b}+\frac{2 \sin [c+d x]}{a(a \cos [c+d x]+b \sin [c+d x])}\right)}{d}-\frac{1}{b d}$$

$$\left(-\left(b \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos \left[c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2\right] \sin \left[c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]\right) / \left(a \sqrt{1+\frac{b^2}{a^2}} \sqrt{1-\cos \left[c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]}\right)$$

$$\left(\sqrt{a \sqrt{\frac{a^2+b^2}{a^2}} \cos \left[c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right] \sqrt{1+\cos \left[c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]}\right) - \frac{\frac{2 a^2 \sqrt{1+\frac{b^2}{a^2}} \cos \left[c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]}{a^2+b^2}-\frac{b \sin \left[c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]}{a \sqrt{1+\frac{b^2}{a^2}}}}{\sqrt{a \sqrt{1+\frac{b^2}{a^2}} \cos \left[c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]}}$$

**Problem 238:** Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a \cos [c+d x]+b \sin [c+d x])^{5/2}} d x$$

Optimal (type 4, 142 leaves, 3 steps):

$$-\frac{2(b \cos [c+d x]-a \sin [c+d x])}{3(a^2+b^2) d(a \cos [c+d x]+b \sin [c+d x])^{3/2}}+\frac{2 \operatorname{EllipticF}\left[\frac{1}{2}(c+d x-\operatorname{ArcTan}[a, b]), 2\right] \sqrt{\frac{a \cos [c+d x]+b \sin [c+d x]}{\sqrt{a^2+b^2}}}}{3(a^2+b^2) d \sqrt{a \cos [c+d x]+b \sin [c+d x]}}$$

Result (type 5, 145 leaves):

$$\frac{1}{3(a^2 + b^2)d} \left( \frac{-b \cos[c + dx] + a \sin[c + dx]}{(a \cos[c + dx] + b \sin[c + dx])^{3/2}} + \left( \sqrt{\cos\left[c + dx + \arctan\left[\frac{a}{b}\right]\right]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin\left[c + dx + \arctan\left[\frac{a}{b}\right]\right]^2 \tan\left[c + dx + \arctan\left[\frac{a}{b}\right]\right]\right) \right) / \left( \sqrt{\sqrt{1 + \frac{a^2}{b^2}} b \sin\left[c + dx + \arctan\left[\frac{a}{b}\right]\right]} \right)$$

**Problem 239: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a \cos[c + dx] + b \sin[c + dx])^{7/2}} dx$$

Optimal (type 4, 197 leaves, 4 steps):

$$\frac{2(b \cos[c + dx] - a \sin[c + dx])}{5(a^2 + b^2)d(a \cos[c + dx] + b \sin[c + dx])^{5/2}} - \frac{6(b \cos[c + dx] - a \sin[c + dx])}{5(a^2 + b^2)^2 d \sqrt{a \cos[c + dx] + b \sin[c + dx]}} - \frac{6 \operatorname{EllipticE}\left[\frac{1}{2}(c + dx - \arctan[a, b]), 2\right] \sqrt{a \cos[c + dx] + b \sin[c + dx]}}{5(a^2 + b^2)^2 d \sqrt{\frac{a \cos[c + dx] + b \sin[c + dx]}{\sqrt{a^2 + b^2}}}}$$

Result (type 5, 277 leaves):

$$\frac{1}{5b(a^2+b^2)d} \left( - \left( (2(3a^2 \cos[c+dx]^3 - ab \sin[c+dx] + 6ab \cos[c+dx]^2 \sin[c+dx] + b^2 \cos[c+dx](1+3\sin[c+dx]^2))) \right) / \right. \\ \left. (a \cos[c+dx] + b \sin[c+dx])^{5/2} + \right. \\ \left. \left( \cos[c+dx - \arctan\left[\frac{b}{a}\right]] \left( 3b \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[c+dx - \arctan\left[\frac{b}{a}\right]]^2\right] \sin[c+dx - \arctan\left[\frac{b}{a}\right]] - \right. \right. \right. \right. \\ \left. \left. \left. 3 \sqrt{\sin[c+dx - \arctan\left[\frac{b}{a}\right]]^2} \left( -2a \cos[c+dx - \arctan\left[\frac{b}{a}\right]] + b \sin[c+dx - \arctan\left[\frac{b}{a}\right]] \right) \right) \right) \right) / \right. \\ \left. \left( \left( a \sqrt{1 + \frac{b^2}{a^2}} \cos[c+dx - \arctan\left[\frac{b}{a}\right]] \right)^{3/2} \sqrt{\sin[c+dx - \arctan\left[\frac{b}{a}\right]]^2} \right) \right) \right)$$

**Problem 240: Result unnecessarily involves higher level functions.**

$$\int (2 \cos[c+dx] + 3 \sin[c+dx])^{7/2} dx$$

Optimal (type 4, 120 leaves, 4 steps):

$$\frac{130 \times 13^{3/4} \operatorname{EllipticF}\left[\frac{1}{2} \left(c+dx - \arctan\left[\frac{3}{2}\right]\right), 2\right]}{21d} - \frac{130(3 \cos[c+dx] - 2 \sin[c+dx]) \sqrt{2 \cos[c+dx] + 3 \sin[c+dx]}}{21d} - \\ \frac{2(3 \cos[c+dx] - 2 \sin[c+dx]) (2 \cos[c+dx] + 3 \sin[c+dx])^{5/2}}{7d}$$

Result (type 5, 153 leaves):

$$\frac{1}{42d} \left( -\sqrt{2 \cos[c+dx] + 3 \sin[c+dx]} (897 \cos[c+dx] + 27 \cos[3(c+dx)] - 598 \sin[c+dx] + 138 \sin[3(c+dx)]) + \right. \\ \left. 260 \times 13^{3/4} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin\left[c+dx + \arctan\left[\frac{2}{3}\right]\right]^2\right] \sec\left[c+dx + \arctan\left[\frac{2}{3}\right]\right] \right. \\ \left. \sqrt{-\left(-1 + \sin\left[c+dx + \arctan\left[\frac{2}{3}\right]\right]\right) \sin\left[c+dx + \arctan\left[\frac{2}{3}\right]\right]} \sqrt{1 + \sin\left[c+dx + \arctan\left[\frac{2}{3}\right]\right]} \right)$$

**Problem 241: Result unnecessarily involves higher level functions and more than twice size of optimal**



antiderivative.

$$\int (2 \cos [c + d x] + 3 \sin [c + d x])^{5/2} dx$$

Optimal (type 4, 75 leaves, 3 steps):

$$\frac{78 \times 13^{1/4} \operatorname{EllipticE}\left[\frac{1}{2} \left(c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{5 d} - \frac{2 (3 \cos [c + d x] - 2 \sin [c + d x]) (2 \cos [c + d x] + 3 \sin [c + d x])^{3/2}}{5 d}$$

Result (type 5, 199 leaves):

$$\frac{1}{5 d} \left( \sqrt{2 \cos [c + d x] + 3 \sin [c + d x]} (52 - 12 \cos [2 (c + d x)] - 5 \sin [2 (c + d x)]) - \frac{13 \times 13^{1/4} (4 \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]] - 3 \sin [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]])}{\sqrt{\cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]}} - \frac{39 \times 13^{1/4} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]^2\right] \sin [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]}{\sqrt{-(-1 + \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]) \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]} \sqrt{1 + \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]}} \right)$$

**Problem 242: Result unnecessarily involves higher level functions.**

$$\int (2 \cos [c + d x] + 3 \sin [c + d x])^{3/2} dx$$

Optimal (type 4, 75 leaves, 3 steps):

$$\frac{2 \times 13^{3/4} \operatorname{EllipticF}\left[\frac{1}{2} \left(c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{3 d} - \frac{2 (3 \cos [c + d x] - 2 \sin [c + d x]) \sqrt{2 \cos [c + d x] + 3 \sin [c + d x]}}{3 d}$$

Result (type 5, 133 leaves):

$$\frac{1}{3d} \left( 2 \left( -3 \cos [c + dx] + 2 \sin [c + dx] \right) \sqrt{2 \cos [c + dx] + 3 \sin [c + dx]} + 2 \times 13^{3/4} \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin \left[ c + dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right]^2 \right] \right. \\ \left. \sec \left[ c + dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right] \sqrt{-\left( -1 + \sin \left[ c + dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right] \right)} \sin \left[ c + dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right] \sqrt{1 + \sin \left[ c + dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right]} \right)$$

**Problem 243:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{2 \cos [c + dx] + 3 \sin [c + dx]} dx$$

Optimal (type 4, 27 leaves, 2 steps):

$$\frac{2 \times 13^{1/4} \text{EllipticE} \left[ \frac{1}{2} \left( c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right), 2 \right]}{d}$$

Result (type 5, 184 leaves):

$$\frac{1}{3d} \left( -4 \times 13^{1/4} \sqrt{\cos \left[ c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]} + 4 \sqrt{2 \cos [c + dx] + 3 \sin [c + dx]} + \frac{3 \times 13^{1/4} \sin \left[ c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]}{\sqrt{\cos \left[ c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right}}} - \right. \\ \left. \frac{3 \times 13^{1/4} \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos \left[ c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]^2 \right] \sin \left[ c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]}{\sqrt{-\left( -1 + \cos \left[ c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right] \right)} \cos \left[ c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right] \sqrt{1 + \cos \left[ c + dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right}}} \right)$$

**Problem 244:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2 \cos [c + dx] + 3 \sin [c + dx]}} dx$$

Optimal (type 4, 27 leaves, 2 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2}\left(c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{13^{1/4} d}$$

Result (type 5, 88 leaves):

$$\frac{1}{13^{1/4} d} {}_2\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \operatorname{Sin}\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]^2\right] \operatorname{Sec}\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right] \\ \sqrt{-\left(-1+\operatorname{Sin}\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]\right) \operatorname{Sin}\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]} \sqrt{1+\operatorname{Sin}\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]}$$

**Problem 245: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2 \operatorname{Cos}[c+dx] + 3 \operatorname{Sin}[c+dx])^{3/2}} dx$$

Optimal (type 4, 73 leaves, 3 steps):

$$-\frac{2 \operatorname{EllipticE}\left[\frac{1}{2}\left(c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{13^{3/4} d} - \frac{2(3 \operatorname{Cos}[c+dx] - 2 \operatorname{Sin}[c+dx])}{13 d \sqrt{2 \operatorname{Cos}[c+dx] + 3 \operatorname{Sin}[c+dx]}}$$

Result (type 5, 190 leaves):

$$\frac{1}{3 d} \left( \frac{4 \sqrt{\operatorname{Cos}\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}}{13^{3/4}} - \frac{2 \operatorname{Cos}[c+dx]}{\sqrt{2 \operatorname{Cos}[c+dx] + 3 \operatorname{Sin}[c+dx]}} - \frac{3 \operatorname{Sin}\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}{13^{3/4} \sqrt{\operatorname{Cos}\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right}} + \right. \\ \left. \frac{3 \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \operatorname{Cos}\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]^2\right] \operatorname{Sin}\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}{13^{3/4} \sqrt{-\left(-1+\operatorname{Cos}\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right)\right) \operatorname{Cos}\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]} \sqrt{1+\operatorname{Cos}\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]} \right)$$

**Problem 246: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2 \operatorname{Cos}[c+dx] + 3 \operatorname{Sin}[c+dx])^{5/2}} dx$$

Optimal (type 4, 75 leaves, 3 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2}\left(c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{39 \times 13^{1/4} d} - \frac{2(3 \cos[c+dx] - 2 \sin[c+dx])}{39 d (2 \cos[c+dx] + 3 \sin[c+dx])^{3/2}}$$

Result (type 5, 157 leaves):

$$\left( -78 \cos[c+dx] + 52 \sin[c+dx] + \sqrt{2} 13^{3/4} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]^2\right] \operatorname{Sec}\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right] (2 \cos[c+dx] + 3 \sin[c+dx])^{3/2} \right. \\ \left. \sqrt{1 + \sin\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]} \sqrt{-1 + \cos\left[2\left(c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right)\right] + 2 \sin\left[c+dx+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]} \right) / (507 d (2 \cos[c+dx] + 3 \sin[c+dx])^{3/2})$$

**Problem 247: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(2 \cos[c+dx] + 3 \sin[c+dx])^{7/2}} dx$$

Optimal (type 4, 120 leaves, 4 steps):

$$-\frac{6 \operatorname{EllipticE}\left[\frac{1}{2}\left(c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{65 \times 13^{3/4} d} - \frac{2(3 \cos[c+dx] - 2 \sin[c+dx])}{65 d (2 \cos[c+dx] + 3 \sin[c+dx])^{5/2}} - \frac{6(3 \cos[c+dx] - 2 \sin[c+dx])}{845 d \sqrt{2 \cos[c+dx] + 3 \sin[c+dx]}}$$

Result (type 5, 224 leaves):

$$\frac{1}{65 d} \left( \frac{4 \sqrt{\cos\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}}{13^{3/4}} + \frac{-33 \cos[c+dx] + 5 \cos[3(c+dx)] - 4(\sin[c+dx] + 3 \sin[3(c+dx)])}{2(2 \cos[c+dx] + 3 \sin[c+dx])^{5/2}} - \right. \\ \left. \frac{3 \sin\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}{13^{3/4} \sqrt{\cos\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]} + \frac{3 \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]^2\right] \sin\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}{13^{3/4} \sqrt{-(-1 + \cos\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]) \cos\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]} \sqrt{1 + \cos\left[c+dx-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]} \right)$$

**Problem 267: Result more than twice size of optimal antiderivative.**

$$\int (a \operatorname{Sec}[x] + b \operatorname{Tan}[x]) dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$a \operatorname{ArcTanh}[\operatorname{Sin}[x]] - b \operatorname{Log}[\operatorname{Cos}[x]]$$

Result (type 3, 42 leaves):

$$-b \operatorname{Log}[\operatorname{Cos}[x]] - a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + a \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right]$$

**Problem 274: Result more than twice size of optimal antiderivative.**

$$\int (\operatorname{Sec}[x] + \operatorname{Tan}[x])^4 dx$$

Optimal (type 3, 30 leaves, 5 steps):

$$x + \frac{2 \operatorname{Cos}[x]^3}{3 (1 - \operatorname{Sin}[x])^3} - \frac{2 \operatorname{Cos}[x]}{1 - \operatorname{Sin}[x]}$$

Result (type 3, 64 leaves):

$$-\frac{-3 (8 + 3x) \operatorname{Cos}\left[\frac{x}{2}\right] + (16 + 3x) \operatorname{Cos}\left[\frac{3x}{2}\right] + 6 (4 + 2x + x \operatorname{Cos}[x]) \operatorname{Sin}\left[\frac{x}{2}\right]}{6 \left(\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right)^3}$$

**Problem 275: Result more than twice size of optimal antiderivative.**

$$\int (\operatorname{Sec}[x] + \operatorname{Tan}[x])^3 dx$$

Optimal (type 3, 18 leaves, 4 steps):

$$\operatorname{Log}[1 - \operatorname{Sin}[x]] + \frac{2}{1 - \operatorname{Sin}[x]}$$

Result (type 3, 38 leaves):

$$2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + \frac{2}{\left(\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right)^2}$$

### Problem 277: Result more than twice size of optimal antiderivative.

$$\int (\sec [x] + \tan [x]) \, dx$$

Optimal (type 3, 13 leaves, 3 steps):

$$-2 \operatorname{Log} \left[ \cos \left[ \frac{1}{4} (\pi + 2x) \right] \right]$$

Result (type 3, 38 leaves):

$$-\operatorname{Log} [\cos [x]] - \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] + \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right]$$

### Problem 278: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sec [x] + \tan [x]} \, dx$$

Optimal (type 3, 5 leaves, 3 steps):

$$\operatorname{Log} [1 + \sin [x]]$$

Result (type 3, 16 leaves):

$$2 \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right]$$

### Problem 280: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(\sec [x] + \tan [x])^3} \, dx$$

Optimal (type 3, 16 leaves, 4 steps):

$$-\operatorname{Log} [1 + \sin [x]] - \frac{2}{1 + \sin [x]}$$

Result (type 3, 34 leaves):

$$-2 \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right] - \frac{2}{\left( \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right)^2}$$

**Problem 281: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(\sec[x] + \tan[x])^4} dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$x - \frac{2 \cos[x]^3}{3 (1 + \sin[x])^3} + \frac{2 \cos[x]}{1 + \sin[x]}$$

Result (type 3, 62 leaves):

$$\frac{3 (-8 + 3x) \cos\left[\frac{x}{2}\right] + (16 - 3x) \cos\left[\frac{3x}{2}\right] + 6 (-4 + 2x + x \cos[x]) \sin\left[\frac{x}{2}\right]}{6 \left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right)^3}$$

**Problem 287: Result more than twice size of optimal antiderivative.**

$$\int (a \cot[x] + b \csc[x]) dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-b \operatorname{ArcTanh}[\cos[x]] + a \operatorname{Log}[\sin[x]]$$

Result (type 3, 25 leaves):

$$-b \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] + b \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] + a \operatorname{Log}[\sin[x]]$$

**Problem 297: Result more than twice size of optimal antiderivative.**

$$\int (\cot[x] + \csc[x]) dx$$

Optimal (type 3, 9 leaves, 3 steps):

$$-\operatorname{ArcTanh}[\cos[x]] + \operatorname{Log}[\sin[x]]$$

Result (type 3, 20 leaves):

$$-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}[\sin[x]]$$

### Problem 306: Result more than twice size of optimal antiderivative.

$$\int (\operatorname{Csc}[x] - \operatorname{Sin}[x]) \, dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\operatorname{ArcTanh}[\operatorname{Cos}[x]] + \operatorname{Cos}[x]$$

Result (type 3, 19 leaves):

$$\operatorname{Cos}[x] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\operatorname{Sin}\left[\frac{x}{2}\right]\right]$$

### Problem 310: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(\operatorname{Csc}[x] - \operatorname{Sin}[x])^4} \, dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$\frac{\operatorname{Tan}[x]^5}{5} + \frac{\operatorname{Tan}[x]^7}{7}$$

Result (type 3, 37 leaves):

$$\frac{2 \operatorname{Tan}[x]}{35} + \frac{1}{35} \operatorname{Sec}[x]^2 \operatorname{Tan}[x] - \frac{8}{35} \operatorname{Sec}[x]^4 \operatorname{Tan}[x] + \frac{1}{7} \operatorname{Sec}[x]^6 \operatorname{Tan}[x]$$

### Problem 312: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(\operatorname{Csc}[x] - \operatorname{Sin}[x])^6} \, dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$\frac{\operatorname{Tan}[x]^7}{7} + \frac{2 \operatorname{Tan}[x]^9}{9} + \frac{\operatorname{Tan}[x]^{11}}{11}$$

Result (type 3, 57 leaves):

$$-\frac{8 \operatorname{Tan}[x]}{693} - \frac{4}{693} \operatorname{Sec}[x]^2 \operatorname{Tan}[x] - \frac{1}{231} \operatorname{Sec}[x]^4 \operatorname{Tan}[x] + \frac{113}{693} \operatorname{Sec}[x]^6 \operatorname{Tan}[x] - \frac{23}{99} \operatorname{Sec}[x]^8 \operatorname{Tan}[x] + \frac{1}{11} \operatorname{Sec}[x]^{10} \operatorname{Tan}[x]$$



**Problem 318: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{\operatorname{Csc}[x] - \operatorname{Sin}[x]}} dx$$

Optimal (type 3, 60 leaves, 8 steps):

$$\frac{\operatorname{ArcTan}[\sqrt{-\operatorname{Sin}[x]}] \operatorname{Cos}[x]}{\sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]} \sqrt{-\operatorname{Sin}[x]}} - \frac{\operatorname{ArcTanh}[\sqrt{-\operatorname{Sin}[x]}] \operatorname{Cos}[x]}{\sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]} \sqrt{-\operatorname{Sin}[x]}}$$

Result (type 5, 37 leaves):

$$2 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Sec}[x]^2\right] \operatorname{Sec}[x] (-\operatorname{Tan}[x]^2)^{1/4}$$

**Problem 319: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(\operatorname{Csc}[x] - \operatorname{Sin}[x])^{3/2}} dx$$

Optimal (type 3, 80 leaves, 9 steps):

$$\frac{\operatorname{Sec}[x]}{2 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]}} + \frac{\operatorname{ArcTan}[\sqrt{-\operatorname{Sin}[x]}] \operatorname{Cot}[x] \sqrt{-\operatorname{Sin}[x]}}{4 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]}} + \frac{\operatorname{ArcTanh}[\sqrt{-\operatorname{Sin}[x]}] \operatorname{Cot}[x] \sqrt{-\operatorname{Sin}[x]}}{4 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]}}$$

Result (type 5, 42 leaves):

$$\frac{\operatorname{Sec}[x] \left( 3 + \frac{\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Sec}[x]^2\right]}{(-\operatorname{Tan}[x]^2)^{1/4}} \right)}{6 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]}}$$

**Problem 320: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(\operatorname{Csc}[x] - \operatorname{Sin}[x])^{5/2}} dx$$

Optimal (type 3, 99 leaves, 10 steps):

$$-\frac{3 \operatorname{ArcTan}[\sqrt{-\operatorname{Sin}[x]}] \operatorname{Cos}[x]}{32 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]} \sqrt{-\operatorname{Sin}[x]}} + \frac{3 \operatorname{ArcTanh}[\sqrt{-\operatorname{Sin}[x]}] \operatorname{Cos}[x]}{32 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]} \sqrt{-\operatorname{Sin}[x]}} - \frac{3 \operatorname{Tan}[x]}{16 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]}} + \frac{\operatorname{Sec}[x]^2 \operatorname{Tan}[x]}{4 \sqrt{\operatorname{Cos}[x] \operatorname{Cot}[x]}}$$

Result (type 5, 57 leaves):

$$\frac{(5 - 3 \cos[2x]) \sec[x]^2 \tan[x] - 6 \cot[x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \sec[x]^2\right] (-\tan[x]^2)^{1/4}}{32 \sqrt{\cos[x] \cot[x]}}$$

**Problem 321: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(\csc[x] - \sin[x])^{7/2}} dx$$

Optimal (type 3, 118 leaves, 11 steps):

$$\frac{5 \sec[x]}{192 \sqrt{\cos[x] \cot[x]}} - \frac{5 \sec[x]^3}{48 \sqrt{\cos[x] \cot[x]}} - \frac{5 \operatorname{ArcTan}[\sqrt{-\sin[x]}] \cot[x] \sqrt{-\sin[x]}}{128 \sqrt{\cos[x] \cot[x]}} - \frac{5 \operatorname{ArcTanh}[\sqrt{-\sin[x]}] \cot[x] \sqrt{-\sin[x]}}{128 \sqrt{\cos[x] \cot[x]}} + \frac{\sec[x]^3 \tan[x]^2}{6 \sqrt{\cos[x] \cot[x]}}$$

Result (type 5, 63 leaves):

$$\frac{1}{192} \sqrt{\cos[x] \cot[x]} \csc[x] \sec[x] \left( -5 + 57 \sec[x]^2 - 84 \sec[x]^4 + 32 \sec[x]^6 + 5 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \sec[x]^2\right] (-\tan[x]^2)^{3/4} \right)$$

**Problem 323: Result more than twice size of optimal antiderivative.**

$$\int (-\cos[x] + \sec[x])^3 dx$$

Optimal (type 3, 34 leaves, 6 steps):

$$-\frac{5}{2} \operatorname{ArcTanh}[\sin[x]] + \frac{5 \sin[x]}{2} + \frac{5 \sin[x]^3}{6} + \frac{1}{2} \sin[x]^3 \tan[x]^2$$

Result (type 3, 85 leaves):

$$\frac{1}{12} \left( 30 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - 30 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + \frac{3}{\left(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right)^2} - \frac{3}{\left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right)^2} + 27 \sin[x] - \sin[3x] \right)$$

**Problem 325: Result more than twice size of optimal antiderivative.**

$$\int (-\cos[x] + \sec[x]) dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$\operatorname{ArcTanh}[\sin[x]] - \sin[x]$$

Result (type 3, 37 leaves):

$$-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - \sin[x]$$

**Problem 328: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^3} dx$$

Optimal (type 3, 17 leaves, 4 steps):

$$\frac{\csc[x]^3}{3} - \frac{\csc[x]^5}{5}$$

Result (type 3, 93 leaves):

$$\frac{11}{240} \cot\left[\frac{x}{2}\right] + \frac{11}{480} \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^2 - \frac{1}{160} \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^4 + \frac{11}{240} \tan\left[\frac{x}{2}\right] + \frac{11}{480} \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] - \frac{1}{160} \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right]$$

**Problem 329: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^4} dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$-\frac{1}{5} \cot[x]^5 - \frac{\cot[x]^7}{7}$$

Result (type 3, 37 leaves):

$$-\frac{2 \cot[x]}{35} - \frac{1}{35} \cot[x] \csc[x]^2 + \frac{8}{35} \cot[x] \csc[x]^4 - \frac{1}{7} \cot[x] \csc[x]^6$$

**Problem 330: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^5} dx$$

Optimal (type 3, 25 leaves, 4 steps):

$$-\frac{1}{5} \csc[x]^5 + \frac{2 \csc[x]^7}{7} - \frac{\csc[x]^9}{9}$$

Result (type 3, 165 leaves):

$$\begin{array}{r}
-\frac{649 \cot\left[\frac{x}{2}\right]}{80640} - \frac{649 \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^2}{161280} - \frac{31 \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^4}{53760} + \frac{37 \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^6}{32256} - \frac{\cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^8}{4608} \\
-\frac{649 \tan\left[\frac{x}{2}\right]}{80640} - \frac{649 \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]}{161280} - \frac{31 \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right]}{53760} + \frac{37 \sec\left[\frac{x}{2}\right]^6 \tan\left[\frac{x}{2}\right]}{32256} - \frac{\sec\left[\frac{x}{2}\right]^8 \tan\left[\frac{x}{2}\right]}{4608}
\end{array}$$

**Problem 331: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^6} dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$-\frac{1}{7} \cot[x]^7 - \frac{2 \cot[x]^9}{9} - \frac{\cot[x]^{11}}{11}$$

Result (type 3, 57 leaves):

$$\frac{8 \cot[x]}{693} + \frac{4}{693} \cot[x] \csc[x]^2 + \frac{1}{231} \cot[x] \csc[x]^4 - \frac{113}{693} \cot[x] \csc[x]^6 + \frac{23}{99} \cot[x] \csc[x]^8 - \frac{1}{11} \cot[x] \csc[x]^{10}$$

**Problem 332: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^7} dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$\frac{\csc[x]^7}{7} - \frac{\csc[x]^9}{3} + \frac{3 \csc[x]^{11}}{11} - \frac{\csc[x]^{13}}{13}$$

Result (type 3, 237 leaves):

$$\begin{array}{r}
\frac{10027 \cot\left[\frac{x}{2}\right]}{6150144} + \frac{10027 \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^2}{12300288} + \frac{755 \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^4}{4100096} - \frac{101 \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^6}{768768} \\
-\frac{101 \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^8}{878592} + \frac{79 \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^{10}}{1171456} - \frac{\cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^{12}}{106496} + \frac{10027 \tan\left[\frac{x}{2}\right]}{6150144} + \frac{10027 \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right]}{12300288} + \\
\frac{755 \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right]}{4100096} - \frac{101 \sec\left[\frac{x}{2}\right]^6 \tan\left[\frac{x}{2}\right]}{768768} - \frac{101 \sec\left[\frac{x}{2}\right]^8 \tan\left[\frac{x}{2}\right]}{878592} + \frac{79 \sec\left[\frac{x}{2}\right]^{10} \tan\left[\frac{x}{2}\right]}{1171456} - \frac{\sec\left[\frac{x}{2}\right]^{12} \tan\left[\frac{x}{2}\right]}{106496}
\end{array}$$

**Problem 337: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{-\cos[x] + \sec[x]}} dx$$

Optimal (type 3, 52 leaves, 8 steps):

$$\frac{\text{ArcTan}[\sqrt{\cos[x]}] \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}} - \frac{\text{ArcTanh}[\sqrt{\cos[x]}] \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}$$

Result (type 5, 37 leaves):

$$-2 (-\cot[x]^2)^{1/4} \csc[x] \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc[x]^2\right] \sqrt{\sin[x] \tan[x]}$$

**Problem 338: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^{3/2}} dx$$

Optimal (type 3, 72 leaves, 9 steps):

$$-\frac{\csc[x]}{2\sqrt{\sin[x] \tan[x]}} + \frac{\text{ArcTan}[\sqrt{\cos[x]}] \sin[x]}{4\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}} + \frac{\text{ArcTanh}[\sqrt{\cos[x]}] \sin[x]}{4\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}$$

Result (type 5, 49 leaves):

$$\frac{1}{6} \csc[x] \left( -3 \cot[x]^2 + (-\cot[x]^2)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \csc[x]^2\right] \right) \sec[x] \sqrt{\sin[x] \tan[x]}$$

**Problem 339: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^{5/2}} dx$$

Optimal (type 3, 91 leaves, 10 steps):

$$\frac{3 \cot[x]}{16\sqrt{\sin[x] \tan[x]}} - \frac{\cot[x] \csc[x]^2}{4\sqrt{\sin[x] \tan[x]}} - \frac{3 \text{ArcTan}[\sqrt{\cos[x]}] \sin[x]}{32\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}} + \frac{3 \text{ArcTanh}[\sqrt{\cos[x]}] \sin[x]}{32\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}$$

Result (type 5, 53 leaves):

$$\frac{\operatorname{Csc}[x] \left( -5 - 3 \operatorname{Cos}[2x] + \frac{6 \operatorname{Cos}[x]^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \operatorname{Csc}[x]^2\right]}{(-\operatorname{Cot}[x]^2)^{7/4}} \right)}{32 (\operatorname{Sin}[x] \operatorname{Tan}[x])^{3/2}}$$

**Problem 340: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(-\operatorname{Cos}[x] + \operatorname{Sec}[x])^{7/2}} dx$$

Optimal (type 3, 110 leaves, 11 steps):

$$-\frac{5 \operatorname{Csc}[x]}{192 \sqrt{\operatorname{Sin}[x] \operatorname{Tan}[x]}} + \frac{5 \operatorname{Csc}[x]^3}{48 \sqrt{\operatorname{Sin}[x] \operatorname{Tan}[x]}} - \frac{\operatorname{Cot}[x]^2 \operatorname{Csc}[x]^3}{6 \sqrt{\operatorname{Sin}[x] \operatorname{Tan}[x]}} - \frac{5 \operatorname{ArcTan}[\sqrt{\operatorname{Cos}[x]}] \operatorname{Sin}[x]}{128 \sqrt{\operatorname{Cos}[x]} \sqrt{\operatorname{Sin}[x] \operatorname{Tan}[x]}} - \frac{5 \operatorname{ArcTanh}[\sqrt{\operatorname{Cos}[x]}] \operatorname{Sin}[x]}{128 \sqrt{\operatorname{Cos}[x]} \sqrt{\operatorname{Sin}[x] \operatorname{Tan}[x]}}$$

Result (type 5, 63 leaves):

$$-\frac{1}{192} \operatorname{Csc}[x] \left( -5 + 57 \operatorname{Csc}[x]^2 - 84 \operatorname{Csc}[x]^4 + 32 \operatorname{Csc}[x]^6 + 5 (-\operatorname{Cot}[x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \operatorname{Csc}[x]^2\right] \right) \operatorname{Sec}[x] \sqrt{\operatorname{Sin}[x] \operatorname{Tan}[x]}$$

**Problem 341: Result more than twice size of optimal antiderivative.**

$$\int (\operatorname{Sin}[x] + \operatorname{Tan}[x])^4 dx$$

Optimal (type 3, 55 leaves, 18 steps):

$$-\frac{61x}{8} - 2 \operatorname{ArcTanh}[\operatorname{Sin}[x]] + \frac{19}{8} \operatorname{Cos}[x] \operatorname{Sin}[x] + \frac{1}{4} \operatorname{Cos}[x]^3 \operatorname{Sin}[x] - \frac{4 \operatorname{Sin}[x]^3}{3} + 5 \operatorname{Tan}[x] + 2 \operatorname{Sec}[x] \operatorname{Tan}[x] + \frac{\operatorname{Tan}[x]^3}{3}$$

Result (type 3, 129 leaves):

$$\frac{1}{768} \operatorname{Sec}[x]^3 \left( -72 \operatorname{Cos}[x] \left( 61x - 16 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + 16 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] \right) - \right. \\ \left. 24 \operatorname{Cos}[3x] \left( 61x - 16 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]\right] + 16 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right] + \operatorname{Sin}\left[\frac{x}{2}\right]\right] \right) + \right. \\ \left. 1395 \operatorname{Sin}[x] + 672 \operatorname{Sin}[2x] + 1265 \operatorname{Sin}[3x] + 129 \operatorname{Sin}[5x] + 32 \operatorname{Sin}[6x] + 3 \operatorname{Sin}[7x] \right)$$

**Problem 343: Result more than twice size of optimal antiderivative.**

$$\int (\operatorname{Sin}[x] + \operatorname{Tan}[x])^2 dx$$

Optimal (type 3, 25 leaves, 9 steps):

$$-\frac{x}{2} + 2 \operatorname{ArcTanh}[\sin[x]] - 2 \sin[x] - \frac{1}{2} \cos[x] \sin[x] + \tan[x]$$

Result (type 3, 60 leaves):

$$-\frac{x}{2} - 2 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + 2 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - 2 \sin[x] - \frac{1}{8} \sec[x] \sin[3x] + \frac{7 \tan[x]}{8}$$

**Problem 351: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \sin[x]}{(b \cos[x] + c \sin[x])^3} dx$$

Optimal (type 3, 116 leaves, 4 steps):

$$-\frac{A \operatorname{ArcTanh}\left[\frac{c \cos[x] - b \sin[x]}{\sqrt{b^2 + c^2}}\right]}{2 (b^2 + c^2)^{3/2}} + \frac{b C - A c \cos[x] + A b \sin[x]}{2 (b^2 + c^2) (b \cos[x] + c \sin[x])^2} - \frac{c^2 C \cos[x] - b c C \sin[x]}{(b^2 + c^2)^2 (b \cos[x] + c \sin[x])}$$

Result (type 3, 132 leaves):

$$\left( 2 A b \sqrt{b^2 + c^2} \operatorname{ArcTanh}\left[\frac{-c + b \tan\left[\frac{x}{2}\right]}{\sqrt{b^2 + c^2}}\right] (b \cos[x] + c \sin[x])^2 + (b^2 + c^2) (-A b c \cos[x] + A b^2 \sin[x] + 2 c^2 C \sin[x]^2 + b C (b + c \sin[2x])) \right) / \left( 2 b (b - i c)^2 (b + i c)^2 (b \cos[x] + c \sin[x])^2 \right)$$

**Problem 354: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[x]}{(b \cos[x] + c \sin[x])^3} dx$$

Optimal (type 3, 116 leaves, 4 steps):

$$-\frac{A \operatorname{ArcTanh}\left[\frac{c \cos[x] - b \sin[x]}{\sqrt{b^2 + c^2}}\right]}{2 (b^2 + c^2)^{3/2}} - \frac{B c + A c \cos[x] - A b \sin[x]}{2 (b^2 + c^2) (b \cos[x] + c \sin[x])^2} - \frac{b B c \cos[x] - b^2 B \sin[x]}{(b^2 + c^2)^2 (b \cos[x] + c \sin[x])}$$

Result (type 3, 118 leaves):

$$\left( 2 A \sqrt{b^2 + c^2} \operatorname{ArcTanh}\left[\frac{-c + b \tan\left[\frac{x}{2}\right]}{\sqrt{b^2 + c^2}}\right] (b \cos[x] + c \sin[x])^2 + (b^2 + c^2) (-A c \cos[x] - B c \cos[2x] + b (A + 2 B \cos[x]) \sin[x]) \right) / \left( 2 (b - i c)^2 (b + i c)^2 (b \cos[x] + c \sin[x])^2 \right)$$

### Problem 355: Result unnecessarily involves imaginary or complex numbers.

$$\int \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^4 dx$$

Optimal (type 3, 246 leaves, 6 steps):

$$\begin{aligned} & \frac{35}{8} (b^2 + c^2)^2 x - \frac{35 c (b^2 + c^2)^{3/2} \cos [d + e x]}{8 e} + \frac{35 b (b^2 + c^2)^{3/2} \sin [d + e x]}{8 e} - \\ & \frac{35 (b^2 + c^2) (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)}{24 e} - \\ & \frac{7 \sqrt{b^2 + c^2} (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^2}{12 e} - \\ & \frac{(c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^3}{4 e} \end{aligned}$$

Result (type 3, 238 leaves):

$$\begin{aligned} & \frac{1}{96 e} \left( 420 (b^2 + c^2)^2 (d + e x) - 672 (b - i c) (b + i c) c \sqrt{b^2 + c^2} \cos [d + e x] - 336 b c (b^2 + c^2) \cos [2 (d + e x)] + \right. \\ & \quad \left. 32 c (-3 b^2 + c^2) \sqrt{b^2 + c^2} \cos [3 (d + e x)] - 12 b c (b^2 - c^2) \cos [4 (d + e x)] + 672 b (b - i c) (b + i c) \sqrt{b^2 + c^2} \sin [d + e x] + \right. \\ & \quad \left. 168 (b^4 - c^4) \sin [2 (d + e x)] + 32 b (b^2 - 3 c^2) \sqrt{b^2 + c^2} \sin [3 (d + e x)] + 3 (b^4 - 6 b^2 c^2 + c^4) \sin [4 (d + e x)] \right) \end{aligned}$$

### Problem 356: Result unnecessarily involves imaginary or complex numbers.

$$\int \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^3 dx$$

Optimal (type 3, 178 leaves, 5 steps):

$$\begin{aligned} & \frac{5}{2} (b^2 + c^2)^{3/2} x - \frac{5 c (b^2 + c^2) \cos [d + e x]}{2 e} + \frac{5 b (b^2 + c^2) \sin [d + e x]}{2 e} - \\ & \frac{5 \sqrt{b^2 + c^2} (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)}{6 e} - \\ & \frac{(c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^2}{3 e} \end{aligned}$$

Result (type 3, 163 leaves):



$$\frac{1}{12 e} \left( 30 (b - i c) (b + i c) \sqrt{b^2 + c^2} (d + e x) - 45 c (b^2 + c^2) \operatorname{Cos}[d + e x] - 18 b c \sqrt{b^2 + c^2} \operatorname{Cos}[2 (d + e x)] + \right. \\ \left. c (-3 b^2 + c^2) \operatorname{Cos}[3 (d + e x)] + 45 b (b^2 + c^2) \operatorname{Sin}[d + e x] + 9 (b^2 - c^2) \sqrt{b^2 + c^2} \operatorname{Sin}[2 (d + e x)] + b (b^2 - 3 c^2) \operatorname{Sin}[3 (d + e x)] \right)$$

**Problem 361: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(\sqrt{b^2 + c^2} + b \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]\right)^3} dx$$

Optimal (type 3, 191 leaves, 3 steps):

$$- \frac{c \operatorname{Cos}[d + e x] - b \operatorname{Sin}[d + e x]}{5 \sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]\right)^3} - \\ \frac{2 (c \operatorname{Cos}[d + e x] - b \operatorname{Sin}[d + e x])}{15 (b^2 + c^2) e \left(\sqrt{b^2 + c^2} + b \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]\right)^2} - \frac{2 (c - \sqrt{b^2 + c^2} \operatorname{Sin}[d + e x])}{15 c (b^2 + c^2) e (c \operatorname{Cos}[d + e x] - b \operatorname{Sin}[d + e x])}$$

Result (type 3, 420 leaves):

$$\frac{1}{120 c (b^2 + c^2) e (c \operatorname{Cos}[d + e x] - b \operatorname{Sin}[d + e x])^5} \\ \left( -76 b^4 c - 152 b^2 c^3 - 76 c^5 + 90 b c (b^2 + c^2)^{3/2} \operatorname{Cos}[d + e x] + 20 c (-b^4 + c^4) \operatorname{Cos}[2 (d + e x)] + 10 b^3 c \sqrt{b^2 + c^2} \operatorname{Cos}[3 (d + e x)] + \right. \\ \left. 10 b c^3 \sqrt{b^2 + c^2} \operatorname{Cos}[3 (d + e x)] - 4 b^3 c \sqrt{b^2 + c^2} \operatorname{Cos}[5 (d + e x)] + 4 b c^3 \sqrt{b^2 + c^2} \operatorname{Cos}[5 (d + e x)] + 10 b^4 \sqrt{b^2 + c^2} \operatorname{Sin}[d + e x] + \right. \\ \left. 110 b^2 c^2 \sqrt{b^2 + c^2} \operatorname{Sin}[d + e x] + 100 c^4 \sqrt{b^2 + c^2} \operatorname{Sin}[d + e x] - 40 b^3 c^2 \operatorname{Sin}[2 (d + e x)] - 40 b c^4 \operatorname{Sin}[2 (d + e x)] - 5 b^4 \sqrt{b^2 + c^2} \operatorname{Sin}[3 (d + e x)] + \right. \\ \left. 5 c^4 \sqrt{b^2 + c^2} \operatorname{Sin}[3 (d + e x)] + b^4 \sqrt{b^2 + c^2} \operatorname{Sin}[5 (d + e x)] - 6 b^2 c^2 \sqrt{b^2 + c^2} \operatorname{Sin}[5 (d + e x)] + c^4 \sqrt{b^2 + c^2} \operatorname{Sin}[5 (d + e x)] \right)$$

**Problem 362: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(\sqrt{b^2 + c^2} + b \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]\right)^4} dx$$

Optimal (type 3, 259 leaves, 4 steps):

$$\begin{aligned}
& - \frac{c \cos [d+e x]-b \sin [d+e x]}{7 \sqrt{b^2+c^2} e\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^4}-\frac{3\left(c \cos [d+e x]-b \sin [d+e x]\right)}{35\left(b^2+c^2\right) e\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^3} \\
& \frac{2\left(c \cos [d+e x]-b \sin [d+e x]\right)}{35\left(b^2+c^2\right)^{3 / 2} e\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^2}-\frac{2\left(c-\sqrt{b^2+c^2} \sin [d+e x]\right)}{35 c\left(b^2+c^2\right)^{3 / 2} e\left(c \cos [d+e x]-b \sin [d+e x]\right)}
\end{aligned}$$

Result (type 3, 533 leaves):

$$\begin{aligned}
& \frac{1}{1120 c\left(b^2+c^2\right) e\left(-c \cos [d+e x]+b \sin [d+e x]\right)^7} \\
& \left(832 b^4 c \sqrt{b^2+c^2}+1664 b^2 c^3 \sqrt{b^2+c^2}+832 c^5 \sqrt{b^2+c^2}-1190 b c\left(b^2+c^2\right)^2 \cos [d+e x]+448 c \sqrt{b^2+c^2}\left(b^4-c^4\right) \cos [2(d+e x)]-\right. \\
& 112 b^5 c \cos [3(d+e x)]+56 b^3 c^3 \cos [3(d+e x)]+168 b c^5 \cos [3(d+e x)]+28 b^5 c \cos [5(d+e x)]-28 b c^5 \cos [5(d+e x)]- \\
& 6 b^5 c \cos [7(d+e x)]+20 b^3 c^3 \cos [7(d+e x)]-6 b c^5 \cos [7(d+e x)]-35 b^6 \sin [d+e x]-1295 b^4 c^2 \sin [d+e x]-2485 b^2 c^4 \sin [d+e x]- \\
& 1225 c^6 \sin [d+e x]+896 b^3 c^2 \sqrt{b^2+c^2} \sin [2(d+e x)]+896 b c^4 \sqrt{b^2+c^2} \sin [2(d+e x)]+21 b^6 \sin [3(d+e x)]- \\
& 189 b^4 c^2 \sin [3(d+e x)]-161 b^2 c^4 \sin [3(d+e x)]+49 c^6 \sin [3(d+e x)]-7 b^6 \sin [5(d+e x)]+35 b^4 c^2 \sin [5(d+e x)]+ \\
& \left.35 b^2 c^4 \sin [5(d+e x)]-7 c^6 \sin [5(d+e x)]+b^6 \sin [7(d+e x)]-15 b^4 c^2 \sin [7(d+e x)]+15 b^2 c^4 \sin [7(d+e x)]-c^6 \sin [7(d+e x)]\right)
\end{aligned}$$

Problem 366: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{2 a+2 a \cos [d+e x]+2 c \sin [d+e x]} d x$$

Optimal (type 3, 25 leaves, 2 steps):

$$\frac{\operatorname{Log}\left[a+c \tan \left[\frac{1}{2}(d+e x)\right]\right]}{2 c e}$$

Result (type 3, 57 leaves):

$$\frac{1}{2}\left(-\frac{\operatorname{Log}\left[\cos \left[\frac{1}{2}(d+e x)\right]\right]}{c e}+\frac{\operatorname{Log}\left[a \cos \left[\frac{1}{2}(d+e x)\right]+c \sin \left[\frac{1}{2}(d+e x)\right]\right]}{c e}\right)$$

Problem 369: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(2 a+2 a \cos [d+e x]+2 c \sin [d+e x]\right)^4} d x$$

Optimal (type 3, 207 leaves, 5 steps):

$$-\frac{a(5a^2 + 3c^2) \operatorname{Log}\left[a + c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right]}{32c^7e} - \frac{c \operatorname{Cos}[d+ex] - a \operatorname{Sin}[d+ex]}{48c^2e(a + a \operatorname{Cos}[d+ex] + c \operatorname{Sin}[d+ex])^3} +$$

$$\frac{5(ac \operatorname{Cos}[d+ex] - a^2 \operatorname{Sin}[d+ex])}{96c^4e(a + a \operatorname{Cos}[d+ex] + c \operatorname{Sin}[d+ex])^2} - \frac{c(15a^2 + 4c^2) \operatorname{Cos}[d+ex] - a(15a^2 + 4c^2) \operatorname{Sin}[d+ex]}{96c^6e(a + a \operatorname{Cos}[d+ex] + c \operatorname{Sin}[d+ex])}$$

Result (type 3, 492 leaves):

$$\frac{1}{384c^7e(a + a \operatorname{Cos}[d+ex] + c \operatorname{Sin}[d+ex])^4} \operatorname{Cos}\left[\frac{1}{2}(d+ex)\right] \left(a \operatorname{Cos}\left[\frac{1}{2}(d+ex)\right] + c \operatorname{Sin}\left[\frac{1}{2}(d+ex)\right]\right)$$

$$\left(192(5a^3 + 3ac^2) \operatorname{Cos}\left[\frac{1}{2}(d+ex)\right]^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(d+ex)\right]\right] \left(a \operatorname{Cos}\left[\frac{1}{2}(d+ex)\right] + c \operatorname{Sin}\left[\frac{1}{2}(d+ex)\right]\right)^3 -\right.$$

$$192(5a^3 + 3ac^2) \operatorname{Cos}\left[\frac{1}{2}(d+ex)\right]^3 \operatorname{Log}\left[a \operatorname{Cos}\left[\frac{1}{2}(d+ex)\right] + c \operatorname{Sin}\left[\frac{1}{2}(d+ex)\right]\right] \left(a \operatorname{Cos}\left[\frac{1}{2}(d+ex)\right] + c \operatorname{Sin}\left[\frac{1}{2}(d+ex)\right]\right)^3 +$$

$$\frac{1}{a}c(150a^5c + 130a^3c^3 + 24ac^5 + 3ac(25a^4 + 25a^2c^2 - 4c^4) \operatorname{Cos}[d+ex] - 6(25a^5c + 15a^3c^3 + 4ac^5) \operatorname{Cos}[2(d+ex)] -$$

$$75a^5c \operatorname{Cos}[3(d+ex)] - 35a^3c^3 \operatorname{Cos}[3(d+ex)] - 4ac^5 \operatorname{Cos}[3(d+ex)] + 150a^6 \operatorname{Sin}[d+ex] + 255a^4c^2 \operatorname{Sin}[d+ex] +$$

$$129a^2c^4 \operatorname{Sin}[d+ex] + 12c^6 \operatorname{Sin}[d+ex] + 120a^6 \operatorname{Sin}[2(d+ex)] + 72a^4c^2 \operatorname{Sin}[2(d+ex)] + 36a^2c^4 \operatorname{Sin}[2(d+ex)] +$$

$$\left.30a^6 \operatorname{Sin}[3(d+ex)] - 37a^4c^2 \operatorname{Sin}[3(d+ex)] - 27a^2c^4 \operatorname{Sin}[3(d+ex)] - 4c^6 \operatorname{Sin}[3(d+ex)]\right)$$

Problem 370: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{2a + 2a \operatorname{Cos}[d+ex] + 2a \operatorname{Sin}[d+ex]} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\operatorname{Log}\left[1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right]}{2ae}$$

Result (type 3, 50 leaves):

$$\frac{-\frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(d+ex)\right]\right]}{e} + \frac{\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(d+ex)\right] + \operatorname{Sin}\left[\frac{1}{2}(d+ex)\right]\right]}{e}}{2a}$$

Problem 378: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(2a - 2a \operatorname{Cos}[d+ex] + 2c \operatorname{Sin}[d+ex])^2} dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$\frac{a \operatorname{Log}\left[a + c \operatorname{Cot}\left[\frac{1}{2}(d + ex)\right]\right]}{4c^3 e} - \frac{c \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex]}{4c^2 e (a - a \operatorname{Cos}[d + ex] + c \operatorname{Sin}[d + ex])}$$

Result (type 3, 229 leaves):

$$\begin{aligned} & - \frac{1}{4c^3 e (a - a \operatorname{Cos}[d + ex] + c \operatorname{Sin}[d + ex])^2} \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right) \\ & \left( \operatorname{Cos}[d + ex] \left( a^2 + 2c^2 - 2a^2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]\right] + 2a^2 \operatorname{Log}\left[ c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right] \right) + \right. \\ & \left. a \left( a \left( -1 + 2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]\right] - 2 \operatorname{Log}\left[ c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right] \right) + \right. \\ & \left. c \left( 1 + 2 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]\right] - 2 \operatorname{Log}\left[ c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right] \right) \operatorname{Sin}[d + ex] \right) \end{aligned}$$

**Problem 379: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2a - 2a \operatorname{Cos}[d + ex] + 2c \operatorname{Sin}[d + ex])^3} dx$$

Optimal (type 3, 134 leaves, 4 steps):

$$- \frac{(3a^2 + c^2) \operatorname{Log}\left[a + c \operatorname{Cot}\left[\frac{1}{2}(d + ex)\right]\right]}{16c^5 e} - \frac{c \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex]}{16c^2 e (a - a \operatorname{Cos}[d + ex] + c \operatorname{Sin}[d + ex])^2} + \frac{3(a c \operatorname{Cos}[d + ex] + a^2 \operatorname{Sin}[d + ex])}{16c^4 e (a - a \operatorname{Cos}[d + ex] + c \operatorname{Sin}[d + ex])}$$

Result (type 3, 350 leaves):

$$\begin{aligned} & \frac{1}{8c^5 e (a - a \operatorname{Cos}[d + ex] + c \operatorname{Sin}[d + ex])^3} \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right) \\ & \left( c^2 (-ia + c)(ia + c) \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]^2 - 6a(a^2 + c^2) \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]^3 \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right) - \right. \\ & \left. c^2 \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right)^2 + 4(3a^2 + c^2) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]\right] \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]^2 \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right)^2 - \right. \\ & \left. 4(3a^2 + c^2) \operatorname{Log}\left[ c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right] \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]^2 \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right)^2 + \right. \\ & \left. 3ac \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right)^2 \operatorname{Sin}[d + ex] \right) \end{aligned}$$

### Problem 380: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(2a - 2a \cos[d + ex] + 2c \sin[d + ex])^4} dx$$

Optimal (type 3, 207 leaves, 5 steps):

$$\frac{a(5a^2 + 3c^2) \operatorname{Log}\left[a + c \operatorname{Cot}\left[\frac{1}{2}(d + ex)\right]\right]}{32c^7 e} - \frac{c \cos[d + ex] + a \sin[d + ex]}{48c^2 e (a - a \cos[d + ex] + c \sin[d + ex])^3} +$$

$$\frac{5(a c \cos[d + ex] + a^2 \sin[d + ex])}{96c^4 e (a - a \cos[d + ex] + c \sin[d + ex])^2} - \frac{c(15a^2 + 4c^2) \cos[d + ex] + a(15a^2 + 4c^2) \sin[d + ex]}{96c^6 e (a - a \cos[d + ex] + c \sin[d + ex])}$$

Result (type 3, 494 leaves):

$$\frac{1}{384c^7 e (a - a \cos[d + ex] + c \sin[d + ex])^4} \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right)$$

$$\left( 150a^6 + 130a^4c^2 + 24a^2c^4 - 225a^6 \cos[d + ex] - 255a^4c^2 \cos[d + ex] - 42a^2c^4 \cos[d + ex] - 24c^6 \cos[d + ex] + \right.$$

$$90a^6 \cos[2(d + ex)] + 174a^4c^2 \cos[2(d + ex)] - 15a^6 \cos[3(d + ex)] - 49a^4c^2 \cos[3(d + ex)] + 18a^2c^4 \cos[3(d + ex)] +$$

$$8c^6 \cos[3(d + ex)] - 192(5a^3 + 3a^2c) \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]\right] \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]^3 \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right)^3 +$$

$$192(5a^3 + 3a^2c) \operatorname{Log}\left[ c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right] \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right]^3 \left( c \operatorname{Cos}\left[\frac{1}{2}(d + ex)\right] + a \operatorname{Sin}\left[\frac{1}{2}(d + ex)\right] \right)^3 +$$

$$75a^5c \sin[d + ex] + 75a^3c^3 \sin[d + ex] - 12a^5c \sin[2(d + ex)] - 60a^5c \sin[2(d + ex)] - 156a^3c^3 \sin[2(d + ex)] -$$

$$\left. 12a^5c \sin[2(d + ex)] + 15a^5c \sin[3(d + ex)] + 79a^3c^3 \sin[3(d + ex)] + 20a^5c \sin[3(d + ex)] \right)$$

### Problem 384: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{2a + 2b \cos[d + ex] + 2a \sin[d + ex]} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$-\frac{\operatorname{Log}\left[a + b \operatorname{Cot}\left[\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right]\right]}{2be}$$

Result (type 3, 93 leaves):

$$\frac{1}{2} \left( \frac{\text{Log} \left[ \text{Cos} \left[ \frac{1}{2} (d + ex) \right] + \text{Sin} \left[ \frac{1}{2} (d + ex) \right] \right]}{be} - \frac{\text{Log} \left[ a \text{Cos} \left[ \frac{1}{2} (d + ex) \right] + b \text{Cos} \left[ \frac{1}{2} (d + ex) \right] + a \text{Sin} \left[ \frac{1}{2} (d + ex) \right] - b \text{Sin} \left[ \frac{1}{2} (d + ex) \right] \right]}{be} \right)$$

**Problem 387: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2a + 2b \text{Cos}[d + ex] + 2a \text{Sin}[d + ex])^4} dx$$

Optimal (type 3, 215 leaves, 5 steps):

$$\frac{a(5a^2 + 3b^2) \text{Log} \left[ a + b \text{Cot} \left[ \frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2} \right] \right]}{32b^7e} - \frac{a \text{Cos}[d + ex] - b \text{Sin}[d + ex]}{48b^2e(a + b \text{Cos}[d + ex] + a \text{Sin}[d + ex])^3} +$$

$$\frac{5(a^2 \text{Cos}[d + ex] - ab \text{Sin}[d + ex])}{96b^4e(a + b \text{Cos}[d + ex] + a \text{Sin}[d + ex])^2} - \frac{a(15a^2 + 4b^2) \text{Cos}[d + ex] - b(15a^2 + 4b^2) \text{Sin}[d + ex]}{96b^6e(a + b \text{Cos}[d + ex] + a \text{Sin}[d + ex])}$$

Result (type 3, 632 leaves):

$$\frac{1}{384b^7e} \left( -12a(5a^2 + 3b^2) \text{Log} \left[ \text{Cos} \left[ \frac{1}{2} (d + ex) \right] + \text{Sin} \left[ \frac{1}{2} (d + ex) \right] \right] + 12a(5a^2 + 3b^2) \text{Log} \left[ (a + b) \text{Cos} \left[ \frac{1}{2} (d + ex) \right] + (a - b) \text{Sin} \left[ \frac{1}{2} (d + ex) \right] \right] + \right.$$

$$\left. (b(150a^6 + 130a^4b^2 + 24a^2b^4 - 3a^2(25a^4 - 50a^3b + 5a^2b^2 - 30ab^3 + 4b^4) \text{Cos}[d + ex] - 6a^2(15a^4 + 20a^3b + 9a^2b^2 + 2ab^3 - 2b^4) \right.$$

$$\text{Cos}[2(d + ex)] + 15a^6 \text{Cos}[3(d + ex)] - 30a^5b \text{Cos}[3(d + ex)] - 41a^4b^2 \text{Cos}[3(d + ex)] - 38a^3b^3 \text{Cos}[3(d + ex)] -$$

$$12a^2b^4 \text{Cos}[3(d + ex)] - 8ab^5 \text{Cos}[3(d + ex)] + 225a^6 \text{Sin}[d + ex] + 75a^5b \text{Sin}[d + ex] + 180a^4b^2 \text{Sin}[d + ex] + 15a^3b^3 \text{Sin}[d + ex] +$$

$$27a^2b^4 \text{Sin}[d + ex] + 12ab^5 \text{Sin}[d + ex] + 12b^6 \text{Sin}[d + ex] - 60a^6 \text{Sin}[2(d + ex)] + 120a^5b \text{Sin}[2(d + ex)] + 54a^4b^2 \text{Sin}[2(d + ex)] +$$

$$102a^3b^3 \text{Sin}[2(d + ex)] + 6a^2b^4 \text{Sin}[2(d + ex)] + 6ab^5 \text{Sin}[2(d + ex)] - 15a^6 \text{Sin}[3(d + ex)] - 45a^5b \text{Sin}[3(d + ex)] -$$

$$4a^4b^2 \text{Sin}[3(d + ex)] + 3a^3b^3 \text{Sin}[3(d + ex)] + 15a^2b^4 \text{Sin}[3(d + ex)] + 4ab^5 \text{Sin}[3(d + ex)] + 4b^6 \text{Sin}[3(d + ex)]) \Big/$$

$$\left( (a + b) \left( \text{Cos} \left[ \frac{1}{2} (d + ex) \right] + \text{Sin} \left[ \frac{1}{2} (d + ex) \right] \right) \right)^3 \left( (a + b) \text{Cos} \left[ \frac{1}{2} (d + ex) \right] + (a - b) \text{Sin} \left[ \frac{1}{2} (d + ex) \right] \right)^3 \Big)$$

**Problem 391: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{2a + 2b \text{Cos}[d + ex] - 2a \text{Sin}[d + ex]} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\text{Log} \left[ a + b \text{Tan} \left[ \frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2} \right] \right]}{2be}$$

Result (type 3, 96 leaves):

$$-\frac{\operatorname{Log}\left[\cos\left[\frac{1}{2}(d+ex)\right]-\sin\left[\frac{1}{2}(d+ex)\right]\right]}{2be} + \frac{\operatorname{Log}\left[a\cos\left[\frac{1}{2}(d+ex)\right]+b\cos\left[\frac{1}{2}(d+ex)\right]-a\sin\left[\frac{1}{2}(d+ex)\right]+b\sin\left[\frac{1}{2}(d+ex)\right]\right]}{2be}$$

**Problem 394: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2a + 2b \cos[d+ex] - 2a \sin[d+ex])^4} dx$$

Optimal (type 3, 215 leaves, 5 steps):

$$-\frac{a(5a^2 + 3b^2) \operatorname{Log}\left[a + b \tan\left[\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right]\right]}{32b^7e} + \frac{a \cos[d+ex] + b \sin[d+ex]}{48b^2e(a + b \cos[d+ex] - a \sin[d+ex])^3} - \frac{5(a^2 \cos[d+ex] + ab \sin[d+ex])}{96b^4e(a + b \cos[d+ex] - a \sin[d+ex])^2} + \frac{a(15a^2 + 4b^2) \cos[d+ex] + b(15a^2 + 4b^2) \sin[d+ex]}{96b^6e(a + b \cos[d+ex] - a \sin[d+ex])}$$

Result (type 3, 636 leaves):

$$\frac{1}{384b^7e} \left( 12a(5a^2 + 3b^2) \operatorname{Log}\left[\cos\left[\frac{1}{2}(d+ex)\right]-\sin\left[\frac{1}{2}(d+ex)\right]\right] - 12a(5a^2 + 3b^2) \operatorname{Log}\left[(a+b)\cos\left[\frac{1}{2}(d+ex)\right] + (-a+b)\sin\left[\frac{1}{2}(d+ex)\right]\right] + (b(-150a^6 - 130a^4b^2 - 24a^2b^4 + 3a^2(25a^4 - 50a^3b + 5a^2b^2 - 30ab^3 + 4b^4)) \cos[d+ex] + 6a^2(15a^4 + 20a^3b + 9a^2b^2 + 2ab^3 - 2b^4) \cos[2(d+ex)] - 15a^6 \cos[3(d+ex)] + 30a^5b \cos[3(d+ex)] + 41a^4b^2 \cos[3(d+ex)] + 38a^3b^3 \cos[3(d+ex)] + 12a^2b^4 \cos[3(d+ex)] + 8ab^5 \cos[3(d+ex)] + 225a^6 \sin[d+ex] + 75a^5b \sin[d+ex] + 180a^4b^2 \sin[d+ex] + 15a^3b^3 \sin[d+ex] + 27a^2b^4 \sin[d+ex] + 12ab^5 \sin[d+ex] + 12b^6 \sin[d+ex] - 60a^6 \sin[2(d+ex)] + 120a^5b \sin[2(d+ex)] + 54a^4b^2 \sin[2(d+ex)] + 102a^3b^3 \sin[2(d+ex)] + 6a^2b^4 \sin[2(d+ex)] + 6ab^5 \sin[2(d+ex)] - 15a^6 \sin[3(d+ex)] - 45a^5b \sin[3(d+ex)] - 4a^4b^2 \sin[3(d+ex)] + 3a^3b^3 \sin[3(d+ex)] + 15a^2b^4 \sin[3(d+ex)] + 4ab^5 \sin[3(d+ex)] + 4b^6 \sin[3(d+ex)]) \right) / \left( (a+b) \left( \cos\left[\frac{1}{2}(d+ex)\right] - \sin\left[\frac{1}{2}(d+ex)\right] \right) \right)^3 \left( (a+b) \cos\left[\frac{1}{2}(d+ex)\right] + (-a+b) \sin\left[\frac{1}{2}(d+ex)\right] \right)^3 \right)$$

**Problem 402: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b \cos[d+ex] + c \sin[d+ex])^4} dx$$

Optimal (type 3, 292 leaves, 6 steps):

$$\frac{a (2 a^2 + 3 (b^2 + c^2)) \operatorname{ArcTan}\left[\frac{c+(a-b) \operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]}{\sqrt{a^2-b^2-c^2}}\right]}{(a^2-b^2-c^2)^{7/2} e} + \frac{c \operatorname{Cos}[d+e x] - b \operatorname{Sin}[d+e x]}{3 (a^2-b^2-c^2) e (a+b \operatorname{Cos}[d+e x] + c \operatorname{Sin}[d+e x])^3} +$$

$$\frac{5 (a c \operatorname{Cos}[d+e x] - a b \operatorname{Sin}[d+e x])}{6 (a^2-b^2-c^2)^2 e (a+b \operatorname{Cos}[d+e x] + c \operatorname{Sin}[d+e x])^2} + \frac{c (11 a^2 + 4 (b^2 + c^2)) \operatorname{Cos}[d+e x] - b (11 a^2 + 4 (b^2 + c^2)) \operatorname{Sin}[d+e x]}{6 (a^2-b^2-c^2)^3 e (a+b \operatorname{Cos}[d+e x] + c \operatorname{Sin}[d+e x])}$$

Result (type 3, 606 leaves):

$$\frac{1}{24 e} \left( \frac{24 a (2 a^2 + 3 (b^2 + c^2)) \operatorname{ArcTan}\left[\frac{c+(a-b) \operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]}{\sqrt{-a^2+b^2+c^2}}\right]}{(-a^2+b^2+c^2)^{7/2}} + \frac{1}{b (-a^2+b^2+c^2)^3 (a+b \operatorname{Cos}[d+e x] + c \operatorname{Sin}[d+e x])^3} \right.$$

$$\left. \begin{aligned} & (44 a^5 c + 82 a^3 b^2 c + 24 a b^4 c + 82 a^3 c^3 + 48 a b^2 c^3 + 24 a c^5 + 30 a^2 b c (2 a^2 + 3 (b^2 + c^2)) \operatorname{Cos}[d+e x] - \\ & 6 a c (-2 b^4 + 2 b^2 c^2 + 4 c^4 + a^2 (7 b^2 + 11 c^2)) \operatorname{Cos}[2 (d+e x)] - 22 a^2 b^3 c \operatorname{Cos}[3 (d+e x)] - 8 b^5 c \operatorname{Cos}[3 (d+e x)] - 22 a^2 b c^3 \operatorname{Cos}[3 (d+e x)] - \\ & 16 b^3 c^3 \operatorname{Cos}[3 (d+e x)] - 8 b c^5 \operatorname{Cos}[3 (d+e x)] + 72 a^4 b^2 \operatorname{Sin}[d+e x] - 9 a^2 b^4 \operatorname{Sin}[d+e x] + 12 b^6 \operatorname{Sin}[d+e x] + 132 a^4 c^2 \operatorname{Sin}[d+e x] + \\ & 72 a^2 b^2 c^2 \operatorname{Sin}[d+e x] + 36 b^4 c^2 \operatorname{Sin}[d+e x] + 81 a^2 c^4 \operatorname{Sin}[d+e x] + 36 b^2 c^4 \operatorname{Sin}[d+e x] + 12 c^6 \operatorname{Sin}[d+e x] + 54 a^3 b^3 \operatorname{Sin}[2 (d+e x)] + \\ & 6 a b^5 \operatorname{Sin}[2 (d+e x)] + 78 a^3 b c^2 \operatorname{Sin}[2 (d+e x)] + 48 a b^3 c^2 \operatorname{Sin}[2 (d+e x)] + 42 a b c^4 \operatorname{Sin}[2 (d+e x)] + 11 a^2 b^4 \operatorname{Sin}[3 (d+e x)] + \\ & 4 b^6 \operatorname{Sin}[3 (d+e x)] + 4 b^4 c^2 \operatorname{Sin}[3 (d+e x)] - 11 a^2 c^4 \operatorname{Sin}[3 (d+e x)] - 4 b^2 c^4 \operatorname{Sin}[3 (d+e x)] - 4 c^6 \operatorname{Sin}[3 (d+e x)] \end{aligned} \right)$$

**Problem 403: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (2 + 3 \operatorname{Cos}[d+e x] + 5 \operatorname{Sin}[d+e x])^{5/2} dx$$

Optimal (type 4, 185 leaves, 7 steps):

$$\frac{796 \sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2} (d+e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]), \frac{2}{15} (17 - \sqrt{34})\right]}{15 e} + \frac{64 \operatorname{EllipticF}\left[\frac{1}{2} (d+e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]), \frac{2}{15} (17 - \sqrt{34})\right]}{\sqrt{2 + \sqrt{34}} e} -$$

$$\frac{32 (5 \operatorname{Cos}[d+e x] - 3 \operatorname{Sin}[d+e x]) \sqrt{2 + 3 \operatorname{Cos}[d+e x] + 5 \operatorname{Sin}[d+e x]}}{15 e} - \frac{2 (5 \operatorname{Cos}[d+e x] - 3 \operatorname{Sin}[d+e x]) (2 + 3 \operatorname{Cos}[d+e x] + 5 \operatorname{Sin}[d+e x])^{3/2}}{5 e}$$

Result (type 6, 536 leaves):



$$\begin{aligned}
& \frac{1}{e} \sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]} \left( \frac{796}{25} - \frac{44}{3} \cos [d + e x] - 6 \cos [2 (d + e x)] + \frac{44}{5} \sin [d + e x] - \frac{16}{5} \sin [2 (d + e x)] \right) + \\
& \frac{1}{15 e} 1276 \sqrt{\frac{34}{17 + \sqrt{34}}} \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{2 + \sqrt{34} \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]}{\sqrt{34} \left( 1 - \sqrt{\frac{2}{17}} \right)}, -\frac{2 + \sqrt{34} \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]}{\sqrt{34} \left( -1 - \sqrt{\frac{2}{17}} \right)} \right] \\
& \sec [d + e x + \operatorname{ArcTan} [\frac{3}{5}]] \sqrt{1 - \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]} \sqrt{-\frac{1 + \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]}{-17 + \sqrt{34}}} \sqrt{2 + \sqrt{34} \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]} + \frac{1}{75 e} \\
& 13532 \left( - \left( \left( 5 \sqrt{\frac{1}{34} (17 + \sqrt{34})} \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}{\sqrt{34} \left( 1 - \sqrt{\frac{2}{17}} \right)}, -\frac{2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}{\sqrt{34} \left( -1 - \sqrt{\frac{2}{17}} \right)} \right] \right) \right. \\
& \left. \left. \sin [d + e x - \operatorname{ArcTan} [\frac{5}{3}]] \right) \sqrt{17 \sqrt{1 - \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]} \sqrt{-\frac{1 + \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}{-17 + \sqrt{34}}}} \right. \\
& \left. \left. \sqrt{2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]} \right) \left( -\frac{\frac{3}{17} (2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]])}{\sqrt{2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}} - \frac{5 \sin [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}{\sqrt{34}} \right) \right)
\end{aligned}$$

**Problem 404:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (2 + 3 \cos [d + e x] + 5 \sin [d + e x])^{3/2} dx$$

Optimal (type 4, 139 leaves, 6 steps):



**Problem 405: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\frac{2 \sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{e}$$

Result (type 6, 326 leaves):

$$\frac{1}{15 e \sqrt{2 + \sqrt{34}} \cos \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right] \sqrt{\sin \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right]^2} \left( -15 \sqrt{30} \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{34} + 17 \cos \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{-17 + \sqrt{34}}, \frac{\sqrt{34} + 17 \cos \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{17 + \sqrt{34}}\right] \sin \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right] + \left( -75 \cos [d + e x] + 45 \sin [d + e x] + 2 \sqrt{30} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\sqrt{34} + 17 \sin \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{-17 + \sqrt{34}}, \frac{\sqrt{34} + 17 \sin \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{17 + \sqrt{34}}\right] \sqrt{\cos \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]^2} \sqrt{2 + \sqrt{34}} \cos \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right] \sec \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right] \sqrt{2 + \sqrt{34}} \sin \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right] \sqrt{\sin \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right]^2} \right)$$

**Problem 406: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]}} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2}\left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15}\left(17 - \sqrt{34}\right)\right]}{\sqrt{2 + \sqrt{34}} e}$$

Result (type 6, 128 leaves):

$$\frac{1}{e} \sqrt{\frac{2}{15}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\sqrt{34} + 17 \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{-17 + \sqrt{34}}, \frac{\sqrt{34} + 17 \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{17 + \sqrt{34}}\right]$$

$$\sqrt{\operatorname{Cos}\left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]^2 \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]} \sqrt{2 + \sqrt{34}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]$$

**Problem 407: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2 + 3 \operatorname{Cos}[d + e x] + 5 \operatorname{Sin}[d + e x])^{3/2}} dx$$

Optimal (type 4, 94 leaves, 3 steps):

$$-\frac{\sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2}\left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15}\left(17 - \sqrt{34}\right)\right]}{15 e} - \frac{5 \operatorname{Cos}[d + e x] - 3 \operatorname{Sin}[d + e x]}{15 e \sqrt{2 + 3 \operatorname{Cos}[d + e x] + 5 \operatorname{Sin}[d + e x]}}$$

Result (type 6, 528 leaves):



$$\frac{4\sqrt{2+\sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2}\left(d+ex - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15}\left(17-\sqrt{34}\right)\right]}{675e} + \frac{\operatorname{EllipticF}\left[\frac{1}{2}\left(d+ex - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15}\left(17-\sqrt{34}\right)\right]}{45\sqrt{2+\sqrt{34}}e}$$

$$\frac{5\cos[d+ex] - 3\sin[d+ex]}{45e(2+3\cos[d+ex]+5\sin[d+ex])^{3/2}} + \frac{4(5\cos[d+ex] - 3\sin[d+ex])}{675e\sqrt{2+3\cos[d+ex]+5\sin[d+ex]}}$$

Result (type 6, 564 leaves):

$$\frac{\sqrt{2+3\cos[d+ex]+5\sin[d+ex]} \left( \frac{136}{10125} + \frac{-115-136\sin[d+ex]}{2025(2+3\cos[d+ex]+5\sin[d+ex])} + \frac{2(5+17\sin[d+ex])}{135(2+3\cos[d+ex]+5\sin[d+ex])^2} \right)}{e} + \frac{1}{675e}$$

$$23 \sqrt{\frac{17}{2(17+\sqrt{34})}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{2+\sqrt{34}\sin[d+ex+\operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34}\left(1-\sqrt{\frac{2}{17}}\right)}, -\frac{2+\sqrt{34}\sin[d+ex+\operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34}\left(-1-\sqrt{\frac{2}{17}}\right)}\right]$$

$$\sec[d+ex+\operatorname{ArcTan}[\frac{3}{5}]] \sqrt{1-\sin[d+ex+\operatorname{ArcTan}[\frac{3}{5}]]} \sqrt{-\frac{1+\sin[d+ex+\operatorname{ArcTan}[\frac{3}{5}]]}{-17+\sqrt{34}}} \sqrt{2+\sqrt{34}\sin[d+ex+\operatorname{ArcTan}[\frac{3}{5}]]} + \frac{1}{3375e}$$

$$68 \left( - \left( \left( \left( 5 \sqrt{\frac{1}{34}(17+\sqrt{34})} \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{2+\sqrt{34}\cos[d+ex-\operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34}\left(1-\sqrt{\frac{2}{17}}\right)}, -\frac{2+\sqrt{34}\cos[d+ex-\operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34}\left(-1-\sqrt{\frac{2}{17}}\right)}\right] \right) \right) \right.$$

$$\left. \left. \left. \sin\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right] \right) \right) \right) \left( \left( 17 \sqrt{1-\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]} \sqrt{-\frac{1+\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{-17+\sqrt{34}}} \right. \right.$$

$$\left. \left. \left. \sqrt{2+\sqrt{34}\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]} \right) \right) \right) - \frac{\frac{3}{17}\left(2+\sqrt{34}\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]\right) - \frac{5\sin\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{\sqrt{34}}}{\sqrt{2+\sqrt{34}\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}}$$

Problem 409: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(2 + 3 \cos [d + e x] + 5 \sin [d + e x])^{7/2}} dx$$

Optimal (type 4, 233 leaves, 8 steps):

$$\frac{199 \sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} (17 - \sqrt{34})\right]}{101250 e} - \frac{8 \operatorname{EllipticF}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} (17 - \sqrt{34})\right]}{3375 \sqrt{2 + \sqrt{34}} e} - \frac{5 \cos [d + e x] - 3 \sin [d + e x]}{75 e (2 + 3 \cos [d + e x] + 5 \sin [d + e x])^{5/2}} + \frac{8 (5 \cos [d + e x] - 3 \sin [d + e x])}{3375 e (2 + 3 \cos [d + e x] + 5 \sin [d + e x])^{3/2}} - \frac{199 (5 \cos [d + e x] - 3 \sin [d + e x])}{101250 e \sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]}}$$

Result (type 6, 598 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]} \\
& \left( -\frac{3383}{759375} + \frac{-305 - 272 \sin [d + e x]}{10125 (2 + 3 \cos [d + e x] + 5 \sin [d + e x])^2} + \frac{2 (5 + 17 \sin [d + e x])}{225 (2 + 3 \cos [d + e x] + 5 \sin [d + e x])^3} + \frac{1595 + 3383 \sin [d + e x]}{151875 (2 + 3 \cos [d + e x] + 5 \sin [d + e x])} \right) - \\
& \frac{1}{50625 e} \sqrt{\frac{17}{2 (17 + \sqrt{34})}} \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{2 + \sqrt{34} \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}, -\frac{2 + \sqrt{34} \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)} \right] \\
& \operatorname{Sec} [d + e x + \operatorname{ArcTan} [\frac{3}{5}]] \sqrt{1 - \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]} \sqrt{-\frac{1 + \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]}{-17 + \sqrt{34}}} \sqrt{2 + \sqrt{34} \sin [d + e x + \operatorname{ArcTan} [\frac{3}{5}]]} - \frac{1}{506250 e} \\
& 3383 \left( - \left( \left( 5 \sqrt{\frac{1}{34} (17 + \sqrt{34})} \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}, -\frac{2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)} \right] \right) \right. \\
& \left. \left. \operatorname{Sin} [d + e x - \operatorname{ArcTan} [\frac{5}{3}]] \right) \sqrt{17} \sqrt{1 - \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]} \sqrt{-\frac{1 + \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}{-17 + \sqrt{34}}} \right. \\
& \left. \left. \sqrt{2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]} \right) - \frac{\frac{3}{17} (2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]] - \frac{5 \sin [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}{\sqrt{34}})}{\sqrt{2 + \sqrt{34} \cos [d + e x - \operatorname{ArcTan} [\frac{5}{3}]]}} \right)
\end{aligned}$$

**Problem 410: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \cos [d + e x] + c \sin [d + e x])^{5/2} dx$$

Optimal (type 4, 347 leaves, 7 steps):



$$\begin{aligned}
& - \frac{16 (a c \cos [d+e x]-a b \sin [d+e x]) \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}}{15 e} \\
& - \frac{2 (c \cos [d+e x]-b \sin [d+e x]) (a+b \cos [d+e x]+c \sin [d+e x])^{3 / 2}}{5 e} \\
& - \left( 2 (23 a^2+9 (b^2+c^2)) \operatorname{EllipticE}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a+b \cos [d+e x]+c \sin [d+e x]} \right) / \\
& - \left( 15 e \sqrt{\frac{a+b \cos [d+e x]+c \sin [d+e x]}{a+\sqrt{b^2+c^2}}} - \frac{16 a (a^2-b^2-c^2) \operatorname{EllipticF}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b \cos [d+e x]+c \sin [d+e x]}{a+\sqrt{b^2+c^2}}}}{15 e \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}} \right)
\end{aligned}$$

Result (type 6, 3767 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{a+b \cos [d+e x]+c \sin [d+e x]} \\
& \left( \frac{2 b (23 a^2+9 b^2+9 c^2)}{15 c} - \frac{22}{15} a c \cos [d+e x] - \frac{2}{5} b c \cos [2 (d+e x)] + \frac{22}{15} a b \sin [d+e x] + \frac{1}{5} (b^2-c^2) \sin [2 (d+e x)] \right) + \frac{1}{\sqrt{1+\frac{b^2}{c^2}} c e} \\
& 2 a^3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}}-c \sqrt{\frac{b^2+c^2}{c^2}} \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{a+c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a+c \sqrt{\frac{b^2+c^2}{c^2}} \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}}+c \sqrt{\frac{b^2+c^2}{c^2}} \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{-a+c \sqrt{\frac{b^2+c^2}{c^2}}}} + \\
& \frac{1}{15 \sqrt{1+\frac{b^2}{c^2}} c e} 34 a b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c}\right]
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \\
& \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{15 \sqrt{1 + \frac{b^2}{c^2}} e} \\
& 34 a c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \\
& \frac{1}{15 c e} 23 a^2 b^2 \left( \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right), -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right) \right) \right)
\end{aligned}$$

$$\left. \left. \left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \right)$$

$$\left. \left. \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) + \frac{1}{5 c e}$$

$$3 b^4 \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right) \right) \right)$$

$$\left. \left. \left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \right)$$

$$\left. \left. \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) + \frac{1}{15 e}$$

$$\begin{aligned}
 & 23 a^2 c \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right) \right) \right. \\
 & \left. \left. \left. \sin \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \right. \\
 & \left. \left. \left. \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right)} - \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}}}{b^2+c^2} \right) \right) + \frac{1}{5e}
 \end{aligned}$$
  

$$\begin{aligned}
 & 6 b^2 c \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right) \right) \right) \right. \\
 & \left. \left. \left. \sin \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \right.
 \end{aligned}$$

$$\begin{aligned}
& \left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}} - \frac{\frac{2b\left(a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]\right)}{b^2+c^2} - \frac{c \sin\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 + \frac{c^2}{b^2}}}}{b\sqrt{1 + \frac{c^2}{b^2}}} \right) + \frac{1}{5e} \\
& 3c^3 \left( \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b\sqrt{1 + \frac{c^2}{b^2}}}\right)}, -\frac{a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b\sqrt{1 + \frac{c^2}{b^2}}}\right)}\right] \right) \right) \right) \\
& \left. \sin\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) \left( b\sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b\sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \right) \\
& \left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}} - \frac{\frac{2b\left(a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]\right)}{b^2+c^2} - \frac{c \sin\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 + \frac{c^2}{b^2}}}}{b\sqrt{1 + \frac{c^2}{b^2}}} \right)
\end{aligned}$$

**Problem 411:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + b \cos[d + ex] + c \sin[d + ex])^{3/2} dx$$

Optimal (type 4, 283 leaves, 6 steps):

$$\begin{aligned}
& - \frac{2 (c \cos [d + e x] - b \sin [d + e x]) \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}}{3 e} + \\
& \frac{8 a \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right] \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}}{3 e \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}}} - \\
& \frac{2 (a^2 - b^2 - c^2) \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right] \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}}}{3 e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}}
\end{aligned}$$

Result (type 6, 2190 leaves):

$$\begin{aligned}
& \frac{\left(\frac{8 a b}{3 c} - \frac{2}{3} c \cos [d + e x] + \frac{2}{3} b \sin [d + e x]\right) \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}}{e} + \frac{1}{\sqrt{1 + \frac{b^2}{c^2} c e}} \\
& 2 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]} \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \\
& \frac{1}{3 \sqrt{1 + \frac{b^2}{c^2} c e}} 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right]
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \\
& \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{3 \sqrt{1 + \frac{b^2}{c^2}} e} \\
& 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \\
& \frac{1}{3 c e} 4 a b^2 \left( \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}\right] \right) \right)
\end{aligned}$$

$$\left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2} \right) + \frac{1}{3e}$$

$$4ac \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right), -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right) \right)$$

$$\left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2} \right) + \frac{1}{3e}$$



**Problem 412:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \cos[d + ex] + c \sin[d + ex]} \, dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$2 \operatorname{EllipticE}\left[\frac{1}{2}(d + ex - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a + b \cos[d + ex] + c \sin[d + ex]}$$

$$e \sqrt{\frac{a + b \cos[d + ex] + c \sin[d + ex]}{a + \sqrt{b^2 + c^2}}}$$

Result (type 6, 1408 leaves):

$$\frac{2b\sqrt{a + b \cos[d + ex] + c \sin[d + ex]}}{ce} + \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} ce}$$

$$2a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + ex + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} +$$

$$\frac{1}{ce} b^2 \left( \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos[d + ex - \operatorname{ArcTan}[\frac{c}{b}]]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right), -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos[d + ex - \operatorname{ArcTan}[\frac{c}{b}]]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right) \right)$$

$$\left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2} \right) + \frac{1}{e}$$

$$c \left( \left( \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right)$$

$$\left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2} \right)$$

**Problem 413:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + b \cos [d + e x] + c \sin [d + e x]}} dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b \cos [d+e x]+c \sin [d+e x]}{a+\sqrt{b^2+c^2}}}}{e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}}$$

Result (type 6, 285 leaves):

$$\frac{1}{\sqrt{1 + \frac{b^2}{c^2} c} e} {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{a + \sqrt{1 + \frac{b^2}{c^2} c} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a - \sqrt{1 + \frac{b^2}{c^2} c}}, \frac{a + \sqrt{1 + \frac{b^2}{c^2} c} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + \sqrt{1 + \frac{b^2}{c^2} c}}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]$$

$$\sqrt{\frac{-\sqrt{1 + \frac{b^2}{c^2} c} (-1 + \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right])}{a + \sqrt{1 + \frac{b^2}{c^2} c}}} \sqrt{\frac{\sqrt{1 + \frac{b^2}{c^2} c} (1 + \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right])}{-a + \sqrt{1 + \frac{b^2}{c^2} c}}} \sqrt{a + \sqrt{1 + \frac{b^2}{c^2} c} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}$$

**Problem 414:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos [d + e x] + c \sin [d + e x])^{3/2}} dx$$

Optimal (type 4, 186 leaves, 3 steps):

$$\frac{2 (c \cos [d + e x] - b \sin [d + e x])}{(a^2 - b^2 - c^2) e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}} + \frac{2 \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}}{(a^2 - b^2 - c^2) e \sqrt{\frac{a+b \cos [d+e x]+c \sin [d+e x]}{a+\sqrt{b^2+c^2}}}}$$

Result (type 6, 1540 leaves):

$$\frac{\sqrt{a + b \cos [d + e x] + c \sin [d + e x]} \left( -\frac{2 (b^2 + c^2)}{b c (-a^2 + b^2 + c^2)} + \frac{2 (a c + b^2 \sin [d + e x] + c^2 \sin [d + e x])}{b (-a^2 + b^2 + c^2) (a + b \cos [d + e x] + c \sin [d + e x])} \right)}{e}$$

$$\left( 2 a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right]]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right]]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right] \right)$$

$$\operatorname{Sec} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right]]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right]]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right]]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \left/ \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2) e \right) - \frac{1}{c (-a^2 + b^2 + c^2) e} \right.$$

$$b^2 \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos [d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right]]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right), -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos [d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right]]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right) \right)$$

$$\operatorname{Sin} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \left/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos [d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right]]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos [d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right]]} \right) \right.$$

$$\begin{aligned}
& \left( \frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} - \frac{1}{(-a^2 + b^2 + c^2) e} \\
& c \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right) \right. \\
& \left. \left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \right) \\
& \left( \frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}}
\end{aligned}$$

Problem 415: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos [d + e x] + c \sin [d + e x])^{5/2}} dx$$

Optimal (type 4, 382 leaves, 7 steps):

$$\frac{2 (c \cos [d + e x] - b \sin [d + e x])}{3 (a^2 - b^2 - c^2) e (a + b \cos [d + e x] + c \sin [d + e x])^{3/2}} + \frac{8 (a c \cos [d + e x] - a b \sin [d + e x])}{3 (a^2 - b^2 - c^2)^2 e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}} +$$

$$\frac{8 a \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right] \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}}{3 (a^2 - b^2 - c^2)^2 e \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}}} -$$

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right] \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}}}{3 (a^2 - b^2 - c^2) e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}}$$

Result (type 6, 2408 leaves):

$$\frac{1}{e} \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}$$

$$\left( \frac{8 a (b^2 + c^2)}{3 b c (a^2 - b^2 - c^2)^2} + \frac{2 (a c + b^2 \sin [d + e x] + c^2 \sin [d + e x])}{3 b (-a^2 + b^2 + c^2) (a + b \cos [d + e x] + c \sin [d + e x])^2} - \frac{2 (3 a^2 c + b^2 c + c^3 + 4 a b^2 \sin [d + e x] + 4 a c^2 \sin [d + e x])}{3 b (-a^2 + b^2 + c^2)^2 (a + b \cos [d + e x] + c \sin [d + e x])} \right) +$$

$$\left[ 2 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \right]$$

$$\operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin [d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \Big/ \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^2 e \right) +$$

$$\left[ 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \right]$$

$$\operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \Big/ \left( 3 \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^2 e \right) +$$

$$\left[ 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \right]$$

$$\text{Sec}\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \left/ \left( 3 \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)^2 e \right) + \frac{1}{3 c (-a^2 + b^2 + c^2)^2 e} \right.$$

$$4 a b^2 \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)} \right] \right) \right.$$

$$\left. \text{Sin}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \left/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right.$$

$$\left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right)}{b^2 + c^2} - \frac{c \text{Sin}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) + \frac{1}{3 (-a^2 + b^2 + c^2)^2 e}$$



$$\begin{aligned}
& 4 a c \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \right) \right. \\
& \left. \left. \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \right. \\
& \left. \left. \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2 b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}} \right) \right) \\
& \left. \sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
\end{aligned}$$

Problem 416: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b \cos [d+e x]+c \sin [d+e x])^{7/2}} dx$$

Optimal (type 4, 490 leaves, 8 steps):

$$\frac{2 (c \cos [d+e x]-b \sin [d+e x])}{5\left(a^2-b^2-c^2\right) e\left(a+b \cos [d+e x]+c \sin [d+e x]\right)^{5 / 2}}+\frac{16\left(a c \cos [d+e x]-a b \sin [d+e x]\right)}{15\left(a^2-b^2-c^2\right)^2 e\left(a+b \cos [d+e x]+c \sin [d+e x]\right)^{3 / 2}}+$$

$$\left(2\left(23 a^2+9\left(b^2+c^2\right)\right) \operatorname{EllipticE}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}\right) /$$

$$\left(15\left(a^2-b^2-c^2\right)^3 e \sqrt{\frac{a+b \cos [d+e x]+c \sin [d+e x]}{a+\sqrt{b^2+c^2}}}\right)-\frac{16 a \operatorname{EllipticF}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b \cos [d+e x]+c \sin [d+e x]}{a+\sqrt{b^2+c^2}}}}{15\left(a^2-b^2-c^2\right)^2 e \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}}+$$

$$\frac{2\left(c\left(23 a^2+9\left(b^2+c^2\right)\right) \cos [d+e x]-b\left(23 a^2+9\left(b^2+c^2\right)\right) \sin [d+e x]\right)}{15\left(a^2-b^2-c^2\right)^3 e \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}}$$

Result (type 6, 4116 leaves):

$$\frac{1}{e} \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}\left(-\frac{2\left(b^2+c^2\right)\left(23 a^2+9 b^2+9 c^2\right)}{15 b c\left(-a^2+b^2+c^2\right)^3}+\right.$$

$$\frac{2\left(a c+b^2 \sin [d+e x]+c^2 \sin [d+e x]\right)}{5 b\left(-a^2+b^2+c^2\right)\left(a+b \cos [d+e x]+c \sin [d+e x]\right)^3}-\frac{2\left(5 a^2 c+3 b^2 c+3 c^3+8 a b^2 \sin [d+e x]+8 a c^2 \sin [d+e x]\right)}{15 b\left(-a^2+b^2+c^2\right)^2\left(a+b \cos [d+e x]+c \sin [d+e x]\right)^2}+$$

$$\left.\left(2\left(15 a^3 c+17 a b^2 c+17 a c^3+23 a^2 b^2 \sin [d+e x]+9 b^4 \sin [d+e x]+23 a^2 c^2 \sin [d+e x]+18 b^2 c^2 \sin [d+e x]+9 c^4 \sin [d+e x]\right)\right) /$$

$$\left.\left(15 b\left(-a^2+b^2+c^2\right)^3\left(a+b \cos [d+e x]+c \sin [d+e x]\right)\right)\right)-$$

$$\left[2 a^3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2},-\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c},-\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c}\right]\right]$$

$$\operatorname{Sec}\left[d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}}-c \sqrt{\frac{b^2+c^2}{c^2}} \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}{a+c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a+c \sqrt{\frac{b^2+c^2}{c^2}} \sin [d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \Big/ \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^3 e \right) -$$

$$\left[ 34 a b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \right]$$

$$\operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \Big/ \left( 15 \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^3 e \right) -$$

$$\left[ 34 a c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \right]$$

$$\text{Sec}\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \left/ \left( 15 \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)^3 e \right) - \frac{1}{15 c (-a^2 + b^2 + c^2)^3 e} \right.$$

$$23 a^2 b^2 \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)} \right] \right) \right)$$

$$\text{Sin}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \left/ \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right)$$

$$\sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \text{Cos}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right)}{b^2 + c^2} - \frac{c \text{Sin}\left[d + e x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) - \frac{1}{5 c (-a^2 + b^2 + c^2)^3 e}$$

$$\begin{aligned}
& 3 b^4 \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]} + \frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}{b\sqrt{1+\frac{c^2}{b^2}} \left(1-\frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}{b\sqrt{1+\frac{c^2}{b^2}} \left(-1-\frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)} \right] \right. \right. \\
& \left. \left. \sin[d+ex-\operatorname{ArcTan}[\frac{c}{b}]] \right) \right) / \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}} \right) \\
& \left. \left( \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]] \right)}{b^2+c^2} - \frac{c \sin[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}{b\sqrt{1+\frac{c^2}{b^2}}} \right) - \frac{1}{15(-a^2+b^2+c^2)^3 e}
\end{aligned}$$

$$\begin{aligned}
& 23 a^2 c \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]} + \frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}{b\sqrt{1+\frac{c^2}{b^2}} \left(1-\frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}{b\sqrt{1+\frac{c^2}{b^2}} \left(-1-\frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)} \right] \right. \right. \\
& \left. \left. \sin[d+ex-\operatorname{ArcTan}[\frac{c}{b}]] \right) \right) / \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{b}]]}} \right)
\end{aligned}$$

$$\left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}} - \frac{2b\left(a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]\right) - \frac{c \sin\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2} \right) - \frac{1}{5(-a^2 + b^2 + c^2)^3 e}$$

$$6b^2c \left( \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b\sqrt{1 + \frac{c^2}{b^2}}}\right)}, -\frac{a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b\sqrt{1 + \frac{c^2}{b^2}}}\right)}\right] \right) \right)$$

$$\left. \sin\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right/ \left( b\sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b\sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \right)$$

$$\left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}} - \frac{2b\left(a + b\sqrt{1 + \frac{c^2}{b^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]\right) - \frac{c \sin\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2} \right) - \frac{1}{5(-a^2 + b^2 + c^2)^3 e}$$

$$\begin{aligned}
& 3c^3 \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex - \operatorname{ArcTan}[\frac{c}{b}]]}{b\sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex - \operatorname{ArcTan}[\frac{c}{b}]]}{b\sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)} \right] \right) \right. \\
& \left. \left. \sin[d+ex - \operatorname{ArcTan}[\frac{c}{b}]] \right) \right) / \left( \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos[d+ex - \operatorname{ArcTan}[\frac{c}{b}]]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos[d+ex - \operatorname{ArcTan}[\frac{c}{b}]]} \right) \right. \\
& \left. \left. \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos[d+ex - \operatorname{ArcTan}[\frac{c}{b}]]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right) \right) - \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex - \operatorname{ArcTan}[\frac{c}{b}]] \right)}{b^2+c^2} - \frac{c \sin[d+ex - \operatorname{ArcTan}[\frac{c}{b}]]}{b\sqrt{1+\frac{c^2}{b^2}}} \right) \\
& \left. \sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos[d+ex - \operatorname{ArcTan}[\frac{c}{b}]]} \right)
\end{aligned}$$

Problem 420: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{5+4\cos[d+ex]+3\sin[d+ex]}} dx$$

Optimal (type 3, 48 leaves, 3 steps):

$$\frac{\sqrt{\frac{2}{5}} \operatorname{ArcTanh} \left[ \frac{\sin[d+ex - \operatorname{ArcTan}[\frac{3}{4}]]}{\sqrt{2} \sqrt{1+\cos[d+ex - \operatorname{ArcTan}[\frac{3}{4}]]}} \right]}{e}$$

Result (type 3, 101 leaves):

$$- \left( \left( \left( \frac{2}{5} + \frac{6i}{5} \right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \operatorname{ArcTan} \left[ \left( \frac{1}{10} + \frac{3i}{10} \right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \left( -1 + 3 \operatorname{Tan} \left[ \frac{1}{4} (d + ex) \right] \right) \right] \left( 3 \operatorname{Cos} \left[ \frac{1}{2} (d + ex) \right] + \operatorname{Sin} \left[ \frac{1}{2} (d + ex) \right] \right) \right) / \right. \\ \left. \left( e \sqrt{5 + 4 \operatorname{Cos} [d + ex] + 3 \operatorname{Sin} [d + ex]} \right) \right)$$

**Problem 421: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(5 + 4 \operatorname{Cos} [d + ex] + 3 \operatorname{Sin} [d + ex])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh} \left[ \frac{\operatorname{Sin} [d + ex - \operatorname{ArcTan} [\frac{3}{4}]]}{\sqrt{2} \sqrt{1 + \operatorname{Cos} [d + ex - \operatorname{ArcTan} [\frac{3}{4}]]}} \right]}{10 \sqrt{10} e} - \frac{3 \operatorname{Cos} [d + ex] - 4 \operatorname{Sin} [d + ex]}{10 e (5 + 4 \operatorname{Cos} [d + ex] + 3 \operatorname{Sin} [d + ex])^{3/2}}$$

Result (type 3, 154 leaves):

$$- \left( \left( \left( \frac{1}{250} - \frac{i}{125} \right) \left( 3 \operatorname{Cos} \left[ \frac{1}{2} (d + ex) \right] + \operatorname{Sin} \left[ \frac{1}{2} (d + ex) \right] \right) \right. \right. \\ \left. \left( (5 + 10i) \left( \operatorname{Cos} \left[ \frac{1}{2} (d + ex) \right] - 3 \operatorname{Sin} \left[ \frac{1}{2} (d + ex) \right] \right) - (1 - i) \sqrt{20 + 15i} \operatorname{ArcTan} \left[ \left( \frac{1}{10} + \frac{3i}{10} \right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \left( -1 + 3 \operatorname{Tan} \left[ \frac{1}{4} (d + ex) \right] \right) \right] \right) \right. \\ \left. \left( 3 \operatorname{Cos} \left[ \frac{1}{2} (d + ex) \right] + \operatorname{Sin} \left[ \frac{1}{2} (d + ex) \right] \right)^2 \right) / \left( e (5 + 4 \operatorname{Cos} [d + ex] + 3 \operatorname{Sin} [d + ex])^{3/2} \right)$$

**Problem 422: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(5 + 4 \operatorname{Cos} [d + ex] + 3 \operatorname{Sin} [d + ex])^{5/2}} dx$$

Optimal (type 3, 142 leaves, 5 steps):



$$\frac{3 \operatorname{ArcTanh}\left[\frac{\sin\left[d+e x-\operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}{\sqrt{2} \sqrt{1+\cos\left[d+e x-\operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}}\right]}{400 \sqrt{10} e} - \frac{3 \cos [d+e x]-4 \sin [d+e x]}{20 e\left(5+4 \cos [d+e x]+3 \sin [d+e x]\right)^{5 / 2}} - \frac{3\left(3 \cos [d+e x]-4 \sin [d+e x]\right)}{400 e\left(5+4 \cos [d+e x]+3 \sin [d+e x]\right)^{3 / 2}}$$

Result (type 3, 180 leaves):

$$-\left(\left(\left(\frac{1}{20000}-\frac{i}{10000}\right)\left(3 \cos \left[\frac{1}{2}(d+e x)\right]+\sin \left[\frac{1}{2}(d+e x)\right]\right)\right.\right. \\ \left.\left.\left(\left(-6+6 i\right) \sqrt{20+15 i} \operatorname{ArcTan}\left[\left(\frac{1}{10}+\frac{3 i}{10}\right) \sqrt{\frac{4}{5}+\frac{3 i}{5}}\left(-1+3 \tan \left[\frac{1}{4}(d+e x)\right]\right)\right]\left(3 \cos \left[\frac{1}{2}(d+e x)\right]+\sin \left[\frac{1}{2}(d+e x)\right]\right)^4+(5+10 i)\right.\right.\right. \\ \left.\left.\left(55 \cos \left[\frac{1}{2}(d+e x)\right]+39 \cos \left[\frac{3}{2}(d+e x)\right]-165 \sin \left[\frac{1}{2}(d+e x)\right]-27 \sin \left[\frac{3}{2}(d+e x)\right]\right)\right)\right) / \left(e\left(5+4 \cos [d+e x]+3 \sin [d+e x]\right)^{5 / 2}\right)$$

**Problem 427: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-5+4 \cos [d+e x]+3 \sin [d+e x]}} d x$$

Optimal (type 3, 49 leaves, 3 steps):

$$\frac{\sqrt{\frac{2}{5}} \operatorname{ArcTan}\left[\frac{\sin\left[d+e x-\operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}{\sqrt{2} \sqrt{-1+\cos\left[d+e x-\operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}}\right]}{e}$$

Result (type 3, 99 leaves):

$$\left(\left(\left(\frac{2}{5}+\frac{6 i}{5}\right) \sqrt{-\frac{4}{5}-\frac{3 i}{5}} \operatorname{ArcTanh}\left[\left(\frac{1}{10}+\frac{3 i}{10}\right) \sqrt{-\frac{4}{5}-\frac{3 i}{5}}\left(3+\tan \left[\frac{1}{4}(d+e x)\right]\right)\right]\left(\cos \left[\frac{1}{2}(d+e x)\right]-3 \sin \left[\frac{1}{2}(d+e x)\right]\right)\right)\right) / \left(e \sqrt{-5+4 \cos [d+e x]+3 \sin [d+e x]}\right)$$

**Problem 428: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(-5+4 \cos [d+e x]+3 \sin [d+e x])^{3 / 2}} d x$$

Optimal (type 3, 96 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sin\left[d+ex-\text{ArcTan}\left[\frac{3}{4}\right]\right]}{\sqrt{2}\sqrt{-1+\cos\left[d+ex-\text{ArcTan}\left[\frac{3}{4}\right]\right]}}\right]}{10\sqrt{10}e} + \frac{3\cos[d+ex] - 4\sin[d+ex]}{10e(-5+4\cos[d+ex]+3\sin[d+ex])^{3/2}}$$

Result (type 3, 152 leaves):

$$\left(\left(\frac{1}{250} - \frac{i}{125}\right)\left(\cos\left[\frac{1}{2}(d+ex)\right] - 3\sin\left[\frac{1}{2}(d+ex)\right]\right)\right. \\ \left.\left(\left(-1+i\right)\sqrt{-20-15i}\text{ArcTanh}\left[\left(\frac{1}{10} + \frac{3i}{10}\right)\sqrt{-\frac{4}{5} - \frac{3i}{5}}\left(3 + \tan\left[\frac{1}{4}(d+ex)\right]\right)\right]\right)\left(\cos\left[\frac{1}{2}(d+ex)\right] - 3\sin\left[\frac{1}{2}(d+ex)\right]\right)^2 + \right. \\ \left.\left.\left(5+10i\right)\left(3\cos\left[\frac{1}{2}(d+ex)\right] + \sin\left[\frac{1}{2}(d+ex)\right]\right)\right)\right) / \left(e(-5+4\cos[d+ex]+3\sin[d+ex])^{3/2}\right)$$

**Problem 429: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(-5+4\cos[d+ex]+3\sin[d+ex])^{5/2}} dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$-\frac{3\text{ArcTan}\left[\frac{\sin\left[d+ex-\text{ArcTan}\left[\frac{3}{4}\right]\right]}{\sqrt{2}\sqrt{-1+\cos\left[d+ex-\text{ArcTan}\left[\frac{3}{4}\right]\right]}}\right]}{400\sqrt{10}e} + \frac{3\cos[d+ex] - 4\sin[d+ex]}{20e(-5+4\cos[d+ex]+3\sin[d+ex])^{5/2}} - \frac{3(3\cos[d+ex] - 4\sin[d+ex])}{400e(-5+4\cos[d+ex]+3\sin[d+ex])^{3/2}}$$

Result (type 3, 178 leaves):

$$\left( \left( \frac{1}{10000} + \frac{i}{20000} \right) \left( \cos \left[ \frac{1}{2} (d + e x) \right] - 3 \sin \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\ \left. \left( (6 + 6i) \sqrt{-20 - 15i} \operatorname{ArcTanh} \left[ \left( \frac{1}{10} + \frac{3i}{10} \right) \sqrt{-\frac{4}{5} - \frac{3i}{5}} \left( 3 + \tan \left[ \frac{1}{4} (d + e x) \right] \right) \right] \left( \cos \left[ \frac{1}{2} (d + e x) \right] - 3 \sin \left[ \frac{1}{2} (d + e x) \right] \right)^4 + (10 - 5i) \right. \right. \\ \left. \left. \left( 165 \cos \left[ \frac{1}{2} (d + e x) \right] - 27 \cos \left[ \frac{3}{2} (d + e x) \right] + 55 \sin \left[ \frac{1}{2} (d + e x) \right] - 39 \sin \left[ \frac{3}{2} (d + e x) \right] \right) \right) \right) / \left( e^{-5 + 4 \cos [d + e x] + 3 \sin [d + e x]} \right)^{5/2}$$

**Problem 430: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^{7/2} dx$$

Optimal (type 3, 258 leaves, 4 steps):

$$\frac{256 (b^2 + c^2)^{3/2} (c \cos [d + e x] - b \sin [d + e x])}{35 e \sqrt{\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}} - \frac{64 (b^2 + c^2) (c \cos [d + e x] - b \sin [d + e x]) \sqrt{\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}}{35 e} \\ - \frac{24 \sqrt{b^2 + c^2} (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^{3/2}}{35 e} \\ - \frac{2 (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^{5/2}}{7 e}$$

Result (type 4, 11888 leaves):

$$\frac{1}{e} \sqrt{b^2 + c^2} \sqrt{\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]} \\ \left( \frac{24 b (b^2 + c^2)}{5 c} - \frac{2}{5} c \sqrt{b^2 + c^2} \cos [d + e x] - \frac{6}{5} b c \cos [2 (d + e x)] + \frac{2}{5} b \sqrt{b^2 + c^2} \sin [d + e x] + \frac{3}{5} (b^2 - c^2) \sin [2 (d + e x)] \right) + \\ \frac{1}{e} \sqrt{\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]} \left( \frac{88 b (b^2 + c^2)^{3/2}}{35 c} - \frac{173}{70} c (b^2 + c^2) \cos [d + e x] - \frac{2}{35} b c \sqrt{b^2 + c^2} \cos [2 (d + e x)] - \right. \\ \left. \frac{1}{14} c (3 b^2 - c^2) \cos [3 (d + e x)] + \frac{173}{70} b (b^2 + c^2) \sin [d + e x] + \frac{1}{35} (b^2 - c^2) \sqrt{b^2 + c^2} \sin [2 (d + e x)] + \frac{1}{14} b (b^2 - 3 c^2) \sin [3 (d + e x)] \right) -$$

$$\begin{aligned}
& \left( 1024 b (-i b + c) (b^2 + c^2)^2 \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])}} \right], 1 \right] - \right. \right. \\
& \quad \left. \left. 2 \text{EllipticPi} \left[ -1, \text{ArcSin} \left[ \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])}} \right], 1 \right] \right) \sqrt{\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]} \right. \\
& \quad \left. (-i + \tan[\frac{1}{2}(d + e x)]) \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])}} \left( c + (-b + \sqrt{b^2 + c^2}) \tan[\frac{1}{2}(d + e x)] \right) \right) \Big/ \\
& \quad \left( 35 (b + i c - \sqrt{b^2 + c^2})^2 (b + i c + \sqrt{b^2 + c^2}) e (1 + \cos[d + e x]) \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]}{(1 + \cos[d + e x])^2}} \right. \\
& \quad \left. \sqrt{\left( (1 + \tan[\frac{1}{2}(d + e x)])^2 \right) \left( b + 2 c \tan[\frac{1}{2}(d + e x)] - b \tan[\frac{1}{2}(d + e x)]^2 + \sqrt{b^2 + c^2} (1 + \tan[\frac{1}{2}(d + e x)]^2) \right) \right) \Big) + \\
& \quad \frac{1}{35 c e (1 + \cos[d + e x]) \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]}{(1 + \cos[d + e x])^2}}} 256 (b^2 + c^2)^{5/2} \sqrt{\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]} \\
& \quad \left( \left( (-b + c \tan[\frac{1}{2}(d + e x)]) \sqrt{b + \sqrt{b^2 + c^2} + 2 c \tan[\frac{1}{2}(d + e x)] - b \tan[\frac{1}{2}(d + e x)]^2 + \sqrt{b^2 + c^2} \tan[\frac{1}{2}(d + e x)]^2} \right. \right. \\
& \quad \left. \left. \sqrt{\left( (1 + \tan[\frac{1}{2}(d + e x)])^2 \right) \left( b + 2 c \tan[\frac{1}{2}(d + e x)] - b \tan[\frac{1}{2}(d + e x)]^2 + \sqrt{b^2 + c^2} (1 + \tan[\frac{1}{2}(d + e x)]^2) \right) \right) \right) \Big/ \\
& \quad \left( (b^2 + c^2) (1 + \tan[\frac{1}{2}(d + e x)]^2) \sqrt{b + 2 c \tan[\frac{1}{2}(d + e x)] - b \tan[\frac{1}{2}(d + e x)]^2 + \sqrt{b^2 + c^2} (1 + \tan[\frac{1}{2}(d + e x)]^2)} \right) -
\end{aligned}$$

$$\left( \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] - b \tan\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right.$$

$$\sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + 2c \tan\left[\frac{1}{2}(d+ex)\right] - b \tan\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \right)}$$

$$\left( 2c^2 \left( -i - \frac{c}{b - \sqrt{b^2+c^2}} \right) \left( -i \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] + 2i \right. \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{i + \frac{c}{b - \sqrt{b^2+c^2}}}{-i + \frac{c}{b - \sqrt{b^2+c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)$$

$$\left. \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \left( -\frac{c}{b - \sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \right) \left/ \left( \left( i - \frac{c}{b - \sqrt{b^2+c^2}} \right) \right. \right.$$

$$\left. \left( i + \frac{c}{b - \sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + (-b + \sqrt{b^2+c^2}) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) \right) +$$

$$\left( 8b^3 \left( -b + ic + \sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})(i+\frac{c}{b-\sqrt{b^2+c^2}})}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right)$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 4b^5 \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1 \right] - 2ic \right. \\
& \left. \operatorname{EllipticPi}\left[ \frac{(b+ic-\sqrt{b^2+c^2}) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{(b-ic-\sqrt{b^2+c^2}) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1 \right] \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 4b^2 c^2 \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1 \right] - 2ic \right. \\
& \left. \operatorname{EllipticPi}\left[ \frac{(b+ic-\sqrt{b^2+c^2}) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{(b-ic-\sqrt{b^2+c^2}) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) - \\
& \left( 8b^2\sqrt{b^2+c^2} \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] - 2i \right. \\
& \left. c \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) - \\
& \left( 8b^4\sqrt{b^2+c^2} \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] - 2i \right. \\
& \left. c \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2}) (-b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(-\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \tan\left[\frac{1}{2}(d+ex)\right]^2\right)\right)} \right) + \\
& \left( 4b(b^2+c^2) \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2}) (-b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(-\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \tan\left[\frac{1}{2}(d+ex)\right]^2\right)\right)} \right) + \\
& \left( 4b^3(b^2+c^2) \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2i \right. \right. \\
& \left. \left. c \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2}) (-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 8b^3 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2}) (b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 4b^5 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( c^2 (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]) + (-b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) + \\
& \left( 4bc^2 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]) + (-b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) - \\
& \left( 4b(b^2+c^2) \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) - \\
& \left( 4b^3(b^2+c^2) \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] - 2i \right. \right. \\
& \left. \left. c \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( c^2 (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 2b^3 \left( -i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \left(c \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \\
& \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} \Bigg) + \\
& \left(2bc \left(-i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)\right) \\
& \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \left(\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \\
& \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} \Bigg) - \\
& \left(2b^2 \sqrt{b^2 + c^2} \left(-i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)\right) \\
& \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \left(c \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \\
& \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left( b c \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right. \right. \\
& \left. \left. i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] + \right. \right. \\
& \left. \left. \frac{2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right]}{(b - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right) \right) \\
& \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \Bigg) \Bigg/ \\
& \left( \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) +
\end{aligned}$$

$$\left( c \sqrt{b^2 + c^2} \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right. \right.$$

$$\left. \left. i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] + \right. \right.$$

$$\left. \left. \frac{2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right]}{(b - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right) \right)$$

$$\left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}}$$

$$\left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left. \right) \left. \right) /$$

$$\left( \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( b + \sqrt{b^2 + c^2} + 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) \left. \right) \left. \right) /$$

$$\left( (b^2 + c^2) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \sqrt{b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2} \right. \\ \left. \sqrt{b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2} \right)$$

**Problem 431: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex] \right)^{5/2} dx$$

Optimal (type 3, 190 leaves, 3 steps):

$$\frac{64 (b^2 + c^2) (c \cos[d + ex] - b \sin[d + ex])}{15 e \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}} - \frac{16 \sqrt{b^2 + c^2} (c \cos[d + ex] - b \sin[d + ex]) \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}{15 e} \\ - \frac{2 (c \cos[d + ex] - b \sin[d + ex]) \left( \sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex] \right)^{3/2}}{5 e}$$

Result (type 4, 11771 leaves):

$$\frac{\sqrt{b^2 + c^2} \left( \frac{4b\sqrt{b^2 + c^2}}{3c} - \frac{4}{3} c \cos[d + ex] + \frac{4}{3} b \sin[d + ex] \right) \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}{e} + \frac{1}{e} \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \\ \left( \frac{44b(b^2 + c^2)}{15c} - \frac{2}{15} c \sqrt{b^2 + c^2} \cos[d + ex] - \frac{2}{5} b c \cos[2(d + ex)] + \frac{2}{15} b \sqrt{b^2 + c^2} \sin[d + ex] + \frac{1}{5} (b^2 - c^2) \sin[2(d + ex)] \right) - \\ \left( 256b(-ib + c)(b^2 + c^2)^{3/2} \left[ \text{EllipticF}\left[ \text{ArcSin}\left[ \sqrt{-\frac{(-b - ic + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(-b + ic + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right]}, 1 \right] \right) -$$

$$\begin{aligned}
& 2 \operatorname{EllipticPi}\left[-1, \operatorname{ArcSin}\left[\frac{(-b - i c + \sqrt{b^2 + c^2}) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right)}{(-b + i c + \sqrt{b^2 + c^2}) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right)}\right], 1\right] \sqrt{\sqrt{b^2 + c^2} + b \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]} \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{(-b - i c + \sqrt{b^2 + c^2}) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right)}{(-b + i c + \sqrt{b^2 + c^2}) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right)} \left(c + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right)} \Bigg/ \\
& \left(15 (b + i c - \sqrt{b^2 + c^2})^2 (b + i c + \sqrt{b^2 + c^2}) e (1 + \operatorname{Cos}[d + e x]) \sqrt{\frac{\sqrt{b^2 + c^2} + b \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]}{(1 + \operatorname{Cos}[d + e x])^2}}\right. \\
& \left. + \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right)^2 \left(b + 2 c \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]^2 + \sqrt{b^2 + c^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]^2\right)\right)}\right)} \Bigg) + \\
& \frac{1}{15 c e (1 + \operatorname{Cos}[d + e x]) \sqrt{\frac{\sqrt{b^2 + c^2} + b \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]}{(1 + \operatorname{Cos}[d + e x])^2}}} - 64 (b^2 + c^2)^2 \sqrt{\sqrt{b^2 + c^2} + b \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]} \\
& \left( \left( (-b + c \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]) \sqrt{b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]^2 + \sqrt{b^2 + c^2} \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]^2} \right. \right. \\
& \left. \left. + \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right)^2 \left(b + 2 c \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]^2 + \sqrt{b^2 + c^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]^2\right)\right)}\right)} \right) \Bigg/ \\
& \left( (b^2 + c^2) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]\right)^2 \sqrt{b + 2 c \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]^2 + \sqrt{b^2 + c^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + e x)\right]^2\right)} \right) -
\end{aligned}$$



$$\left( \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] - b \tan\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right.$$

$$\sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + 2c \tan\left[\frac{1}{2}(d+ex)\right] - b \tan\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \right)}$$

$$\left( 2c^2 \left( -i - \frac{c}{b - \sqrt{b^2+c^2}} \right) \left( -i \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] + 2i \right. \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{i + \frac{c}{b - \sqrt{b^2+c^2}}}{-i + \frac{c}{b - \sqrt{b^2+c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)$$

$$\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \left( -\frac{c}{b - \sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( \left( i - \frac{c}{b - \sqrt{b^2+c^2}} \right) \right.$$

$$\left. \left( i + \frac{c}{b - \sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + (-b + \sqrt{b^2+c^2}) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) \Bigg) +$$

$$\left( 8b^3 \left( -b + ic + \sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})(i+\frac{c}{b-\sqrt{b^2+c^2}})}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right)$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 4b^5 \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 4b^2 c^2 \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) - \\
& \left( 8b^2\sqrt{b^2+c^2} \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] - 2i \right. \\
& \left. c \operatorname{EllipticPi}\left[ \frac{(b+ic-\sqrt{b^2+c^2}) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{(b-ic-\sqrt{b^2+c^2}) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) - \\
& \left( 8b^4\sqrt{b^2+c^2} \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] - 2i \right. \\
& \left. c \operatorname{EllipticPi}\left[ \frac{(b+ic-\sqrt{b^2+c^2}) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{(b-ic-\sqrt{b^2+c^2}) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2}) (-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 (b+\sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 4b(b^2+c^2) \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2}) (-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 (b+\sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 4b^3(b^2+c^2) \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2i \right. \right. \\
& \left. \left. c \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 8b^3 \left( -b+ic-\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] - 2ic \right. \\
& \left. \operatorname{EllipticPi}\left[ \frac{(b+ic+\sqrt{b^2+c^2}) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{(b-ic+\sqrt{b^2+c^2}) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 4b^5 \left( -b+ic-\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] - 2ic \right. \\
& \left. \operatorname{EllipticPi}\left[ \frac{(b+ic+\sqrt{b^2+c^2}) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{(b-ic+\sqrt{b^2+c^2}) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( c^2 (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]) + (-b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) + \\
& \left( 4bc^2 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]) + (-b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) - \\
& \left( 4b(b^2+c^2) \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) - \\
& \left( 4b^3(b^2+c^2) \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] - 2i \right. \right. \\
& \left. \left. c \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( c^2 (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 2b^3 \left( -i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \left(c \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \\
& \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} \Bigg) + \\
& \left(2bc \left(-i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)\right) \\
& \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \left(\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \\
& \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} \Bigg) - \\
& \left(2b^2 \sqrt{b^2 + c^2} \left(-i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)\right) \\
& \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \left(c \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \\
& \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} \Bigg) -
\end{aligned}$$



$$\left( b c \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c - i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right] \right), 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right. \right.$$

$$\left. \left. i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c - i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right] \right], 1 \right] + \right.$$

$$\left. \frac{2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c - i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right] \right], 1 \right]}{(b - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right)$$

$$\left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c - i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}}$$

$$\left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left/ \right.$$

$$\left( \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) +$$

$$\left( c \sqrt{b^2 + c^2} \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right. \right.$$

$$\left. \left. i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] + \right. \right.$$

$$\left. \left. \frac{2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] \right)}{(b - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right)$$

$$\left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}}$$

$$\left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left. \right) \left. \right) /$$

$$\left( \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \left. \right) \left. \right) /$$

$$\left( (b^2 + c^2) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \sqrt{b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2} \right. \\ \left. \sqrt{b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2} \right)$$

**Problem 432: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex] \right)^{3/2} dx$$

Optimal (type 3, 126 leaves, 2 steps):

$$\frac{8 \sqrt{b^2 + c^2} (c \cos[d + ex] - b \sin[d + ex])}{3 e \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}} - \frac{2 (c \cos[d + ex] - b \sin[d + ex]) \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}{3 e}$$

Result (type 4, 11679 leaves):

$$\frac{2 b \sqrt{b^2 + c^2} \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}{c e} + \\ \frac{\left( \frac{2 b \sqrt{b^2 + c^2}}{3 c} - \frac{2}{3} c \cos[d + ex] + \frac{2}{3} b \sin[d + ex] \right) \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}{e} - \\ \left( 32 b (-i b + c) (b^2 + c^2) \left( \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + ex)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + ex)])}}\right]}, 1\right] - \right. \\ \left. 2 \text{EllipticPi}\left[-1, \text{ArcSin}\left[\sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + ex)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + ex)])}}\right]}, 1\right] \right) \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{-\frac{(-b-ic+\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(-b+ic+\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \left( c + (-b+\sqrt{b^2+c^2}) \tan\left[\frac{1}{2}(d+ex)\right] \right)} \right) / \\
& \left( 3 \left( b+ic-\sqrt{b^2+c^2} \right)^2 \left( b+ic+\sqrt{b^2+c^2} \right) e^{(1+\cos[d+ex])} \sqrt{\frac{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}{(1+\cos[d+ex])^2}} \right. \\
& \left. \sqrt{\left( \left( 1+\tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b+2c\tan\left[\frac{1}{2}(d+ex)\right]-b\tan\left[\frac{1}{2}(d+ex)\right]^2+\sqrt{b^2+c^2} \left( 1+\tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right)} \right) \right) + \\
& \frac{1}{3ce^{(1+\cos[d+ex])} \sqrt{\frac{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}{(1+\cos[d+ex])^2}}} 8(b^2+c^2)^{3/2} \sqrt{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]} \\
& \left( \left( \left( -b+c\tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]-b\tan\left[\frac{1}{2}(d+ex)\right]^2+\sqrt{b^2+c^2}\tan\left[\frac{1}{2}(d+ex)\right]^2} \right. \right. \\
& \left. \left. \sqrt{\left( \left( 1+\tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b+2c\tan\left[\frac{1}{2}(d+ex)\right]-b\tan\left[\frac{1}{2}(d+ex)\right]^2+\sqrt{b^2+c^2} \left( 1+\tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right)} \right) \right) / \\
& \left( (b^2+c^2) \left( 1+\tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \sqrt{b+2c\tan\left[\frac{1}{2}(d+ex)\right]-b\tan\left[\frac{1}{2}(d+ex)\right]^2+\sqrt{b^2+c^2} \left( 1+\tan\left[\frac{1}{2}(d+ex)\right] \right)^2} \right) - \\
& \left( \sqrt{\left( \left( 1+\tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]-b\tan\left[\frac{1}{2}(d+ex)\right]^2+\sqrt{b^2+c^2}\tan\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + 2c \tan\left[\frac{1}{2}(d+ex)\right] - b \tan\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \right)} \\
& \left( \left( 2c^2 \left( -i - \frac{c}{b - \sqrt{b^2+c^2}} \right) \left( -i \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \right]}, 1\right] + 2i \right. \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{i + \frac{c}{b - \sqrt{b^2+c^2}}}{-i + \frac{c}{b - \sqrt{b^2+c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \right]}, 1\right] \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \right. \\
& \left. \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \left( -\frac{c}{b - \sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \right) / \left( \left( i - \frac{c}{b - \sqrt{b^2+c^2}} \right) \right. \\
& \left. \left( i + \frac{c}{b - \sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + (-b + \sqrt{b^2+c^2}) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 8b^3 \left( (-b + ic + \sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \right]}, 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})(i+\frac{c}{b-\sqrt{b^2+c^2}})}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \right]}, 1\right] \right) \right. \\
& \left. \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \left( -\frac{c}{b - \sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \right) / \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b - \sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} + \\
& \left( 4b^5 \left( -b + ic + \sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \\
& \left. \text{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})(i+\frac{c}{b-\sqrt{b^2+c^2}})}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \text{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} + \\
& \left( 4b^2 \left( -b + ic + \sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \\
& \left. \text{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})(i+\frac{c}{b-\sqrt{b^2+c^2}})}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \text{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} - \\
& \left( 8b^2\sqrt{b^2+c^2} \left( -b + ic + \sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1 \right] - 2i \right. \\
& \quad \left. c \text{EllipticPi}\left[\frac{\left( b + ic - \sqrt{b^2+c^2} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{\left( b - ic - \sqrt{b^2+c^2} \right) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1 \right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( \left( b - ic - \sqrt{b^2+c^2} \right) \left( -b - ic + \sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} - \\
& \left( 8b^4\sqrt{b^2+c^2} \left( -b + ic + \sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1 \right] - 2i \right. \\
& \quad \left. c \text{EllipticPi}\left[\frac{\left( b + ic - \sqrt{b^2+c^2} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{\left( b - ic - \sqrt{b^2+c^2} \right) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1 \right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( c^2 \left( b - ic - \sqrt{b^2+c^2} \right) \left( -b - ic + \sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} + \\
& \left( 4b(b^2+c^2) \left( \left( -b + ic + \sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{\left( b+ic-\sqrt{b^2+c^2} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{\left( b-ic-\sqrt{b^2+c^2} \right) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \text{ArcSin}\left[\sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( \left( b-ic-\sqrt{b^2+c^2} \right) \left( -b-ic+\sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} + \\
& \left( 4b^3(b^2+c^2) \left( \left( -b + ic + \sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1\right] - 2i \right. \right. \\
& \left. \left. c \text{EllipticPi}\left[\frac{\left( b+ic-\sqrt{b^2+c^2} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{\left( b-ic-\sqrt{b^2+c^2} \right) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \text{ArcSin}\left[\sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 \left( b-ic-\sqrt{b^2+c^2} \right) \left( -b-ic+\sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} + \\
& \left( 8b^3 \left( \left( -b + ic - \sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{\left( b+ic+\sqrt{b^2+c^2} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{\left( b-ic+\sqrt{b^2+c^2} \right) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( \left( -b-ic-\sqrt{b^2+c^2} \right) \left( b-ic+\sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right) \\
& \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} + \\
& \left( 4b^5 \left( \left( -b + ic - \sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{\left( b+ic+\sqrt{b^2+c^2} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{\left( b-ic+\sqrt{b^2+c^2} \right) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -ib+c+i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( c^2 \left( -b-ic-\sqrt{b^2+c^2} \right) \left( b-ic+\sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} + \\
& \left( 4bc^2 \left( -b + ic - \sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\right], 1\right] - 2ic \right. \\
& \left. \text{EllipticPi}\left[\frac{\left(b+ic+\sqrt{b^2+c^2}\right)\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{\left(b-ic+\sqrt{b^2+c^2}\right)\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \text{ArcSin}\left[\sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\right], 1\right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( \left( -b - ic - \sqrt{b^2+c^2} \right) \left( b - ic + \sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} - \\
& \left( 4b(b^2+c^2) \left( -b + ic - \sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\right], 1\right] - 2ic \right. \\
& \left. \text{EllipticPi}\left[\frac{\left(b+ic+\sqrt{b^2+c^2}\right)\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{\left(b-ic+\sqrt{b^2+c^2}\right)\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \text{ArcSin}\left[\sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\right], 1\right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( \left( -b - ic - \sqrt{b^2+c^2} \right) \left( b - ic + \sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} - \\
& \left( 4b^3(b^2+c^2) \left( \left( -b + ic - \sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right], 1 \right] - 2i \right. \right. \\
& \quad \left. \left. c \operatorname{EllipticPi}\left[\frac{\left( b + ic + \sqrt{b^2+c^2} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}{\left( b - ic + \sqrt{b^2+c^2} \right) \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right], 1 \right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -ib + c + i\sqrt{b^2+c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib + c - i\sqrt{b^2+c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right) / \\
& \left( c^2 \left( -b - ic - \sqrt{b^2+c^2} \right) \left( b - ic + \sqrt{b^2+c^2} \right) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right) \\
& \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} + \\
& \left( 2b^3 \left( -i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right], 1 \right] \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \right) \\
& \sqrt{\frac{\left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right) / \left( c \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \right) \\
& \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} +
\end{aligned}$$

$$\begin{aligned}
& \left( 2 b c \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right) / \left( \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \right) - \\
& \left( 2 b^2 \sqrt{b^2 + c^2} \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right) / \left( c \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \right) - \\
& \left( b c \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right. \right. \\
& \left. \left. i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] + \right. \right.
\end{aligned}$$

$$\left. \frac{2 c \operatorname{EllipticPi}\left[\frac{i+\frac{c}{b-\sqrt{b^2+c^2}}}{-i+\frac{c}{b-\sqrt{b^2+c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(-i b+c+i \sqrt{b^2+c^2})\left(i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)}{(i b+c-i \sqrt{b^2+c^2})\left(-i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)}}\right], 1\right]}{\left(b-\sqrt{b^2+c^2}\right)\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)} \right)$$

$$\left(-i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right) \sqrt{\frac{(-i b+c+i \sqrt{b^2+c^2})\left(i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)}{(i b+c-i \sqrt{b^2+c^2})\left(-i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)}}$$

$$\left.\left.\left.\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)+\left(i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)^2\right)\right)\right) /$$

$$\left(\sqrt{\left(\left(1+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)^2\right)\left(b+\sqrt{b^2+c^2}+2 c \operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]+\left(-b+\sqrt{b^2+c^2}\right) \operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]^2\right)}\right) +$$

$$\left(c \sqrt{b^2+c^2}\left(2 i\left(-\frac{1}{2} i\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i b+c+i \sqrt{b^2+c^2})\left(i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)}{(i b+c-i \sqrt{b^2+c^2})\left(-i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)}}\right], 1\right]-\frac{1}{2\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}\right.\right.$$

$$\left.\left.+i\left(i\left(-i-\frac{c}{b-\sqrt{b^2+c^2}}\right)+i\left(i-\frac{c}{b-\sqrt{b^2+c^2}}\right)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i b+c+i \sqrt{b^2+c^2})\left(i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)}{(i b+c-i \sqrt{b^2+c^2})\left(-i+\operatorname{Tan}\left[\frac{1}{2}(d+e x)\right]\right)}}\right], 1\right]+$$

$$\left. \frac{2 c \operatorname{EllipticPi} \left[ \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right]}{(b - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right)$$

$$\left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}}$$

$$\left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left/ \right.$$

$$\left( \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \left/ \right.$$

$$\left( (b^2 + c^2) \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \sqrt{b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] - b \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2}$$

$$\left( \sqrt{b + 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] - b \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2} \right)$$

**Problem 433: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} dx$$

Optimal (type 3, 55 leaves, 1 step):

$$\frac{2(c \cos[d + ex] - b \sin[d + ex])}{e \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}$$

Result (type 4, 11 586 leaves):

$$\frac{2 b \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}{c e} - \left( 8 b (-i b + c) \sqrt{b^2 + c^2} \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(-b + i c + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right]}, 1 \right] - 2 \text{EllipticPi} \left[ -1, \text{ArcSin} \left[ \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(-b + i c + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right]}, 1 \right] \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \right. \\ \left. (-i + \tan[\frac{1}{2}(d + ex)]) \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(-b + i c + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} (c + (-b + \sqrt{b^2 + c^2}) \tan[\frac{1}{2}(d + ex)]) \right) \Bigg) / \\ \left( (b + i c - \sqrt{b^2 + c^2})^2 (b + i c + \sqrt{b^2 + c^2}) e (1 + \cos[d + ex]) \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}} \right. \\ \left. \sqrt{\left( \left( 1 + \tan[\frac{1}{2}(d + ex)] \right)^2 \right) \left( b + 2 c \tan[\frac{1}{2}(d + ex)] - b \tan[\frac{1}{2}(d + ex)]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan[\frac{1}{2}(d + ex)] \right)^2 \right) \right) \right) + \\ \frac{1}{c e (1 + \cos[d + ex])} \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}} - 2 (b^2 + c^2) \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}$$

$$\left( \left( -b + c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2} \right.$$

$$\left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) /$$

$$\left( (b^2+c^2) \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \sqrt{b + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) -$$

$$\left( \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right)$$

$$\sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] - b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)}$$

$$\left( 2c^2 \left( -i - \frac{c}{b - \sqrt{b^2+c^2}} \right) \left( -i \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c + i \sqrt{b^2+c^2})(i + \operatorname{Tan}[\frac{1}{2}(d+ex)])}{(i b + c - i \sqrt{b^2+c^2})(-i + \operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] + 2i \right) \right.$$

$$\left. \operatorname{EllipticPi}\left[\frac{i + \frac{c}{b - \sqrt{b^2+c^2}}}{-i + \frac{c}{b - \sqrt{b^2+c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c + i \sqrt{b^2+c^2})(i + \operatorname{Tan}[\frac{1}{2}(d+ex)])}{(i b + c - i \sqrt{b^2+c^2})(-i + \operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)$$



$$\begin{aligned}
& \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{(i b + c - i \sqrt{b^2 + c^2}) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right)} \left/\left(\left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right.\right. \\
& \left.\left. \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + e x)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2}(d + e x)\right] + (-b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + e x)\right]^2\right)\right)}\right) + \right. \\
& \left. \left(8 b^3 \left(-b + i c + \sqrt{b^2 + c^2}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{(i b + c - i \sqrt{b^2 + c^2}) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}}\right], 1\right] - 2 i c \right. \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{(b + i c - \sqrt{b^2 + c^2}) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{(b - i c - \sqrt{b^2 + c^2}) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \text{ArcSin}\left[\sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{(i b + c - i \sqrt{b^2 + c^2}) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}}\right], 1\right]\right) \right. \\
& \left. \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{(i b + c - i \sqrt{b^2 + c^2}) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right)}\right)} \right/ \\
& \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right)\right) \\
& \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + e x)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2}(d + e x)\right] + (-b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + e x)\right]^2\right)\right)} + \\
& \left(4 b^5 \left(-b + i c + \sqrt{b^2 + c^2}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{(i b + c - i \sqrt{b^2 + c^2}) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}}\right], 1\right] - 2 i c \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{(b + i c - \sqrt{b^2 + c^2}) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{(b - i c - \sqrt{b^2 + c^2}) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \text{ArcSin}\left[\sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{(i b + c - i \sqrt{b^2 + c^2}) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}}\right], 1\right]\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2}) (-b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(-\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]) + (-b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) + \\
& \left( 4bc^2 \left( -b+ic+\sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \\
& \left. \text{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \text{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2}) (-b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(-\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]) + (-b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) - \\
& \left( 8b^2\sqrt{b^2+c^2} \left( -b+ic+\sqrt{b^2+c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2i \right. \\
& \left. c \text{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b-\sqrt{b^2+c^2}})}, \text{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) - \\
& \left( 8b^4\sqrt{b^2+c^2} \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2i \right. \\
& \left. c \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 4b(b^2+c^2) \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 4b^3(b^2+c^2) \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2i \right. \right. \\
& \left. \left. c \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) + \\
& \left( 8b^3 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b + \sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 4b^5 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b + \sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 4b^2 c^2 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]) + (-b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) - \\
& \left( 4b(b^2+c^2) \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(i - \frac{c}{b-\sqrt{b^2+c^2}}\right) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 (b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]) + (-b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) - \\
& \left( 4b^3(b^2+c^2) \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2i \right. \right. \\
& \left. \left. c \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c-i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c+i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c-i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2(-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b + \sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 2b^3 \left( -i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right], 1 \right] \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \right. \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right] \Bigg/ \left( c \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \right. \\
& \left. \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b + \sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) \Bigg) + \\
& \left( 2bc \left( -i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right], 1 \right] \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \right. \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)}{\left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \right] \Bigg/ \left( \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (-b + \sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left( 2 b^2 \sqrt{b^2 + c^2} \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right) / \left( c \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( b + \sqrt{b^2 + c^2} + 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \right) - \\
& \left( b c \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right. \right. \\
& \left. \left. i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] + \right. \\
& \left. \frac{2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right]}{\left( b - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right) \\
& \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}}
\end{aligned}$$



$$\left. \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) + \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) /$$

$$\left( \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)^2 \left( b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} +$$

$$\left( c \sqrt{b^2 + c^2} \left( 2i \left( -\frac{1}{2}i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right) \right)$$

$$+ i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] +$$

$$\frac{2c \operatorname{EllipticPi}\left[\frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right]}{\left( b - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}$$

$$\left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}}$$

$$\left. \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) + \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) /$$

$$\left. \left( \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right)} \right) /$$

$$\left( (b^2 + c^2) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \sqrt{b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2}$$

$$\left. \sqrt{b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2} \right)$$

**Problem 434:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}} dx$$

Optimal (type 3, 88 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{(b^2 + c^2)^{1/4} \sin[d + ex - \operatorname{ArcTan}[b, c]]}{\sqrt{2} \sqrt{\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos[d + ex - \operatorname{ArcTan}[b, c]]}}\right]}{(b^2 + c^2)^{1/4} e}$$

Result (type 4, 63264 leaves): Display of huge result suppressed!

Problem 435: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]\right)^{3/2}} dx$$

Optimal (type 3, 160 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{(b^2+c^2)^{1/4} \sin [d+e x-\text{ArcTan}[b,c]]}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} + \sqrt{b^2+c^2}} \cos [d+e x-\text{ArcTan}[b,c]]}\right]}{2 \sqrt{2} (b^2+c^2)^{3/4} e} - \frac{c \cos [d+e x] - b \sin [d+e x]}{2 \sqrt{b^2+c^2} e \left(\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

Problem 436: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]\right)^{5/2}} dx$$

Optimal (type 3, 226 leaves, 5 steps):

$$\frac{3 \text{ArcTanh}\left[\frac{(b^2+c^2)^{1/4} \sin [d+e x-\text{ArcTan}[b,c]]}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} + \sqrt{b^2+c^2}} \cos [d+e x-\text{ArcTan}[b,c]]}\right]}{16 \sqrt{2} (b^2+c^2)^{5/4} e} - \frac{c \cos [d+e x] - b \sin [d+e x]}{4 \sqrt{b^2+c^2} e \left(\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]\right)^{5/2}} - \frac{3 (c \cos [d+e x] - b \sin [d+e x])}{16 (b^2+c^2) e \left(\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

**Problem 437: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( -\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^{5/2} dx$$

Optimal (type 3, 196 leaves, 3 steps):

$$\frac{64 (b^2 + c^2) (c \cos [d + e x] - b \sin [d + e x])}{15 e \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}} + \frac{16 \sqrt{b^2 + c^2} (c \cos [d + e x] - b \sin [d + e x]) \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}}{15 e} - \frac{2 (c \cos [d + e x] - b \sin [d + e x]) \left( -\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^{3/2}}{5 e}$$

Result (type 4, 11 602 leaves):

$$\frac{\sqrt{b^2 + c^2} \left( \frac{4b\sqrt{b^2+c^2}}{3c} + \frac{4}{3} c \cos [d + e x] - \frac{4}{3} b \sin [d + e x] \right) \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}}{e} + \frac{1}{e} \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]} \left( \frac{44 b (b^2 + c^2)}{15 c} + \frac{2}{15} c \sqrt{b^2 + c^2} \cos [d + e x] - \frac{2}{5} b c \cos [2 (d + e x)] - \frac{2}{15} b \sqrt{b^2 + c^2} \sin [d + e x] + \frac{1}{5} (b^2 - c^2) \sin [2 (d + e x)] \right) - \left( 256 b c (b^2 + c^2)^{5/2} \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan [\frac{1}{2} (d + e x)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan [\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 \text{EllipticPi} \left[ -1, \text{ArcSin} \left[ \sqrt{-\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan [\frac{1}{2} (d + e x)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan [\frac{1}{2} (d + e x)])}} \right], 1 \right] \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]} \right. \left. - (-i + \tan [\frac{1}{2} (d + e x)]) \left( -\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan [\frac{1}{2} (d + e x)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan [\frac{1}{2} (d + e x)])} \right)^{3/2} \left( -c + (b + \sqrt{b^2 + c^2}) \tan [\frac{1}{2} (d + e x)] \right) \left( 1 + \tan [\frac{1}{2} (d + e x)]^2 \right) \right) \Big/ \left( 15 (b + i c + \sqrt{b^2 + c^2})^3 (b^2 + c^2 - b \sqrt{b^2 + c^2}) e (1 + \cos [d + e x]) \sqrt{\frac{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}{(1 + \cos [d + e x])^2}} (i + \tan [\frac{1}{2} (d + e x)])^2 \right)$$

$$\begin{aligned}
& \sqrt{\left(-\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)\left(-2c\tan\left[\frac{1}{2}(d+ex)\right]+b\left(-1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)+\sqrt{b^2+c^2}\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)} + \\
& \frac{1}{15ce(1+\cos[d+ex])\sqrt{\frac{-\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}{(1+\cos[d+ex])^2}}} 64(b^2+c^2)^2\sqrt{-\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]} \\
& \left( \left( \left( -b+c\tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{-b+\sqrt{b^2+c^2}-2c\tan\left[\frac{1}{2}(d+ex)\right]+b\tan\left[\frac{1}{2}(d+ex)\right]^2+\sqrt{b^2+c^2}\tan\left[\frac{1}{2}(d+ex)\right]^2} \right. \right. \\
& \left. \left. \sqrt{\left(-\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)\left(-2c\tan\left[\frac{1}{2}(d+ex)\right]+b\left(-1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)+\sqrt{b^2+c^2}\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2}\right)} \right) / \\
& \left( (b^2+c^2)\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\sqrt{-2c\tan\left[\frac{1}{2}(d+ex)\right]+b\left(-1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2+\sqrt{b^2+c^2}\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2} \right) + \\
& \left( \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)\left(-b+\sqrt{b^2+c^2}-2c\tan\left[\frac{1}{2}(d+ex)\right]+b\tan\left[\frac{1}{2}(d+ex)\right]^2+\sqrt{b^2+c^2}\tan\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) \\
& \sqrt{\left(-\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)\left(-2c\tan\left[\frac{1}{2}(d+ex)\right]+b\left(-1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)+\sqrt{b^2+c^2}\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)}
\end{aligned}$$

$$\left( \left( 2 c^2 \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] + 2 i \right. \right.$$

$$\left. \left. \operatorname{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right.$$

$$\left. \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) / \left( \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right.$$

$$\left. \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \right) +$$

$$\left( 8 b^3 \left( -b + i c + \sqrt{b^2 + c^2} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i c \right.$$

$$\left. \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right)$$

$$\left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) /$$

$$\left( (b - i c - \sqrt{b^2 + c^2}) \left( -b - i c + \sqrt{b^2 + c^2} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right.$$

$$\left. \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \right) +$$

$$\begin{aligned}
& \left( 4 b^5 \left( (-b + i c + \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]} \right)} \right], 1 \right] - 2 i c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]} \right)} \right], 1 \right] \right) \right. \\
& \quad \left. \left. (-i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]} \right)} \right] \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) / \\
& \quad \left( c^2 (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) + \\
& \left( 4 b c^2 \left( (-b + i c + \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]} \right)} \right], 1 \right] - 2 i c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]} \right)} \right], 1 \right] \right) \right. \\
& \quad \left. \left. (-i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]} \right)} \right] \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) / \\
& \quad \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b (b^2 + c^2) \left( (-b + i c + \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) (i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - i c - \sqrt{b^2 + c^2}) (-i + \frac{c}{b + \sqrt{b^2 + c^2}})} \right], \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \\
& \quad \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \Bigg/ \\
& \quad \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) - \\
& \quad \left( 4 b^3 (b^2 + c^2) \left( (-b + i c + \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i \right. \right. \\
& \quad \left. \left. c \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) (i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - i c - \sqrt{b^2 + c^2}) (-i + \frac{c}{b + \sqrt{b^2 + c^2}})} \right], \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \\
& \quad \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \Bigg/ \\
& \quad \left( c^2 (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) +
\end{aligned}$$



$$\begin{aligned}
& \left( 8 b^3 \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \right. \\
& \quad \left. (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)]) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}[\frac{1}{2} (d + e x)] \right) \right) \Bigg/ \\
& \quad \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan}[\frac{1}{2} (d + e x)] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}[\frac{1}{2} (d + e x)] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan}[\frac{1}{2} (d + e x)]^2) \right) \right) + \\
& \left( 4 b^5 \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \right. \\
& \quad \left. (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)]) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}[\frac{1}{2} (d + e x)] \right) \right) \Bigg/ \\
& \quad \left( c^2 (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan}[\frac{1}{2} (d + e x)] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}[\frac{1}{2} (d + e x)] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan}[\frac{1}{2} (d + e x)]^2) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b c^2 \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i c \right. \right. \\
& \quad \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \right. \\
& \quad \left. (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)]) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} [\frac{1}{2} (d + e x)] \right) \right) \Bigg/ \\
& \quad \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} [\frac{1}{2} (d + e x)] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} [\frac{1}{2} (d + e x)] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} [\frac{1}{2} (d + e x)]^2) \right) \right) + \\
& \quad \left( 8 b^2 \sqrt{b^2 + c^2} \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i \right. \right. \\
& \quad \left. \left. c \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \right. \\
& \quad \left. (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)]) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} [\frac{1}{2} (d + e x)] \right) \right) \Bigg/ \\
& \quad \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} [\frac{1}{2} (d + e x)] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} [\frac{1}{2} (d + e x)] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} [\frac{1}{2} (d + e x)]^2) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 8 b^4 \sqrt{b^2 + c^2} \left( -b + i c - \sqrt{b^2 + c^2} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i \right. \\
& \quad \left. c \text{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) (i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - i c + \sqrt{b^2 + c^2}) (-i + \frac{c}{b + \sqrt{b^2 + c^2}})}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \\
& \quad \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \Bigg/ \\
& \quad \left( c^2 (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2) \right)} \right) + \\
& \quad \left( 4 b (b^2 + c^2) \left( -b + i c - \sqrt{b^2 + c^2} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i c \right. \\
& \quad \left. \text{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) (i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - i c + \sqrt{b^2 + c^2}) (-i + \frac{c}{b + \sqrt{b^2 + c^2}})}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \\
& \quad \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \text{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \text{Tan}[\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \Bigg/ \\
& \quad \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2) \right)} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b^3 (b^2 + c^2) \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 i \right. \right. \\
& \quad \left. \left. c \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) (i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - i c + \sqrt{b^2 + c^2}) (-i + \frac{c}{b + \sqrt{b^2 + c^2}})}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \right) \\
& \quad \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \Bigg/ \\
& \quad \left( c^2 (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2) \right)} \right) + \\
& \quad \left( 2 b^3 \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i + \frac{c}{b + \sqrt{b^2 + c^2}}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i + \frac{c}{b + \sqrt{b^2 + c^2}}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\
& \quad \left. \sqrt{\frac{(-i + \frac{c}{b + \sqrt{b^2 + c^2}}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i + \frac{c}{b + \sqrt{b^2 + c^2}}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right] \Bigg/ \left( c \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2) \right)} \right) \Bigg) + \\
& \quad \left( 2 b c \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i + \frac{c}{b + \sqrt{b^2 + c^2}}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i + \frac{c}{b + \sqrt{b^2 + c^2}}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} / \left(\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)\right) \\
& \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(-b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left(b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} + \\
& \left(2b^2 \sqrt{b^2 + c^2} \left(-i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)\right) \\
& \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} / \left(c \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)\right) \\
& \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(-b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left(b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} - \\
& \left(b c \left(2i \left(-\frac{1}{2}i \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(-ib + c - i\sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(ib + c + i\sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] - \frac{1}{2 \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}\right)\right) \\
& i \left(i \left(-i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) + i \left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-ib + c - i\sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(ib + c + i\sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] +
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{2 c \operatorname{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right]}{(b + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right) \\
& \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \\
& \left. \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) / \\
& \left( \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) - \\
& \left( c \sqrt{b^2 + c^2} \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right. \right. \right. \\
& \left. \left. \left. i \left( i \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] + \right. \right.
\end{aligned}$$

$$\left. \frac{2 c \operatorname{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right]}{(b + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right)$$

$$\left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}}$$

$$\left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left/ \right.$$

$$\left( \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \left/ \right.$$

$$\left( (b^2 + c^2) \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \sqrt{-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + b \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2}$$

$$\left( \sqrt{-2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 + \sqrt{b^2 + c^2} \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2} \right)$$

**Problem 438: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( -\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^{3/2} dx$$

Optimal (type 3, 130 leaves, 2 steps):

$$\frac{8 \sqrt{b^2 + c^2} (c \cos [d + e x] - b \sin [d + e x])}{3 e \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}} - \frac{2 (c \cos [d + e x] - b \sin [d + e x]) \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}}{3 e}$$

Result (type 4, 11512 leaves):

$$\begin{aligned} & - \frac{2 b \sqrt{b^2 + c^2} \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}}{c e} + \\ & \frac{\left( -\frac{2 b \sqrt{b^2 + c^2}}{3 c} - \frac{2}{3} c \cos [d + e x] + \frac{2}{3} b \sin [d + e x] \right) \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}}{e} + \\ & \left( 32 b c (b^2 + c^2)^2 \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan [\frac{1}{2} (d + e x)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan [\frac{1}{2} (d + e x)])}} \right], 1 \right] - \right. \right. \\ & \left. \left. 2 \text{EllipticPi} \left[ -1, \text{ArcSin} \left[ \sqrt{-\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan [\frac{1}{2} (d + e x)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan [\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]} \right. \\ & \left. \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan [\frac{1}{2} (d + e x)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan [\frac{1}{2} (d + e x)])} \right)^{3/2} \left( -c + (b + \sqrt{b^2 + c^2}) \tan \left[ \frac{1}{2} (d + e x) \right] \right) \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) / \\ & \left( 3 (b + i c + \sqrt{b^2 + c^2})^3 (b^2 + c^2 - b \sqrt{b^2 + c^2}) e (1 + \cos [d + e x]) \sqrt{\frac{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}{(1 + \cos [d + e x])^2}} (i + \tan \left[ \frac{1}{2} (d + e x) \right])^2 \right. \\ & \left. \left. \sqrt{\left( -\left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( -2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) + \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right)} \right) \right) - \end{aligned}$$



$$\begin{aligned}
& \frac{1}{3 c e (1 + \cos [d + e x])} \sqrt{\frac{-\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]}{(1+\cos [d+e x])^2}} - 8 (b^2 + c^2)^{3/2} \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]} \\
& \left( \left( -b + c \tan \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{-b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \tan \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \tan \left[ \frac{1}{2} (d + e x) \right]^2} \right. \\
& \left. \sqrt{\left( -\left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) + \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2} \right) / \\
& \left( (b^2 + c^2) \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \sqrt{-2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 + \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2} \right) + \\
& \left( \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \tan \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right. \\
& \left. \sqrt{\left( -\left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) + \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2} \right) \\
& \left( 2 c^2 \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right) + 2 i \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])}} \right], 1 \right] \left( -i + \tan\left[\frac{1}{2}(d + e x)\right] \right) \\
& \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right] \right)}{\left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right) \\
& \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + e x)\right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan\left[\frac{1}{2}(d + e x)\right] + (b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + e x)\right]^2 \right)} \right) + \\
& \left( 8 b^3 \left( (-b + i c + \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])}} \right], 1 \right] - 2 i c \right. \right. \\
& \left. \left. \text{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])}} \right], 1 \right] \right) \right. \\
& \left. \left( -i + \tan\left[\frac{1}{2}(d + e x)\right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right] \right)} \right) \right) / \\
& \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + e x)\right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan\left[\frac{1}{2}(d + e x)\right] + (b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + e x)\right]^2 \right)} \right) + \\
& \left( 4 b^5 \left( (-b + i c + \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + e x)])}} \right], 1 \right] - 2 i c \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \\
& \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right) / \\
& \left( c^2 (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) + \\
& \left( 4 b c^2 \left( (-b + i c + \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i c \right. \right. \\
& \left. \left. \text{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right. \right. \\
& \left. \left. \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right) \right) / \\
& \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) - \\
& \left( 4 b (b^2 + c^2) \left( (-b + i c + \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i c \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \text{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right) \\
& \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right) / \\
& \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2) \right)} \right) - \\
& \left( 4 b^3 (b^2 + c^2) \left( (-b + i c + \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i \right. \right. \\
& \left. \left. c \text{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right) \right) \\
& \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right) / \\
& \left( c^2 (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2) \right)} \right) + \\
& \left( 8 b^3 \left( (-b + i c - \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i c \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \\
& \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right) / \\
& \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) + \\
& \left( 4 b^5 \left( (-b + i c - \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i c \right. \right. \\
& \left. \left. \text{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right. \right. \\
& \left. \left. \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right) \right) / \\
& \left( c^2 (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) + \\
& \left( 4 b c^2 \left( (-b + i c - \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i c \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \\
& \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \\
& \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2)} \right) \Bigg) + \\
& \left( 8 b^2 \sqrt{b^2 + c^2} \left( (-b + i c - \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i \right. \right. \\
& \left. \left. c \text{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right) \right. \\
& \left. \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right) \Bigg/ \\
& \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2)} \right) \Bigg) + \\
& \left( 8 b^4 \sqrt{b^2 + c^2} \left( (-b + i c - \sqrt{b^2 + c^2}) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i \right. \right.
\end{aligned}$$

$$\begin{aligned}
& c \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \\
& \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Big/ \\
& \left( c^2 (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2)} \right) \Big/ + \\
& \left( 4 b (b^2 + c^2) \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i c \right. \right. \\
& \left. \left. \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right) \right) \\
& \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Big/ \\
& \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2)} \right) \Big/ + \\
& \left( 4 b^3 (b^2 + c^2) \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 i \right. \right.
\end{aligned}$$

$$\begin{aligned}
& c \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right) \\
& \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{(i b + c + i \sqrt{b^2 + c^2}) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \\
& \left( c^2 (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2)} \right) \Bigg) + \\
& \left( 2 b^3 \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right] \Bigg/ \left( c \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2)} \right) \Bigg) + \\
& \left( 2 b c \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right.
\end{aligned}$$



$$\begin{aligned}
& \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} / \left(\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)\right) \\
& \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(-b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left(b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} + \\
& \left(2b^2 \sqrt{b^2 + c^2} \left(-i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)\right) \\
& \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)} / \left(c \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)\right) \\
& \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)^2 \left(-b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left(b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} - \\
& \left(b c \left(2i \left(-\frac{1}{2}i \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(-ib + c - i\sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(ib + c + i\sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] - \frac{1}{2 \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}\right)\right) \\
& i \left(i \left(-i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) + i \left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right)\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-ib + c - i\sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}{\left(ib + c + i\sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]\right)}\right], 1\right] +
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{2 c \operatorname{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right]}{(b + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right) \\
& \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \\
& \left. \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) / \\
& \left( \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) - \\
& \left( c \sqrt{b^2 + c^2} \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right. \right. \right. \\
& \left. \left. \left. i \left( i \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right] + \right. \right.
\end{aligned}$$

$$\left. \frac{2 c \operatorname{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}} \right], 1 \right]}{(b + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right)$$

$$\left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}[\frac{1}{2} (d + e x)])}}$$

$$\left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) + \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left/ \right.$$

$$\left( \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \left/ \right.$$

$$\left( (b^2 + c^2) \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \sqrt{-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + b \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2}$$

$$\left( \sqrt{-2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 + \sqrt{b^2 + c^2} \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2} \right)$$

**Problem 439: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \, dx$$

Optimal (type 3, 57 leaves, 1 step):

$$\frac{2(c \cos[d + ex] - b \sin[d + ex])}{e \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}$$

Result (type 4, 11415 leaves):

$$\begin{aligned} & \frac{2b \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}{ce} - \left( 8bc(b^2 + c^2)^{3/2} \left( \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(b + ic + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(b - ic + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}}\right]}, 1\right] - \right. \\ & \quad \left. 2 \text{EllipticPi}\left[-1, \text{ArcSin}\left[\sqrt{-\frac{(b + ic + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(b - ic + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}}\right]}, 1\right] \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \right. \\ & \quad \left. (-i + \tan[\frac{1}{2}(d + ex)]) \left( -\frac{(b + ic + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(b - ic + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])} \right)^{3/2} (-c + (b + \sqrt{b^2 + c^2}) \tan[\frac{1}{2}(d + ex)]) (1 + \tan[\frac{1}{2}(d + ex)]^2) \right) \Bigg) / \\ & \quad \left( (b + ic + \sqrt{b^2 + c^2})^3 (b^2 + c^2 - b \sqrt{b^2 + c^2}) e (1 + \cos[d + ex]) \sqrt{\frac{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}} (i + \tan[\frac{1}{2}(d + ex)])^2 \right. \\ & \quad \left. \sqrt{\left( -\left(1 + \tan[\frac{1}{2}(d + ex)]^2\right) \left(-2c \tan[\frac{1}{2}(d + ex)] + b \left(-1 + \tan[\frac{1}{2}(d + ex)]^2\right) + \sqrt{b^2 + c^2} \left(1 + \tan[\frac{1}{2}(d + ex)]^2\right)\right) \right)} \right) + \\ & \quad \frac{1}{ce(1 + \cos[d + ex])} \sqrt{\frac{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}} 2(b^2 + c^2) \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \end{aligned}$$

$$\left( \left( -b + c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{-b + \sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2} \right.$$

$$\left. \sqrt{\left( -\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2 \left( -2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2 \right) + \sqrt{b^2+c^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2} \right) \right) /$$

$$\left( (b^2+c^2) \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2 \sqrt{-2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2 + \sqrt{b^2+c^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2} \right) +$$

$$\left( \sqrt{\left( \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2 \left( -b + \sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right.$$

$$\left. \sqrt{\left( -\left(1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2 \left( -2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2 \right) + \sqrt{b^2+c^2} \left(1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2} \right) \right)$$

$$\left( 2c^2 \left( -i - \frac{c}{b + \sqrt{b^2+c^2}} \right) \left( -i \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2+c^2}) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}{(i b + c + i \sqrt{b^2+c^2}) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1 \right] + 2i \right) \right.$$

$$\left. \operatorname{EllipticPi}\left[ \frac{i + \frac{c}{b + \sqrt{b^2+c^2}}}{-i + \frac{c}{b + \sqrt{b^2+c^2}}}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2+c^2}) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}{(i b + c + i \sqrt{b^2+c^2}) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1 \right] \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \right)$$

$$\begin{aligned}
& \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan[\frac{1}{2} (d + e x)] \right)} \Bigg/ \left( \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan[\frac{1}{2} (d + e x)] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan[\frac{1}{2} (d + e x)] + (b + \sqrt{b^2 + c^2}) \tan[\frac{1}{2} (d + e x)]^2 \right)} \right) \Bigg) + \\
& \left( 8 b^3 \left( (-b + i c + \sqrt{b^2 + c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right], 1\right] - 2 i c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b + i c - \sqrt{b^2 + c^2}) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{(b - i c - \sqrt{b^2 + c^2}) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right], 1\right] \right) \right) \\
& \left. (-i + \tan[\frac{1}{2} (d + e x)]) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan[\frac{1}{2} (d + e x)] \right)} \right) \Bigg/ \\
& \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \tan[\frac{1}{2} (d + e x)] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan[\frac{1}{2} (d + e x)] + (b + \sqrt{b^2 + c^2}) \tan[\frac{1}{2} (d + e x)]^2 \right)} \right) \Bigg) + \\
& \left( 4 b^5 \left( (-b + i c + \sqrt{b^2 + c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right], 1\right] - 2 i c \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b + i c - \sqrt{b^2 + c^2}) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{(b - i c - \sqrt{b^2 + c^2}) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])} \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) (-b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2) \right)} \right) + \\
& \left( 4bc^2 \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] - 2ic \right. \\
& \left. \operatorname{EllipticPi}\left[ \frac{(b+ic-\sqrt{b^2+c^2}) \left( i + \frac{c}{b+\sqrt{b^2+c^2}} \right)}{(b-ic-\sqrt{b^2+c^2}) \left( -i + \frac{c}{b+\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] \right) \Big/ \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])} \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) (-b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2) \right)} \right) - \\
& \left( 4b(b^2+c^2) \left( -b+ic+\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] - 2ic \right. \\
& \left. \operatorname{EllipticPi}\left[ \frac{(b+ic-\sqrt{b^2+c^2}) \left( i + \frac{c}{b+\sqrt{b^2+c^2}} \right)}{(b-ic-\sqrt{b^2+c^2}) \left( -i + \frac{c}{b+\sqrt{b^2+c^2}} \right)}, \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1 \right] \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( -b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) - \\
& \left( 4b^3(b^2+c^2) \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2i \right. \right. \\
& \left. \left. c \operatorname{EllipticPi}\left[\frac{(b+ic-\sqrt{b^2+c^2})\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{(b-ic-\sqrt{b^2+c^2})\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( c^2 (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( -b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 8b^3 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])} \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( -b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 4b^5 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])} \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Big/ \\
& \left( c^2 (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) \left( -b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) + \\
& \left( 4b^2 c^2 \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right])}} \right], 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) (-b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2)} \right) + \\
& \left( 8b^2\sqrt{b^2+c^2} \left( -b+ic-\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2i \right. \\
& \left. c \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)} \Bigg/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right)^2 \right) (-b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2)} \right) + \\
& \left( 8b^4\sqrt{b^2+c^2} \left( -b+ic-\sqrt{b^2+c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] - 2i \right. \\
& \left. c \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}} \right], 1\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b+\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Big/ \\
& \left( c^2 (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \left(i - \frac{c}{b+\sqrt{b^2+c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right) (-b+\sqrt{b^2+c^2} - 2c\tan\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2)} \right) + \right. \\
& \left. \left( 4b(b^2+c^2) \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2ic \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b+\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right)} \Big/ \\
& \left( (-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \left(i - \frac{c}{b+\sqrt{b^2+c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right) (-b+\sqrt{b^2+c^2} - 2c\tan\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2})\tan\left[\frac{1}{2}(d+ex)\right]^2)} \right) + \right. \\
& \left. \left( 4b^3(b^2+c^2) \left( (-b+ic-\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] - 2i \right. \right. \\
& \left. \left. c \operatorname{EllipticPi}\left[\frac{(b+ic+\sqrt{b^2+c^2})\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{(b-ic+\sqrt{b^2+c^2})\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \operatorname{ArcSin}\left[\sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\right]}, 1\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)} \Bigg/ \\
& \left( c^2(-b-ic-\sqrt{b^2+c^2})(b-ic+\sqrt{b^2+c^2}) \left(\frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c}\right) \left(i - \frac{c}{b+\sqrt{b^2+c^2}}\right) \right. \\
& \left. \sqrt{\left(\left(1+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2\right) \left(-b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) + \\
& \left( 2b^3 \left(-i - \frac{c}{b+\sqrt{b^2+c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-i + \frac{c}{b+\sqrt{b^2+c^2}}\right)(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{\left(i + \frac{c}{b+\sqrt{b^2+c^2}}\right)(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}\right], 1\right] \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right) \right. \\
& \left. \sqrt{\frac{\left(-i + \frac{c}{b+\sqrt{b^2+c^2}}\right)(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{\left(i + \frac{c}{b+\sqrt{b^2+c^2}}\right)(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)} \right/ \left(c \left(-i + \frac{c}{b+\sqrt{b^2+c^2}}\right)\right) \\
& \left(i + \frac{c}{b+\sqrt{b^2+c^2}}\right) \sqrt{\left(\left(1+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2\right) \left(-b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2\right)} \Bigg) + \\
& \left( 2bc \left(-i - \frac{c}{b+\sqrt{b^2+c^2}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(-i + \frac{c}{b+\sqrt{b^2+c^2}}\right)(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{\left(i + \frac{c}{b+\sqrt{b^2+c^2}}\right)(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}\right], 1\right] \left(-i + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right) \right. \\
& \left. \sqrt{\frac{\left(-i + \frac{c}{b+\sqrt{b^2+c^2}}\right)(i+\operatorname{Tan}[\frac{1}{2}(d+ex)])}{\left(i + \frac{c}{b+\sqrt{b^2+c^2}}\right)(-i+\operatorname{Tan}[\frac{1}{2}(d+ex)])} \left(-\frac{c}{b+\sqrt{b^2+c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)} \right/ \left(\left(-i + \frac{c}{b+\sqrt{b^2+c^2}}\right) \left(i + \frac{c}{b+\sqrt{b^2+c^2}}\right)\right) \\
& \left. \sqrt{\left(\left(1+\operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2\right) \left(-b+\sqrt{b^2+c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2\right)} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 2 b^2 \sqrt{b^2 + c^2} \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{\left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right] / \left( c \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \\
& \left. \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right) \right) - \\
& \left( b c \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right. \right. \\
& \left. \left. i \left( i \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] + \right. \\
& \left. \frac{2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \text{ArcSin} \left[ \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right]}{\left( b + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right) \\
& \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}}
\end{aligned}$$

$$\left. \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) + \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) /$$

$$\left( \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)^2 \left( -b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} -$$

$$\left( c \sqrt{b^2 + c^2} \left( 2i \left( -\frac{1}{2}i \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - \frac{1}{2 \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right) \right)$$

$$i \left( i \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] +$$

$$\frac{2c \operatorname{EllipticPi}\left[\frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \operatorname{ArcSin}\left[\sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right]}{\left( b + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}$$

$$\left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}}$$

$$\left. \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) + \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) /$$

$$\left. \left( \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left( b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right)} \right) /$$

$$\left( (b^2 + c^2) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \sqrt{-b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] + b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2}$$

$$\left. \sqrt{-2c \tan\left[\frac{1}{2}(d + ex)\right] + b \left( -1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2} \right)$$

**Problem 440:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}} dx$$

Optimal (type 3, 91 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{(b^2 + c^2)^{1/4} \sin[d + ex - \operatorname{ArcTan}[b, c]]}{\sqrt{2} \sqrt{-\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos[d + ex - \operatorname{ArcTan}[b, c]]}}\right]}{(b^2 + c^2)^{1/4} e}$$

Result (type 4, 61 904 leaves): Display of huge result suppressed!

Problem 441: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]\right)^{3/2}} dx$$

Optimal (type 3, 164 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{(b^2+c^2)^{1/4} \sin [d+e x-\text{ArcTan}[b,c]]}{\sqrt{2} \sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}} \cos [d+e x-\text{ArcTan}[b,c]]}\right]}{2 \sqrt{2} (b^2+c^2)^{3/4} e} + \frac{c \cos [d+e x] - b \sin [d+e x]}{2 \sqrt{b^2+c^2} e \left(-\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

Problem 442: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]\right)^{5/2}} dx$$

Optimal (type 3, 232 leaves, 5 steps):

$$-\frac{3 \text{ArcTan}\left[\frac{(b^2+c^2)^{1/4} \sin [d+e x-\text{ArcTan}[b,c]]}{\sqrt{2} \sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}} \cos [d+e x-\text{ArcTan}[b,c]]}\right]}{16 \sqrt{2} (b^2+c^2)^{5/4} e} + \frac{c \cos [d+e x] - b \sin [d+e x]}{4 \sqrt{b^2+c^2} e \left(-\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]\right)^{5/2}} - \frac{3 (c \cos [d+e x] - b \sin [d+e x])}{16 (b^2+c^2) e \left(-\sqrt{b^2+c^2} + b \cos [d+e x] + c \sin [d+e x]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???



**Problem 448:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2}}{\operatorname{Sec}[d + e x]^{3/2}} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\begin{aligned} & - \frac{2 (c \operatorname{Cos}[d + e x] - a \operatorname{Sin}[d + e x]) (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2}}{3 e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])} + \\ & \frac{8 b \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2}}{3 e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]) \sqrt{\frac{b+a \operatorname{Cos}[d+e x]+c \operatorname{Sin}[d+e x]}{b+\sqrt{a^2+c^2}}}} + \\ & \left( \frac{2 (a^2 - b^2 + c^2) \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{\frac{b+a \operatorname{Cos}[d+e x]+c \operatorname{Sin}[d+e x]}{b+\sqrt{a^2+c^2}}}}{(3 e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^2)} \right) \end{aligned}$$

Result (type 6, 2490 leaves):

$$\begin{aligned} & \frac{\left(\frac{8ab}{3c} - \frac{2}{3} c \operatorname{Cos}[d + e x] + \frac{2}{3} a \operatorname{Sin}[d + e x]\right) (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2}}{e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])} + \\ & \left( 2 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \right. \\ & \left. \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right) \end{aligned}$$

$$\left. (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2} \right/ \left( 3 \sqrt{1 + \frac{a^2}{c^2}} c e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{3/2} \right) +$$

$$\left( 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \right)$$

$$\sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}}$$

$$\left. (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2} \right/ \left( \sqrt{1 + \frac{a^2}{c^2}} c e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{3/2} \right) +$$

$$\left( 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \right)$$

$$\begin{aligned}
& \left( \frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}} \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \right. \\
& \left. \frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}} \right) \\
& \left. \left( a + b \operatorname{Sec}\left[d + e x\right] + c \operatorname{Tan}\left[d + e x\right] \right)^{3/2} \right) / \left( 3 \sqrt{1 + \frac{a^2}{c^2}} e \operatorname{Sec}\left[d + e x\right]^{3/2} (b + a \operatorname{Cos}\left[d + e x\right] + c \operatorname{Sin}\left[d + e x\right])^{3/2} \right) + \\
& \left( 4 a^2 b \left( \left( \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)}\right], -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(-1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)} \right) \right. \right. \\
& \left. \left. \operatorname{Sin}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \right) \right) / \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} - a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{b + a \sqrt{\frac{a^2+c^2}{a^2}}}} \sqrt{b + a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]} \right) \\
& \left. \left( \frac{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}} \right) \right) - \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \right)}{a^2 + c^2} - \frac{c \operatorname{Sin}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}}} \right) \\
& \left. \sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]} \right)
\end{aligned}$$

$$\left. \left. \left. (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2} \right/ \left( 3 c e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{3/2} \right) + \right. \right.$$

$$\left. \left. \left. 4 b c \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right], -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right) \right) \right) \right.$$

$$\left. \left. \left. \operatorname{Sin}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]] \right/ \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}} \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]} \right) \right.$$

$$\left. \left. \left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) - \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]] \right) - \frac{c \operatorname{Sin}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}}}}{\sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}} \right.$$

$$\left. \frac{(a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2}}{\sqrt{\operatorname{Sec}[d + e x]}} \right/ \left( 3 e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{3/2} \right)$$

**Problem 449: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]}}{\sqrt{\operatorname{Sec}[d + e x]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{2 \operatorname{EllipticE}\left[\frac{1}{2}(d + e x - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]}}{e \sqrt{\operatorname{Sec}[d + e x]} \sqrt{\frac{b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}}}$$

Result (type 6, 1580 leaves):

$$\frac{2 a \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]}}{c e \sqrt{\operatorname{Sec}[d + e x]}} + \left( 2 b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}\right] \operatorname{Sec}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]] \right. \\ \left. \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right)$$

$$\left. \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]} \right/ \left( \sqrt{1 + \frac{a^2}{c^2}} c e \sqrt{\operatorname{Sec}[d + e x]} \sqrt{b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]} \right) +$$

$$\left( a^2 \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \right)$$

$$\left. \operatorname{Sin}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]] \right/ \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}} \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]} \right)$$

$$\left( \frac{\sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) - \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]] \right) - c \operatorname{Sin}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a^2 + c^2} - \frac{c \operatorname{Sin}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}}}{\sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}}$$

$$\left. \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]} \right/ \left( c e \sqrt{\operatorname{Sec}[d + e x]} \sqrt{b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]} \right) +$$

$$\left( \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}}{a\sqrt{1+\frac{c^2}{a^2}} \left( 1-\frac{b}{a\sqrt{1+\frac{c^2}{a^2}}} \right)}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}}{a\sqrt{1+\frac{c^2}{a^2}} \left( -1-\frac{b}{a\sqrt{1+\frac{c^2}{a^2}}} \right)} \right] \right) \right) \right)$$

$$\left. \operatorname{Sin} \left[ d+ex-\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right/ \left( a\sqrt{1+\frac{c^2}{a^2}} \sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}}-a\sqrt{\frac{a^2+c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}{b+a\sqrt{\frac{a^2+c^2}{a^2}}}} \sqrt{b+a\sqrt{\frac{a^2+c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]} \right)$$

$$\left. \sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}}+a\sqrt{\frac{a^2+c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}{-b+a\sqrt{\frac{a^2+c^2}{a^2}}}} \right) - \frac{2a \left( b+a\sqrt{1+\frac{c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]] \right)}{a^2+c^2} - \frac{c \operatorname{Sin} \left[ d+ex-\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a\sqrt{1+\frac{c^2}{a^2}}}$$

$$\left. \sqrt{a+b \operatorname{Sec} [d+ex] + c \operatorname{Tan} [d+ex]} \right/ \left( e \sqrt{\operatorname{Sec} [d+ex]} \sqrt{b+a \operatorname{Cos} [d+ex] + c \operatorname{Sin} [d+ex]} \right)$$

Problem 450: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\operatorname{Sec} [d+ex]}}{\sqrt{a+b \operatorname{Sec} [d+ex] + c \operatorname{Tan} [d+ex]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2}(d+ex - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{\operatorname{Sec}[d+ex]} \sqrt{\frac{b+a\cos[d+ex]+c\sin[d+ex]}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+b\operatorname{Sec}[d+ex]+c\tan[d+ex]}}$$

Result (type 6, 339 leaves):

$$\left( 2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{b+\sqrt{1+\frac{a^2}{c^2}}c\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]]}{b-\sqrt{1+\frac{a^2}{c^2}}c}, \frac{b+\sqrt{1+\frac{a^2}{c^2}}c\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]]}{b+\sqrt{1+\frac{a^2}{c^2}}c}\right] \right. \\ \left. \sqrt{\operatorname{Sec}[d+ex]} \operatorname{Sec}[d+ex+\operatorname{ArcTan}[\frac{a}{c}]] \sqrt{b+a\cos[d+ex]+c\sin[d+ex]}} \sqrt{-\frac{\sqrt{1+\frac{a^2}{c^2}}c(-1+\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]])}{b+\sqrt{1+\frac{a^2}{c^2}}c}} \right) \\ \left( \sqrt{\frac{\sqrt{1+\frac{a^2}{c^2}}c(1+\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]])}{-b+\sqrt{1+\frac{a^2}{c^2}}c}} \sqrt{b+\sqrt{1+\frac{a^2}{c^2}}c\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]]} \right) \left( \sqrt{1+\frac{a^2}{c^2}}c e\sqrt{a+b\operatorname{Sec}[d+ex]+c\tan[d+ex]} \right)$$

**Problem 451: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[d+ex]^{3/2}}{(a+b\operatorname{Sec}[d+ex]+c\tan[d+ex])^{3/2}} dx$$

Optimal (type 4, 240 leaves, 4 steps):



$$\frac{2 \operatorname{Sec}[d+e x]^{3/2} (c \operatorname{Cos}[d+e x] - a \operatorname{Sin}[d+e x]) (b+a \operatorname{Cos}[d+e x] + c \operatorname{Sin}[d+e x])}{(a^2 - b^2 + c^2) e (a+b \operatorname{Sec}[d+e x] + c \operatorname{Tan}[d+e x])^{3/2}}$$

$$\frac{2 \operatorname{EllipticE}\left[\frac{1}{2}(d+e x - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \operatorname{Sec}[d+e x]^{3/2} (b+a \operatorname{Cos}[d+e x] + c \operatorname{Sin}[d+e x])^2}{(a^2 - b^2 + c^2) e \sqrt{\frac{b+a \operatorname{Cos}[d+e x]+c \operatorname{Sin}[d+e x]}{b+\sqrt{a^2+c^2}}} (a+b \operatorname{Sec}[d+e x] + c \operatorname{Tan}[d+e x])^{3/2}}$$

Result (type 6, 1732 leaves):

$$\frac{\operatorname{Sec}[d+e x]^{3/2} (b+a \operatorname{Cos}[d+e x] + c \operatorname{Sin}[d+e x])^2 \left(-\frac{2(a^2+c^2)}{a c (a^2-b^2+c^2)} + \frac{2(b c+a^2 \operatorname{Sin}[d+e x]+c^2 \operatorname{Sin}[d+e x])}{a (a^2-b^2+c^2) (b+a \operatorname{Cos}[d+e x]+c \operatorname{Sin}[d+e x])}\right)}{e (a+b \operatorname{Sec}[d+e x] + c \operatorname{Tan}[d+e x])^{3/2}}$$

$$\left( 2 b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b+\sqrt{1+\frac{a^2}{c^2}} c \operatorname{Sin}[d+e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1+\frac{a^2}{c^2}} \left(1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}} c}\right) c}, -\frac{b+\sqrt{1+\frac{a^2}{c^2}} c \operatorname{Sin}[d+e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1+\frac{a^2}{c^2}} \left(-1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}} c}\right) c}\right] \operatorname{Sec}[d+e x]^{3/2} \right.$$

$$\operatorname{Sec}\left[d+e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] (b+a \operatorname{Cos}[d+e x] + c \operatorname{Sin}[d+e x])^{3/2} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d+e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b+c \sqrt{\frac{a^2+c^2}{c^2}}}}$$

$$\sqrt{\frac{b+c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d+e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d+e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \operatorname{Sin}\left[d+e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b+c \sqrt{\frac{a^2+c^2}{c^2}}}}$$

$$\left( \sqrt{1 + \frac{a^2}{c^2}} c (a^2 - b^2 + c^2) e^{(a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2}} - a^2 \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{3/2} \right)$$

$$\left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(-1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)}\right] \right)$$

$$\operatorname{Sin}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \left/ \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}} \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]]} \right) \right.$$

$$\left( \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} - \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]] \right) - \frac{c \operatorname{Sin}[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]]}{a \sqrt{1 + \frac{c^2}{a^2}}}}{\sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]]}} \right) \left/ \right.$$

$$\left( c (a^2 - b^2 + c^2) e (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2} \right) - \left( c \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{3/2} \right)$$

$$\left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \right) \right)$$

$$\operatorname{Sin}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]] \left/ \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) \right.$$

$$\left( \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) -$$

$$\left( \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]] \right)}{a^2 + c^2} - \frac{c \operatorname{Sin}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]}{a \sqrt{1 + \frac{c^2}{a^2}}} \right) \left/ \left( (a^2 - b^2 + c^2) e (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2} \right) \right.$$

$$\left( \sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{c}{a}]]} \right)$$

**Problem 452: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[d + e x]^{5/2}}{(a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{5/2}} dx$$

Optimal (type 4, 492 leaves, 8 steps):

$$\begin{aligned} & - \frac{2 \operatorname{Sec}[d + e x]^{5/2} (c \operatorname{Cos}[d + e x] - a \operatorname{Sin}[d + e x]) (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])}{3 (a^2 - b^2 + c^2) e (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{5/2}} + \\ & \frac{8 \operatorname{Sec}[d + e x]^{5/2} (b c \operatorname{Cos}[d + e x] - a b \operatorname{Sin}[d + e x]) (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^2}{3 (a^2 - b^2 + c^2)^2 e (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{5/2}} + \\ & \frac{8 b \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \operatorname{Sec}[d + e x]^{5/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^3}{3 (a^2 - b^2 + c^2)^2 e \sqrt{\frac{b+a \operatorname{Cos}[d+e x]+c \operatorname{Sin}[d+e x]}{b+\sqrt{a^2+c^2}}} (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{5/2}} + \\ & \left( 2 \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \operatorname{Sec}[d + e x]^{5/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^2 \sqrt{\frac{b+a \operatorname{Cos}[d+e x]+c \operatorname{Sin}[d+e x]}{b+\sqrt{a^2+c^2}}} \right) / \\ & \left( 3 (a^2 - b^2 + c^2) e (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{5/2} \right) \end{aligned}$$

Result (type 6, 2708 leaves):

$$\begin{aligned} & \left( \operatorname{Sec}[d + e x]^{5/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^3 \right. \\ & \left. \left( \frac{8 b (a^2 + c^2)}{3 a c (-a^2 + b^2 - c^2)^2} + \frac{2 (b c + a^2 \operatorname{Sin}[d + e x] + c^2 \operatorname{Sin}[d + e x])}{3 a (a^2 - b^2 + c^2) (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^2} - \frac{2 (a^2 c + 3 b^2 c + c^3 + 4 a^2 b \operatorname{Sin}[d + e x] + 4 b c^2 \operatorname{Sin}[d + e x])}{3 a (a^2 - b^2 + c^2)^2 (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])} \right) \right) / \\ & \left( e (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{5/2} \right) + \\ & \left( 2 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}\right] \operatorname{Sec}[d + e x]^{5/2} \right) \end{aligned}$$

$$\begin{aligned}
& \text{Sec}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right] (b + a \text{Cos}[d + e x] + c \text{Sin}[d + e x])^{5/2} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \\
& \left( \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right) / \\
& \left( 3 \sqrt{1 + \frac{a^2}{c^2}} c (a^2 - b^2 + c^2)^2 e (a + b \text{Sec}[d + e x] + c \text{Tan}[d + e x])^{5/2} + \right. \\
& \left. 2 b^2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}\right] \text{Sec}[d + e x]^{5/2} \right) \\
& \text{Sec}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right] (b + a \text{Cos}[d + e x] + c \text{Sin}[d + e x])^{5/2} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \\
& \left( \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \text{Sin}\left[d + e x + \text{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{1 + \frac{a^2}{c^2}} c (a^2 - b^2 + c^2)^2 e (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{5/2} \right) + \\
& \left( 2 c \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c} \right] \operatorname{Sec}[d + e x]^{5/2} \right. \\
& \operatorname{Sec}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]] (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{5/2} \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \\
& \left. \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]} \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) / \\
& \left( 3 \sqrt{1 + \frac{a^2}{c^2}} (a^2 - b^2 + c^2)^2 e (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{5/2} \right) + \left( 4 a^2 b \operatorname{Sec}[d + e x]^{5/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{5/2} \right)
\end{aligned}$$

$$\left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}{a\sqrt{1+\frac{c^2}{a^2}} \left(1-\frac{b}{a\sqrt{1+\frac{c^2}{a^2}}}\right)}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}{a\sqrt{1+\frac{c^2}{a^2}} \left(-1-\frac{b}{a\sqrt{1+\frac{c^2}{a^2}}}\right)} \right] \right) \right) \right)$$

$$\left. \left. \left. \sin \left[ d+ex-\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right) \right/ \left( a\sqrt{1+\frac{c^2}{a^2}} \sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}}-a\sqrt{\frac{a^2+c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}{b+a\sqrt{\frac{a^2+c^2}{a^2}}} \sqrt{b+a\sqrt{\frac{a^2+c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]} \right) \right)$$

$$\left. \left. \left. \sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}}+a\sqrt{\frac{a^2+c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}{-b+a\sqrt{\frac{a^2+c^2}{a^2}}}} \right) \right) - \left. \left. \left. \frac{2a \left( b+a\sqrt{1+\frac{c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]] \right) - \frac{c \sin[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}{a\sqrt{1+\frac{c^2}{a^2}}}}{\sqrt{b+a\sqrt{1+\frac{c^2}{a^2}} \cos[d+ex-\operatorname{ArcTan}[\frac{c}{a}]]}} \right) \right) \right)$$

$$\left( 3c(a^2-b^2+c^2)^2 e^{(a+b \operatorname{Sec}[d+ex]+c \operatorname{Tan}[d+ex])^{5/2}} + 4bc \operatorname{Sec}[d+ex]^{5/2} (b+a \operatorname{Cos}[d+ex]+c \operatorname{Sin}[d+ex])^{5/2} \right)$$

$$\left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a\sqrt{1+\frac{c^2}{a^2}} \left( 1 - \frac{b}{a\sqrt{1+\frac{c^2}{a^2}}} \right)}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a\sqrt{1+\frac{c^2}{a^2}} \left( -1 - \frac{b}{a\sqrt{1+\frac{c^2}{a^2}}} \right)} \right] \right) \right)$$

$$\left. \left. \left. \operatorname{Sin} \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right) \right/ \left( a\sqrt{1+\frac{c^2}{a^2}} \sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}} - a\sqrt{\frac{a^2+c^2}{a^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b+a\sqrt{\frac{a^2+c^2}{a^2}}}} \right) \right)$$

$$\left. \left. \left. \sqrt{\frac{b+a\sqrt{\frac{a^2+c^2}{a^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a\sqrt{\frac{a^2+c^2}{a^2}} + a\sqrt{\frac{a^2+c^2}{a^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b+a\sqrt{\frac{a^2+c^2}{a^2}}}} \right) \right)$$

$$\left. \left. \left. \frac{2a \left( b+a\sqrt{1+\frac{c^2}{a^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right)}{a^2+c^2} - \frac{c \operatorname{Sin} \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a\sqrt{1+\frac{c^2}{a^2}}} \right) \right) \right/ \left( 3 \left( a^2 - b^2 + c^2 \right)^2 e \left( a + b \operatorname{Sec} \left[ d+ex \right] + c \operatorname{Tan} \left[ d+ex \right] \right)^{5/2} \right)$$

$$\left. \left. \left. \sqrt{b+a\sqrt{1+\frac{c^2}{a^2}} \cos \left[ d+ex - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \right) \right)$$

Problem 453: Attempted integration timed out after 120 seconds.

$$\int \cos [d+ex]^{3/2} (a+b \operatorname{Sec} [d+ex] + c \operatorname{Tan} [d+ex])^{3/2} dx$$

Optimal (type 4, 371 leaves, 7 steps):



$$\begin{aligned}
& - \frac{2 \operatorname{Cos}[d + e x]^{3/2} (c \operatorname{Cos}[d + e x] - a \operatorname{Sin}[d + e x]) (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2}}{3 e (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])} + \\
& \left( 8 b \operatorname{Cos}[d + e x]^{3/2} \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2} \right) / \\
& \left( 3 e (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]) \sqrt{\frac{b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) + \\
& \left( 2 (a^2 - b^2 + c^2) \operatorname{Cos}[d + e x]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] \sqrt{\frac{b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}} \right. \\
& \left. (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2} \right) / (3 e (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^2)
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 454: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{\operatorname{Cos}[d + e x]} \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{2 \sqrt{\operatorname{Cos}[d + e x]} \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]}}{e \sqrt{\frac{b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}}}$$

Result (type 1, 1 leaves):

???

**Problem 455: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{\operatorname{Cos}[d + e x]} \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{\frac{b+a \cos[d+e x]+c \sin[d+e x]}{b+\sqrt{a^2+c^2}}}}{e \sqrt{\cos[d+e x]} \sqrt{a+b \sec[d+e x]+c \tan[d+e x]}}$$

Result (type 4, 506 leaves):

$$\left( 4 \left( i a - i b + c + \sqrt{a^2 - b^2 + c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i a + i b + c + \sqrt{a^2 - b^2 + c^2}) (-\cos[d+e x] + i \sin[d+e x])}{i a - i b + c + \sqrt{a^2 - b^2 + c^2}}}\right], \frac{b + i \sqrt{a^2 - b^2 + c^2}}{b - i \sqrt{a^2 - b^2 + c^2}}\right] \right. \\ \left. \sqrt{\frac{(-i a + i b + c + \sqrt{a^2 - b^2 + c^2}) (-\cos[d+e x] + i \sin[d+e x])}{i a - i b + c + \sqrt{a^2 - b^2 + c^2}}} (\cos[d+e x] + i \sin[d+e x]) \right. \\ \left. \sqrt{-\frac{i (-c + \sqrt{a^2 - b^2 + c^2} + (a - b) \tan[\frac{1}{2} (d + e x)])}{(-i a + i b - c + \sqrt{a^2 - b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \sqrt{-\frac{i (c + \sqrt{a^2 - b^2 + c^2} + (-a + b) \tan[\frac{1}{2} (d + e x)])}{(i a - i b + c + \sqrt{a^2 - b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right) \\ \left( \left( a + i \left( i b + c + \sqrt{a^2 - b^2 + c^2} \right) \right) e \sqrt{\cos[d+e x]} \sqrt{a+b \sec[d+e x]+c \tan[d+e x]} \right)$$

Problem 456: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\cos[d+e x]^{3/2} (a+b \sec[d+e x]+c \tan[d+e x])^{3/2}} dx$$

Optimal (type 4, 240 leaves, 4 steps):

$$\frac{2 (c \cos[d+e x] - a \sin[d+e x]) (b + a \cos[d+e x] + c \sin[d+e x])}{(a^2 - b^2 + c^2) e \cos[d+e x]^{3/2} (a + b \sec[d+e x] + c \tan[d+e x])^{3/2}} - \\ \frac{2 \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] (b + a \cos[d+e x] + c \sin[d+e x])^2}{(a^2 - b^2 + c^2) e \cos[d+e x]^{3/2} \sqrt{\frac{b+a \cos[d+e x]+c \sin[d+e x]}{b+\sqrt{a^2+c^2}}} (a + b \sec[d+e x] + c \tan[d+e x])^{3/2}}$$

Result (type 1, 1 leaves):

???

### Problem 457: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\cos [d+e x]^{5 / 2} (a+b \operatorname{Sec}[d+e x]+c \operatorname{Tan}[d+e x])^{5 / 2}} d x$$

Optimal (type 4, 492 leaves, 8 steps):

$$\begin{aligned} & -\frac{2(c \cos [d+e x]-a \sin [d+e x])(b+a \cos [d+e x]+c \sin [d+e x])}{3\left(a^2-b^2+c^2\right) e \cos [d+e x]^{5 / 2}(a+b \operatorname{Sec}[d+e x]+c \operatorname{Tan}[d+e x])^{5 / 2}}+\frac{8(b c \cos [d+e x]-a b \sin [d+e x])(b+a \cos [d+e x]+c \sin [d+e x])^2}{3\left(a^2-b^2+c^2\right)^2 e \cos [d+e x]^{5 / 2}(a+b \operatorname{Sec}[d+e x]+c \operatorname{Tan}[d+e x])^{5 / 2}}+ \\ & \frac{8 b \operatorname{EllipticE}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right](b+a \cos [d+e x]+c \sin [d+e x])^3}{3\left(a^2-b^2+c^2\right)^2 e \cos [d+e x]^{5 / 2} \sqrt{\frac{b+a \cos [d+e x]+c \sin [d+e x]}{b+\sqrt{a^2+c^2}}}(a+b \operatorname{Sec}[d+e x]+c \operatorname{Tan}[d+e x])^{5 / 2}}+ \\ & \left(2 \operatorname{EllipticF}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right](b+a \cos [d+e x]+c \sin [d+e x])^2 \sqrt{\frac{b+a \cos [d+e x]+c \sin [d+e x]}{b+\sqrt{a^2+c^2}}}\right) / \\ & \left(3\left(a^2-b^2+c^2\right) e \cos [d+e x]^{5 / 2}(a+b \operatorname{Sec}[d+e x]+c \operatorname{Tan}[d+e x])^{5 / 2}\right) \end{aligned}$$

Result (type 1, 1 leaves):

???

### Problem 461: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[x]}{2+2 \operatorname{Cot}[x]+3 \operatorname{Csc}[x]} d x$$

Optimal (type 3, 21 leaves, 4 steps):

$$x+2 \operatorname{ArcTan}\left[\frac{\cos [x]-\sin [x]}{2+\cos [x]+\sin [x]}\right]$$

Result (type 3, 51 leaves):

$$-\operatorname{ArcTan}\left[\frac{\cos \left[\frac{x}{2}\right]}{2 \cos \left[\frac{x}{2}\right]+\sin \left[\frac{x}{2}\right]}\right]+\operatorname{ArcTan}\left[\operatorname{Sec}\left[\frac{x}{2}\right]\left(2 \cos \left[\frac{x}{2}\right]+\sin \left[\frac{x}{2}\right]\right)\right]$$

**Problem 462:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2}}{\operatorname{Csc} [d + e x]^{3/2}} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\frac{8 b (a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} \operatorname{EllipticE}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right]}{3 e \operatorname{Csc} [d + e x]^{3/2} (b + c \cos [d + e x] + a \sin [d + e x]) \sqrt{\frac{b+c \cos [d+e x]+a \sin [d+e x]}{b+\sqrt{a^2+c^2}}}} +$$

$$\left( 2 (a^2 - b^2 + c^2) (a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{\frac{b+c \cos [d+e x]+a \sin [d+e x]}{b+\sqrt{a^2+c^2}}} \right) /$$

$$\left( 3 e \operatorname{Csc} [d + e x]^{3/2} (b + c \cos [d + e x] + a \sin [d + e x])^2 \right) - \frac{2 (a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} (a \cos [d + e x] - c \sin [d + e x])}{3 e \operatorname{Csc} [d + e x]^{3/2} (b + c \cos [d + e x] + a \sin [d + e x])}$$

Result (type 6, 2490 leaves):

$$\frac{(a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} \left( \frac{8bc}{3a} - \frac{2}{3} a \cos [d + e x] + \frac{2}{3} c \sin [d + e x] \right)}{e \operatorname{Csc} [d + e x]^{3/2} (b + c \cos [d + e x] + a \sin [d + e x])} + \left( 4 a b (a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} \right.$$

$$\left. - \left( \left( \left( a \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos [d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos [d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c} \right) \right) \right) \right)$$



$$\left( \frac{\sqrt{b+c} \sqrt{\frac{a^2+c^2}{c^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{\frac{c\sqrt{\frac{a^2+c^2}{c^2}} + c\sqrt{\frac{a^2+c^2}{c^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b+c\sqrt{\frac{a^2+c^2}{c^2}}}} \right) -$$

$$\left( \frac{2c \left( \frac{b+\sqrt{1+\frac{a^2}{c^2}} c \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{a^2+c^2} - \frac{a \sin\left[d+ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1+\frac{a^2}{c^2}} c} \right)}{\sqrt{b+\sqrt{1+\frac{a^2}{c^2}} c \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right}}} \right) / \left( 3ae \operatorname{Csc}\left[d+ex\right]^{3/2} \left( b+c \cos\left[d+ex\right] + a \sin\left[d+ex\right] \right)^{3/2} \right) +$$

$$\left[ 2a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \sin\left[d+ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a\sqrt{1+\frac{c^2}{a^2}} \left(1 - \frac{b}{a\sqrt{1+\frac{c^2}{a^2}}}\right)}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \sin\left[d+ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a\sqrt{1+\frac{c^2}{a^2}} \left(-1 - \frac{b}{a\sqrt{1+\frac{c^2}{a^2}}}\right)} \right]$$

$$\left( a+c \cot\left[d+ex\right] + b \operatorname{Csc}\left[d+ex\right] \right)^{3/2} \operatorname{Sec}\left[d+ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]$$

$$\sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}} - a\sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d+ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{b+a\sqrt{\frac{a^2+c^2}{a^2}}}}$$

$$\sqrt{b+a\sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d+ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}$$

$$\sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}}}$$

$$\left( 3 \sqrt{1 + \frac{c^2}{a^2}} e \operatorname{Csc}[d + ex]^{3/2} (b + c \cos[d + ex] + a \sin[d + ex])^{3/2} \right) +$$

$$\left( 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(-1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)}\right] \right)$$

$$(a + c \cot[d + ex] + b \operatorname{Csc}[d + ex])^{3/2} \operatorname{Sec}\left[d + ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} - a \sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{b + a \sqrt{\frac{a^2+c^2}{a^2}}}}$$

$$\sqrt{b + a \sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]} \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}}}$$

$$\left( a \sqrt{1 + \frac{c^2}{a^2}} e \operatorname{Csc}[d + ex]^{3/2} (b + c \cos[d + ex] + a \sin[d + ex])^{3/2} \right) +$$

$$\left( 2 c^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}}\sin[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]}{a\sqrt{1+\frac{c^2}{a^2}}\left(1-\frac{b}{a\sqrt{1+\frac{c^2}{a^2}}}\right)}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}}\sin[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]}{a\sqrt{1+\frac{c^2}{a^2}}\left(-1-\frac{b}{a\sqrt{1+\frac{c^2}{a^2}}}\right)}\right], \right.$$

$$\left. (a+c\cot[d+ex]+b\csc[d+ex])^{3/2}\sec[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]\sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}}-a\sqrt{\frac{a^2+c^2}{a^2}}\sin[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]}{b+a\sqrt{\frac{a^2+c^2}{a^2}}}} \right)$$

$$\left( \sqrt{b+a\sqrt{\frac{a^2+c^2}{a^2}}\sin[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]}\sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}}+a\sqrt{\frac{a^2+c^2}{a^2}}\sin[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]}{-b+a\sqrt{\frac{a^2+c^2}{a^2}}}} \right)$$

$$\left( 3 a \sqrt{1+\frac{c^2}{a^2}} e \csc[d+ex]^{3/2} (b+c\cos[d+ex]+a\sin[d+ex])^{3/2} \right)$$

**Problem 463:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+c\cot[d+ex]+b\csc[d+ex]}}{\sqrt{\csc[d+ex]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{2\sqrt{a+c\cot[d+ex]+b\csc[d+ex]}\operatorname{EllipticE}\left[\frac{1}{2}(d+ex-\operatorname{ArcTan}[c,a]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right]}{e\sqrt{\csc[d+ex]}\sqrt{\frac{b+c\cos[d+ex]+a\sin[d+ex]}{b+\sqrt{a^2+c^2}}}}$$

Result (type 6, 1580 leaves):



$$\begin{aligned}
& \frac{2c\sqrt{a+c\cot[d+ex]+b\csc[d+ex]}}{ae\sqrt{\csc[d+ex]}} + \left( a\sqrt{a+c\cot[d+ex]+b\csc[d+ex]} \right. \\
& \left. - \left( \left( a \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b+\sqrt{1+\frac{a^2}{c^2}}c\cos[d+ex-\operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1+\frac{a^2}{c^2}}\left(1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}}c}\right)c}, -\frac{b+\sqrt{1+\frac{a^2}{c^2}}c\cos[d+ex-\operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1+\frac{a^2}{c^2}}\left(-1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}}c}\right)c}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Sin}\left[d+ex-\operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \right) \sqrt{\frac{\sqrt{1+\frac{a^2}{c^2}}c\sqrt{\frac{a^2+c^2}{c^2}-c\sqrt{\frac{a^2+c^2}{c^2}}\cos[d+ex-\operatorname{ArcTan}[\frac{a}{c}]]}}{b+c\sqrt{\frac{a^2+c^2}{c^2}}}\sqrt{b+c\sqrt{\frac{a^2+c^2}{c^2}}\cos[d+ex-\operatorname{ArcTan}[\frac{a}{c}]]}} \right.} \\
& \left. \left. \sqrt{\frac{c\sqrt{\frac{a^2+c^2}{c^2}}+c\sqrt{\frac{a^2+c^2}{c^2}}\cos[d+ex-\operatorname{ArcTan}[\frac{a}{c}]]}{-b+c\sqrt{\frac{a^2+c^2}{c^2}}}} - \frac{2c\left(b+\sqrt{1+\frac{a^2}{c^2}}c\cos[d+ex-\operatorname{ArcTan}[\frac{a}{c}]]\right)-a\operatorname{Sin}\left[d+ex-\operatorname{ArcTan}[\frac{a}{c}]\right]}{\sqrt{1+\frac{a^2}{c^2}}c}\right) \sqrt{b+\sqrt{1+\frac{a^2}{c^2}}c\cos[d+ex-\operatorname{ArcTan}[\frac{a}{c}]]}} \right) \\
& \left. \left( e\sqrt{\csc[d+ex]}\sqrt{b+c\cos[d+ex]+a\operatorname{Sin}[d+ex]} \right) + \left( c^2\sqrt{a+c\cot[d+ex]+b\csc[d+ex]} \right)
\end{aligned}$$

$$\left( \left( \left( a \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right)} \right], -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right)} \right] \right) \right)$$

$$\left. \operatorname{Sin} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right/ \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right)$$

$$\left. \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) -$$

$$\left( \frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \operatorname{Sin} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) \left/ \left( a e \sqrt{\operatorname{Csc} \left[ d + e x \right]} \sqrt{b + c \operatorname{Cos} \left[ d + e x \right]} + a \operatorname{Sin} \left[ d + e x \right] \right) + \right.$$

$$\left. \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right)$$

$$\left( 2 b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right], -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right)$$

$$\sqrt{a + c \cot [d + e x] + b \csc [d + e x]} \operatorname{Sec} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]$$

$$\sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} - a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2+c^2}{a^2}}}}$$

$$\sqrt{b + a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}$$

$$\sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}}}$$

$$\left( a \sqrt{1 + \frac{c^2}{a^2}} e^{\sqrt{\csc [d + e x]} \sqrt{b + c \cos [d + e x] + a \sin [d + e x]}} \right)$$

**Problem 464:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\csc [d + e x]}}{\sqrt{a + c \cot [d + e x] + b \csc [d + e x]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{2 \sqrt{\csc [d + e x]} \operatorname{EllipticF} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan} [c, a]), \frac{2 \sqrt{a^2+c^2}}{b + \sqrt{a^2+c^2}} \right] \sqrt{\frac{b+c \cos [d+e x]+a \sin [d+e x]}{b + \sqrt{a^2+c^2}}}}{e \sqrt{a + c \cot [d + e x] + b \csc [d + e x]}}$$

Result (type 6, 339 leaves):

$$\left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{b+a\sqrt{1+\frac{c^2}{a^2}} \operatorname{Sin}[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]}{b-a\sqrt{1+\frac{c^2}{a^2}}}, \frac{b+a\sqrt{1+\frac{c^2}{a^2}} \operatorname{Sin}[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]}{b+a\sqrt{1+\frac{c^2}{a^2}}} \right], \right. \\
\left. \sqrt{\operatorname{Csc}[d+ex]} \operatorname{Sec}[d+ex+\operatorname{ArcTan}[\frac{c}{a}]] \sqrt{b+c \operatorname{Cos}[d+ex]+a \operatorname{Sin}[d+ex]} \sqrt{\frac{a\sqrt{1+\frac{c^2}{a^2}}(-1+\operatorname{Sin}[d+ex+\operatorname{ArcTan}[\frac{c}{a}]])}{b+a\sqrt{1+\frac{c^2}{a^2}}}} \right. \\
\left. \sqrt{\frac{a\sqrt{1+\frac{c^2}{a^2}}(1+\operatorname{Sin}[d+ex+\operatorname{ArcTan}[\frac{c}{a}]])}{-b+a\sqrt{1+\frac{c^2}{a^2}}}} \sqrt{b+a\sqrt{1+\frac{c^2}{a^2}} \operatorname{Sin}[d+ex+\operatorname{ArcTan}[\frac{c}{a}]]} \right) / \left( a\sqrt{1+\frac{c^2}{a^2}} e^{\sqrt{a+c \operatorname{Cot}[d+ex]+b \operatorname{Csc}[d+ex]}} \right)$$

**Problem 465:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[d+ex]^{3/2}}{(a+c \operatorname{Cot}[d+ex]+b \operatorname{Csc}[d+ex])^{3/2}} dx$$

Optimal (type 4, 240 leaves, 4 steps):

$$\frac{2 \operatorname{Csc}[d+ex]^{3/2} \operatorname{EllipticE} \left[ \frac{1}{2} (d+ex - \operatorname{ArcTan}[c, a]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}} \right] (b+c \operatorname{Cos}[d+ex]+a \operatorname{Sin}[d+ex])^2}{(a^2-b^2+c^2) e (a+c \operatorname{Cot}[d+ex]+b \operatorname{Csc}[d+ex])^{3/2} \sqrt{\frac{b+c \operatorname{Cos}[d+ex]+a \operatorname{Sin}[d+ex]}{b+\sqrt{a^2+c^2}}}} \\
\frac{2 \operatorname{Csc}[d+ex]^{3/2} (b+c \operatorname{Cos}[d+ex]+a \operatorname{Sin}[d+ex]) (a \operatorname{Cos}[d+ex]-c \operatorname{Sin}[d+ex])}{(a^2-b^2+c^2) e (a+c \operatorname{Cot}[d+ex]+b \operatorname{Csc}[d+ex])^{3/2}}$$

Result (type 6, 1732 leaves):

$$\frac{\operatorname{Csc}[d+ex]^{3/2} (b+c \operatorname{Cos}[d+ex]+a \operatorname{Sin}[d+ex])^2 \left( -\frac{2(a^2+c^2)}{a(c(a^2-b^2+c^2))} + \frac{2(a b+a^2 \operatorname{Sin}[d+ex]+c^2 \operatorname{Sin}[d+ex])}{c(a^2-b^2+c^2)(b+c \operatorname{Cos}[d+ex]+a \operatorname{Sin}[d+ex])} \right)}{e (a+c \operatorname{Cot}[d+ex]+b \operatorname{Csc}[d+ex])^{3/2}}$$

$$a \operatorname{Csc}[d + e x]^{3/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^{3/2}$$

$$- \left( \left( \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}\right] \right) \right)$$

$$\operatorname{Sin}\left[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \left/ \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}} \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]} \right) \right)$$

$$\sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \left( \frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]] \right)}{a^2 + c^2} - \frac{a \operatorname{Sin}[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) \left/ \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos}[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]]} \right)$$

$$\left( (a^2 - b^2 + c^2) e (a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} \right) - \left( c^2 \operatorname{Csc} [d + e x]^{3/2} (b + c \cos [d + e x] + a \sin [d + e x])^{3/2} \right)$$

$$\left( \left( \left( a \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos [d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos [d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \right) \right)$$

$$\left. \operatorname{Sin} \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right) / \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \cos [d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right]]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right)$$

$$\left( \frac{\sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos [d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right]]}}{\sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos [d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right]]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}}} \right) -$$

$$\left( \frac{2 c \left( \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos [d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right]]}{a^2 + c^2} - \frac{a \operatorname{Sin} [d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right]]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right)}{\sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos [d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right]]}} \right) / \left( (a (a^2 - b^2 + c^2) e (a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} \right) -$$

$$\left( 2 b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \right.$$

$$\left. \operatorname{Csc} [d + e x]^{3/2} \operatorname{Sec} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] (b + c \operatorname{Cos} [d + e x] + a \operatorname{Sin} [d + e x])^{3/2} \right.$$

$$\left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}} \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \right.$$

$$\left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}} \right)$$

$$\left( a (a^2 - b^2 + c^2) \sqrt{1 + \frac{c^2}{a^2}} e (a + c \operatorname{Cot} [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} \right)$$

**Problem 466:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc} [d + e x]^{5/2}}{(a + c \operatorname{Cot} [d + e x] + b \operatorname{Csc} [d + e x])^{5/2}} dx$$

Optimal (type 4, 492 leaves, 8 steps):

$$\begin{aligned}
& \frac{8 b \operatorname{Csc}[d+e x]^{5/2} \operatorname{EllipticE}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right](b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x])^3}{3\left(a^2-b^2+c^2\right)^2 e\left(a+c \operatorname{Cot}[d+e x]+b \operatorname{Csc}[d+e x]\right)^{5/2} \sqrt{\frac{b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x]}{b+\sqrt{a^2+c^2}}}}+ \\
& \left(2 \operatorname{Csc}[d+e x]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right](b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x])^2 \sqrt{\frac{b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x]}{b+\sqrt{a^2+c^2}}}\right) / \\
& \left(3\left(a^2-b^2+c^2\right) e\left(a+c \operatorname{Cot}[d+e x]+b \operatorname{Csc}[d+e x]\right)^{5/2}\right)-\frac{2 \operatorname{Csc}[d+e x]^{5/2}(b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x])(a \operatorname{Cos}[d+e x]-c \operatorname{Sin}[d+e x])}{3\left(a^2-b^2+c^2\right) e\left(a+c \operatorname{Cot}[d+e x]+b \operatorname{Csc}[d+e x]\right)^{5/2}}+ \\
& \frac{8 \operatorname{Csc}[d+e x]^{5/2}(b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x])^2(a b \operatorname{Cos}[d+e x]-b c \operatorname{Sin}[d+e x])}{3\left(a^2-b^2+c^2\right)^2 e\left(a+c \operatorname{Cot}[d+e x]+b \operatorname{Csc}[d+e x]\right)^{5/2}}
\end{aligned}$$

Result (type 6, 2708 leaves):

$$\begin{aligned}
& \left(\operatorname{Csc}[d+e x]^{5/2}(b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x])^3\right. \\
& \left.\left(\frac{8 b\left(a^2+c^2\right)}{3 a c\left(-a^2+b^2-c^2\right)^2}+\frac{2\left(a b+a^2 \operatorname{Sin}[d+e x]+c^2 \operatorname{Sin}[d+e x]\right)}{3 c\left(a^2-b^2+c^2\right)(b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x])^2}-\frac{2\left(a^3+3 a b^2+a c^2+4 a^2 b \operatorname{Sin}[d+e x]+4 b c^2 \operatorname{Sin}[d+e x]\right)}{3 c\left(a^2-b^2+c^2\right)^2(b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x])}\right)\right) / \\
& \left(e\left(a+c \operatorname{Cot}[d+e x]+b \operatorname{Csc}[d+e x]\right)^{5/2}\right)+\left(4 a b \operatorname{Csc}[d+e x]^{5/2}(b+c \operatorname{Cos}[d+e x]+a \operatorname{Sin}[d+e x])^{5/2}\right. \\
& \left.\left(-\left(\left(\operatorname{AppellF1}\left[-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}, \frac{1}{2},-\frac{b+\sqrt{1+\frac{a^2}{c^2}} c \operatorname{Cos}\left[d+e x-\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1+\frac{a^2}{c^2}}\left(1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}} c}\right) c},-\frac{b+\sqrt{1+\frac{a^2}{c^2}} c \operatorname{Cos}\left[d+e x-\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1+\frac{a^2}{c^2}}\left(-1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}} c}\right) c}\right]\right)\right)
\end{aligned}$$



$$\left. \left. \left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right) \right) / \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}} \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right) \right)$$

$$\left. \left. \left. \left. \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \frac{2c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right) - a \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{a^2 + c^2} - \frac{a \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) \right) \right) / \left( \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right)$$

$$\left( 3 (a^2 - b^2 + c^2)^2 e (a + c \cot [d + e x] + b \csc [d + e x])^{5/2} \right) + \left( 4 b c^2 \csc [d + e x]^{5/2} (b + c \cos [d + e x] + a \sin [d + e x])^{5/2} \right)$$

$$\left( \left( \left( a \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \right) \right) \right)$$

$$\left. \left. \left. \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right) \right) / \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right)$$

$$\left( \frac{\sqrt{b+c} \sqrt{\frac{a^2+c^2}{c^2}} \cos\left[d+ex - \text{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{\frac{c\sqrt{\frac{a^2+c^2}{c^2}} + c\sqrt{\frac{a^2+c^2}{c^2}} \cos\left[d+ex - \text{ArcTan}\left[\frac{a}{c}\right]\right]}{-b+c\sqrt{\frac{a^2+c^2}{c^2}}}} \right) -$$

$$\left( \frac{2c \left( \frac{b+\sqrt{1+\frac{a^2}{c^2}} c \cos\left[d+ex - \text{ArcTan}\left[\frac{a}{c}\right]\right]}{a^2+c^2} \right) - \frac{a \sin\left[d+ex - \text{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1+\frac{a^2}{c^2}} c}}{\sqrt{b+\sqrt{1+\frac{a^2}{c^2}} c \cos\left[d+ex - \text{ArcTan}\left[\frac{a}{c}\right]\right}}} \right) \left/ \left( 3a(a^2-b^2+c^2)^2 e^{(a+c \cot[d+ex] + b \csc[d+ex])^{5/2}} \right) + \right.$$

$$\left[ 2a \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \sin\left[d+ex + \text{ArcTan}\left[\frac{c}{a}\right]\right]}{a\sqrt{1+\frac{c^2}{a^2}} \left(1 - \frac{b}{a\sqrt{1+\frac{c^2}{a^2}}}\right)}, -\frac{b+a\sqrt{1+\frac{c^2}{a^2}} \sin\left[d+ex + \text{ArcTan}\left[\frac{c}{a}\right]\right]}{a\sqrt{1+\frac{c^2}{a^2}} \left(-1 - \frac{b}{a\sqrt{1+\frac{c^2}{a^2}}}\right)}\right] \right]$$

$$\csc[d+ex]^{5/2} \sec\left[d+ex + \text{ArcTan}\left[\frac{c}{a}\right]\right] (b+c \cos[d+ex] + a \sin[d+ex])^{5/2}$$

$$\sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}} - a\sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d+ex + \text{ArcTan}\left[\frac{c}{a}\right]\right]}{b+a\sqrt{\frac{a^2+c^2}{a^2}}}} \sqrt{b+a\sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d+ex + \text{ArcTan}\left[\frac{c}{a}\right]\right]}$$

$$\sqrt{\frac{a\sqrt{\frac{a^2+c^2}{a^2}} + a\sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d+ex + \text{ArcTan}\left[\frac{c}{a}\right]\right]}{-b+a\sqrt{\frac{a^2+c^2}{a^2}}}}$$

$$\begin{aligned}
& \left( 3 (a^2 - b^2 + c^2)^2 \sqrt{1 + \frac{c^2}{a^2}} e (a + c \cot [d + e x] + b \csc [d + e x])^{5/2} \right) + \\
& \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \csc [d + e x]^{5/2} \right. \\
& \left. \sec [d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right]] (b + c \cos [d + e x] + a \sin [d + e x])^{5/2} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right]]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right. \\
& \left. \sqrt{\frac{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right]]}{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right]]}}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) / \\
& \left( a (a^2 - b^2 + c^2)^2 \sqrt{1 + \frac{c^2}{a^2}} e (a + c \cot [d + e x] + b \csc [d + e x])^{5/2} \right) + \\
& \left( 2 c^2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin [d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right]]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \csc [d + e x]^{5/2} \right)
\end{aligned}$$

$$\begin{aligned} & \left( \frac{\operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \left(b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]\right)^{5/2}}{\sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} - a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{b + a \sqrt{\frac{a^2+c^2}{a^2}}}} \right. \\ & \left. \sqrt{\frac{b + a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}}} \right) \\ & \left( 3 a \left(a^2 - b^2 + c^2\right)^2 \sqrt{1 + \frac{c^2}{a^2}} e \left(a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x]\right)^{5/2} \right) \end{aligned}$$

**Problem 467: Attempted integration timed out after 120 seconds.**

$$\int (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{3/2} \operatorname{Sin}[d + e x]^{3/2} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\begin{aligned} & \left( 8 b \left(a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x]\right)^{3/2} \operatorname{EllipticE}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}[c, a]\right), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] \operatorname{Sin}[d + e x]^{3/2} \right) / \\ & \left( 3 e \left(b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]\right) \sqrt{\frac{b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) + \\ & \left( 2 \left(a^2 - b^2 + c^2\right) \left(a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x]\right)^{3/2} \operatorname{EllipticF}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}[c, a]\right), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] \right. \\ & \left. \operatorname{Sin}[d + e x]^{3/2} \sqrt{\frac{b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) / \left( 3 e \left(b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]\right)^2 \right) - \\ & \frac{2 \left(a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x]\right)^{3/2} \operatorname{Sin}[d + e x]^{3/2} \left(a \operatorname{Cos}[d + e x] - c \operatorname{Sin}[d + e x]\right)}{3 e \left(b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]\right)} \end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 468: Attempted integration timed out after 120 seconds.**

$$\int \sqrt{a + c \cot [d + e x] + b \operatorname{Csc} [d + e x]} \sqrt{\sin [d + e x]} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{2 \sqrt{a + c \cot [d + e x] + b \operatorname{Csc} [d + e x]} \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan} [c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \sqrt{\sin [d + e x]}}{e \sqrt{\frac{b + c \cos [d + e x] + a \sin [d + e x]}{b + \sqrt{a^2 + c^2}}}}$$

Result (type 1, 1 leaves):

???

**Problem 469: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{a + c \cot [d + e x] + b \operatorname{Csc} [d + e x]} \sqrt{\sin [d + e x]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{2 \operatorname{EllipticF} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan} [c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \sqrt{\frac{b + c \cos [d + e x] + a \sin [d + e x]}{b + \sqrt{a^2 + c^2}}}}{e \sqrt{a + c \cot [d + e x] + b \operatorname{Csc} [d + e x]} \sqrt{\sin [d + e x]}}$$

Result (type 4, 719 leaves):

$$\begin{aligned}
& \left( 4 \left( i a + b - c - i \sqrt{a^2 - b^2 + c^2} \right) (1 + \cos [d + e x]) \sqrt{\csc [d + e x]} \right. \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-a - i b + i c + \sqrt{a^2 - b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(-a + i b - i c + \sqrt{a^2 - b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], \frac{i b + \sqrt{a^2 - b^2 + c^2}}{i b - \sqrt{a^2 - b^2 + c^2}} \right] \\
& \sqrt{\frac{b + c \cos [d + e x] + a \sin [d + e x]}{(1 + \cos [d + e x])^2}} \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \sqrt{\frac{(-a - i b + i c + \sqrt{a^2 - b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(-a + i b - i c + \sqrt{a^2 - b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \sqrt{\cot \left[ \frac{1}{2} (d + e x) \right] + \tan \left[ \frac{1}{2} (d + e x) \right]} \sqrt{\frac{i \left( a - \sqrt{a^2 - b^2 + c^2} + b \tan \left[ \frac{1}{2} (d + e x) \right] - c \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(-a + i b - i c + \sqrt{a^2 - b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \sqrt{-\frac{i \left( a + \sqrt{a^2 - b^2 + c^2} + b \tan \left[ \frac{1}{2} (d + e x) \right] - c \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{(a - i b + i c + \sqrt{a^2 - b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \sqrt{\frac{\tan \left[ \frac{1}{2} (d + e x) \right]}{1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2}} \right) / \\
& \left( \left( a + i b - i c - \sqrt{a^2 - b^2 + c^2} \right) e^{\sqrt{a + c \cot [d + e x] + b \csc [d + e x]}} \right. \\
& \left. \sqrt{\left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( b + c + 2 a \tan \left[ \frac{1}{2} (d + e x) \right] + b \tan \left[ \frac{1}{2} (d + e x) \right]^2 - c \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \right)
\end{aligned}$$

**Problem 470: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{(a + c \cot [d + e x] + b \csc [d + e x])^{3/2} \sin [d + e x]^{3/2}} dx$$

Optimal (type 4, 240 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{EllipticE}\left[\frac{1}{2}(d+ex - \operatorname{ArcTan}[c, a]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] (b+c \cos[d+ex] + a \sin[d+ex])^2}{(a^2 - b^2 + c^2) e (a+c \cot[d+ex] + b \csc[d+ex])^{3/2} \sin[d+ex]^{3/2} \sqrt{\frac{b+c \cos[d+ex]+a \sin[d+ex]}{b+\sqrt{a^2+c^2}}}} \\
& \frac{2 (b+c \cos[d+ex] + a \sin[d+ex]) (a \cos[d+ex] - c \sin[d+ex])}{(a^2 - b^2 + c^2) e (a+c \cot[d+ex] + b \csc[d+ex])^{3/2} \sin[d+ex]^{3/2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 471: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{(a+c \cot[d+ex] + b \csc[d+ex])^{5/2} \sin[d+ex]^{5/2}} dx$$

Optimal (type 4, 492 leaves, 8 steps):

$$\begin{aligned}
& \frac{8 b \operatorname{EllipticE}\left[\frac{1}{2}(d+ex - \operatorname{ArcTan}[c, a]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] (b+c \cos[d+ex] + a \sin[d+ex])^3}{3 (a^2 - b^2 + c^2)^2 e (a+c \cot[d+ex] + b \csc[d+ex])^{5/2} \sin[d+ex]^{5/2} \sqrt{\frac{b+c \cos[d+ex]+a \sin[d+ex]}{b+\sqrt{a^2+c^2}}}} + \\
& \left( \frac{2 \operatorname{EllipticF}\left[\frac{1}{2}(d+ex - \operatorname{ArcTan}[c, a]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] (b+c \cos[d+ex] + a \sin[d+ex])^2 \sqrt{\frac{b+c \cos[d+ex]+a \sin[d+ex]}{b+\sqrt{a^2+c^2}}}}{3 (a^2 - b^2 + c^2) e (a+c \cot[d+ex] + b \csc[d+ex])^{5/2} \sin[d+ex]^{5/2}} - \right. \\
& \left. \frac{2 (b+c \cos[d+ex] + a \sin[d+ex]) (a \cos[d+ex] - c \sin[d+ex])}{3 (a^2 - b^2 + c^2) e (a+c \cot[d+ex] + b \csc[d+ex])^{5/2} \sin[d+ex]^{5/2}} + \frac{8 (b+c \cos[d+ex] + a \sin[d+ex])^2 (a b \cos[d+ex] - b c \sin[d+ex])}{3 (a^2 - b^2 + c^2)^2 e (a+c \cot[d+ex] + b \csc[d+ex])^{5/2} \sin[d+ex]^{5/2}} \right) /
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 475: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos[x]^2 - \sin[x]^2} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{1}{2} \text{ArcTanh}[2 \text{Cos}[x] \text{Sin}[x]]$$

Result (type 3, 23 leaves):

$$-\frac{1}{2} \text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \frac{1}{2} \text{Log}[\text{Cos}[x] + \text{Sin}[x]]$$

**Problem 503: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{d + e \text{Sin}[x]}{a + b \text{Sin}[x] + c \text{Sin}[x]^2} dx$$

Optimal (type 3, 242 leaves, 7 steps):

$$\frac{\sqrt{2} \left( e + \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[ \frac{2c + (b - \sqrt{b^2-4ac}) \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - b\sqrt{b^2-4ac}}} \right]}{\sqrt{b^2 - 2c(a+c) - b\sqrt{b^2-4ac}}} + \frac{\sqrt{2} \left( e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \text{ArcTan} \left[ \frac{2c + (b + \sqrt{b^2-4ac}) \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + b\sqrt{b^2-4ac}}} \right]}{\sqrt{b^2 - 2c(a+c) + b\sqrt{b^2-4ac}}}$$

Result (type 3, 286 leaves):

$$\frac{1}{\sqrt{-\frac{b^2}{2} + 2ac}} \left( \frac{(-2i cd + (ib + \sqrt{-b^2+4ac}) e) \text{ArcTan} \left[ \frac{2c + (b - i\sqrt{-b^2+4ac}) \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2-2c(a+c) - ib\sqrt{-b^2+4ac}}} \right]}{\sqrt{b^2 - 2c(a+c) - ib\sqrt{-b^2+4ac}}} + \frac{(2i cd + (-ib + \sqrt{-b^2+4ac}) e) \text{ArcTan} \left[ \frac{2c + (b + i\sqrt{-b^2+4ac}) \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2-2c(a+c) + ib\sqrt{-b^2+4ac}}} \right]}{\sqrt{b^2 - 2c(a+c) + ib\sqrt{-b^2+4ac}}} \right)$$

**Problem 510: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \text{Tan}[d + ex]) (b^2 + 2ab \text{Tan}[d + ex] + a^2 \text{Tan}[d + ex]^2) dx$$

Optimal (type 3, 144 leaves, 7 steps):



$$a (a^2 - 3b^2) (a^2 + b^2) x + \frac{b (3a^2 - b^2) (a^2 + b^2) \operatorname{Log}[\operatorname{Cos}[d + ex]]}{e} - \frac{a (a^4 - b^4) \operatorname{Tan}[d + ex]}{e} +$$

$$\frac{b (a^2 + b^2) (b + a \operatorname{Tan}[d + ex])^2}{2e} + \frac{(a^2 + b^2) (b + a \operatorname{Tan}[d + ex])^3}{3e} + \frac{b (b + a \operatorname{Tan}[d + ex])^4}{4e}$$

Result (type 3, 578 leaves):

$$\frac{a^4 b \operatorname{Cos}[d + ex] (b + a \operatorname{Tan}[d + ex])^4 (a + b \operatorname{Tan}[d + ex])}{4e (b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex])^4 (a \operatorname{Cos}[d + ex] + b \operatorname{Sin}[d + ex])} + \frac{a^2 b (a^2 + 3b^2) \operatorname{Cos}[d + ex]^3 (b + a \operatorname{Tan}[d + ex])^4 (a + b \operatorname{Tan}[d + ex])}{e (b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex])^4 (a \operatorname{Cos}[d + ex] + b \operatorname{Sin}[d + ex])} -$$

$$\frac{a (-i a + b) (i a + b) (-a^2 + 3b^2) (d + ex) \operatorname{Cos}[d + ex]^5 (b + a \operatorname{Tan}[d + ex])^4 (a + b \operatorname{Tan}[d + ex])}{e (b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex])^4 (a \operatorname{Cos}[d + ex] + b \operatorname{Sin}[d + ex])} +$$

$$\frac{(3a^4 b + 2a^2 b^3 - b^5) \operatorname{Cos}[d + ex]^5 \operatorname{Log}[\operatorname{Cos}[d + ex]] (b + a \operatorname{Tan}[d + ex])^4 (a + b \operatorname{Tan}[d + ex])}{e (b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex])^4 (a \operatorname{Cos}[d + ex] + b \operatorname{Sin}[d + ex])} +$$

$$\frac{\operatorname{Cos}[d + ex]^2 (a^5 \operatorname{Sin}[d + ex] + 4a^3 b^2 \operatorname{Sin}[d + ex]) (b + a \operatorname{Tan}[d + ex])^4 (a + b \operatorname{Tan}[d + ex])}{3e (b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex])^4 (a \operatorname{Cos}[d + ex] + b \operatorname{Sin}[d + ex])} +$$

$$\left( \frac{2 \operatorname{Cos}[d + ex]^4 (-2a^5 \operatorname{Sin}[d + ex] + a^3 b^2 \operatorname{Sin}[d + ex] + 6a b^4 \operatorname{Sin}[d + ex]) (b + a \operatorname{Tan}[d + ex])^4 (a + b \operatorname{Tan}[d + ex])}{(3e (b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex])^4 (a \operatorname{Cos}[d + ex] + b \operatorname{Sin}[d + ex]))} \right) /$$

**Problem 512: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + b \operatorname{Tan}[d + ex]}{b^2 + 2ab \operatorname{Tan}[d + ex] + a^2 \operatorname{Tan}[d + ex]^2} dx$$

Optimal (type 3, 101 leaves, 4 steps):

$$- \frac{a (a^2 - 3b^2) x}{(a^2 + b^2)^2} + \frac{b (3a^2 - b^2) \operatorname{Log}[b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex]]}{(a^2 + b^2)^2 e} - \frac{a^2 - b^2}{(a^2 + b^2) e (b + a \operatorname{Tan}[d + ex])}$$

Result (type 3, 219 leaves):

$$\frac{1}{2b (a^2 + b^2)^2 e (b + a \operatorname{Tan}[d + ex])} \left( b^2 (-2(a - ib)^3 (d + ex) - b(-3a^2 + b^2) \operatorname{Log}[(b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex])^2]) \right) +$$

$$a \left( 2(a - ib) (a^3 - a^2 b (-i + d + ex) + b^3 (-i + d + ex) + i a b^2 (i + 2d + 2ex)) - b^2 (-3a^2 + b^2) \operatorname{Log}[(b \operatorname{Cos}[d + ex] + a \operatorname{Sin}[d + ex])^2] \right)$$

$$\operatorname{Tan}[d + ex] + 2i b^2 (-3a^2 + b^2) \operatorname{ArcTan}[\operatorname{Tan}[d + ex]] (b + a \operatorname{Tan}[d + ex])$$

### Problem 513: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{Tan}[d + e x]}{(b^2 + 2 a b \operatorname{Tan}[d + e x] + a^2 \operatorname{Tan}[d + e x]^2)^2} dx$$

Optimal (type 3, 197 leaves, 6 steps):

$$\frac{a (a^4 - 10 a^2 b^2 + 5 b^4) x}{(a^2 + b^2)^4} - \frac{b (5 a^4 - 10 a^2 b^2 + b^4) \operatorname{Log}[b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]]}{(a^2 + b^2)^4 e} - \frac{a^2 - b^2}{3 (a^2 + b^2) e (b + a \operatorname{Tan}[d + e x])^3} - \frac{b (3 a^2 - b^2)}{2 (a^2 + b^2)^2 e (b + a \operatorname{Tan}[d + e x])^2} + \frac{a^4 - 6 a^2 b^2 + b^4}{(a^2 + b^2)^3 e (b + a \operatorname{Tan}[d + e x])}$$

Result (type 3, 1098 leaves):

$$\begin{aligned} & \left( (-5 i a^{11} b + 5 a^{10} b^2 - 5 i a^9 b^3 + 5 a^8 b^4 + 14 i a^7 b^5 - 14 a^6 b^6 + 22 i a^5 b^7 - 22 a^4 b^8 + 7 i a^3 b^9 - 7 a^2 b^{10} - i a b^{11} + b^{12}) \right. \\ & \quad \left. (d + e x) \operatorname{Sec}[d + e x]^3 (b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^4 (a + b \operatorname{Tan}[d + e x]) \right) / \\ & \left( (a - i b)^3 (a + i b)^4 (-i a + b)^4 (i a + b)^4 e (a \operatorname{Cos}[d + e x] + b \operatorname{Sin}[d + e x]) (b + a \operatorname{Tan}[d + e x])^4 \right) - \\ & \left( i (-5 a^4 b + 10 a^2 b^3 - b^5) \operatorname{ArcTan}[\operatorname{Tan}[d + e x]] \operatorname{Sec}[d + e x]^3 (b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^4 (a + b \operatorname{Tan}[d + e x]) \right) / \\ & \left( (a^2 + b^2)^4 e (a \operatorname{Cos}[d + e x] + b \operatorname{Sin}[d + e x]) (b + a \operatorname{Tan}[d + e x])^4 \right) + \\ & \left( (-5 a^4 b + 10 a^2 b^3 - b^5) \operatorname{Log}[(b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^2] \operatorname{Sec}[d + e x]^3 (b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^4 (a + b \operatorname{Tan}[d + e x]) \right) / \\ & \left( 2 (a^2 + b^2)^4 e (a \operatorname{Cos}[d + e x] + b \operatorname{Sin}[d + e x]) (b + a \operatorname{Tan}[d + e x])^4 \right) + \\ & \left( \operatorname{Sec}[d + e x]^3 (b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]) (-12 a^8 b \operatorname{Cos}[d + e x] + 24 a^6 b^3 \operatorname{Cos}[d + e x] + 36 a^4 b^5 \operatorname{Cos}[d + e x] + 9 a^7 b^2 (d + e x) \operatorname{Cos}[d + e x] - \right. \\ & \quad 81 a^5 b^4 (d + e x) \operatorname{Cos}[d + e x] - 45 a^3 b^6 (d + e x) \operatorname{Cos}[d + e x] + 45 a b^8 (d + e x) \operatorname{Cos}[d + e x] + 8 a^8 b \operatorname{Cos}[3 (d + e x)]) - \\ & \quad 54 a^6 b^3 \operatorname{Cos}[3 (d + e x)] - 44 a^4 b^5 \operatorname{Cos}[3 (d + e x)] + 18 a^2 b^7 \operatorname{Cos}[3 (d + e x)] - 9 a^7 b^2 (d + e x) \operatorname{Cos}[3 (d + e x)] + \\ & \quad 93 a^5 b^4 (d + e x) \operatorname{Cos}[3 (d + e x)] - 75 a^3 b^6 (d + e x) \operatorname{Cos}[3 (d + e x)] + 15 a b^8 (d + e x) \operatorname{Cos}[3 (d + e x)] - 12 a^9 \operatorname{Sin}[d + e x] + \\ & \quad 51 a^7 b^2 \operatorname{Sin}[d + e x] + 81 a^5 b^4 \operatorname{Sin}[d + e x] + 9 a^3 b^6 \operatorname{Sin}[d + e x] - 9 a b^8 \operatorname{Sin}[d + e x] + 9 a^8 b (d + e x) \operatorname{Sin}[d + e x] - \\ & \quad 81 a^6 b^3 (d + e x) \operatorname{Sin}[d + e x] - 45 a^4 b^5 (d + e x) \operatorname{Sin}[d + e x] + 45 a^2 b^7 (d + e x) \operatorname{Sin}[d + e x] + 4 a^9 \operatorname{Sin}[3 (d + e x)] - \\ & \quad 31 a^7 b^2 \operatorname{Sin}[3 (d + e x)] + 5 a^5 b^4 \operatorname{Sin}[3 (d + e x)] + 31 a^3 b^6 \operatorname{Sin}[3 (d + e x)] - 9 a b^8 \operatorname{Sin}[3 (d + e x)] - 3 a^8 b (d + e x) \operatorname{Sin}[3 (d + e x)] + \\ & \quad \left. 39 a^6 b^3 (d + e x) \operatorname{Sin}[3 (d + e x)] - 105 a^4 b^5 (d + e x) \operatorname{Sin}[3 (d + e x)] + 45 a^2 b^7 (d + e x) \operatorname{Sin}[3 (d + e x)] \right) (a + b \operatorname{Tan}[d + e x]) / \\ & \left( 12 b (-i a + b)^4 (i a + b)^4 e (a \operatorname{Cos}[d + e x] + b \operatorname{Sin}[d + e x]) (b + a \operatorname{Tan}[d + e x])^4 \right) \end{aligned}$$

### Problem 517: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{Tan}[d + e x]}{(b^2 + 2 a b \operatorname{Tan}[d + e x] + a^2 \operatorname{Tan}[d + e x]^2)^{3/2}} dx$$

Optimal (type 3, 316 leaves, 5 steps):

$$-\frac{(a^2 - b^2)(b + a \tan[d + ex])}{2(a^2 + b^2)e(b^2 + 2ab \tan[d + ex] + a^2 \tan^2[d + ex])^{3/2}} - \frac{(a^4 - 6a^2b^2 + b^4) \log[b \cos[d + ex] + a \sin[d + ex]](b + a \tan[d + ex])^3}{(a^2 + b^2)^3 e(b^2 + 2ab \tan[d + ex] + a^2 \tan^2[d + ex])^{3/2}} -$$

$$\frac{4b(a^2 - b^2)x(a b + a^2 \tan[d + ex])^3}{a^2(a^2 + b^2)^3(b^2 + 2ab \tan[d + ex] + a^2 \tan^2[d + ex])^{3/2}} - \frac{b(3a^2 - b^2)(a b + a^2 \tan[d + ex])^3}{(a^2 + b^2)^2 e(a^3 b + a^4 \tan[d + ex])(b^2 + 2ab \tan[d + ex] + a^2 \tan^2[d + ex])^{3/2}}$$

Result (type 3, 293 leaves):

$$\frac{1}{2(a^2 + b^2)^3 e(b + a \tan[d + ex]) \sqrt{(b + a \tan[d + ex])^2}}$$

$$\left( (-a^6 + a^2 b^4) \sec^2[d + ex] + 2i(a^4 - 6a^2 b^2 + b^4) \operatorname{ArcTan}[\tan[d + ex]](b + a \tan[d + ex])^2 + \right.$$

$$\left. (b + a \tan[d + ex]) \left( b(-2i(a - ib)^4(d + ex) - (a^4 - 6a^2 b^2 + b^4) \log[(b \cos[d + ex] + a \sin[d + ex])^2]) \right) + \right.$$

$$\left. a(2(a - ib)(a^2 b(4i - 3d - 3ex) + b^3(-2i + d + ex) - ia^3(4i + d + ex) + ia b^2(2i + 3d + 3ex)) - \right.$$

$$\left. (a^4 - 6a^2 b^2 + b^4) \log[(b \cos[d + ex] + a \sin[d + ex])^2] \right) \tan[d + ex]$$

**Problem 518: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[d + ex]) (b^2 + 2ab \sec[d + ex] + a^2 \sec^2[d + ex])^2 dx$$

Optimal (type 3, 184 leaves, 8 steps):

$$a b^4 x + \frac{b(19a^4 + 56a^2 b^2 + 8b^4) \operatorname{ArcTanh}[\sin[d + ex]]}{8e} + \frac{a(4a^4 + 50a^2 b^2 + 19b^4) \tan[d + ex]}{6e} +$$

$$\frac{a^2 b(41a^2 + 26b^2) \sec[d + ex] \tan[d + ex]}{24e} + \frac{(4a^2 + 7b^2)(a b + a^2 \sec[d + ex])^2 \tan[d + ex]}{12ae} + \frac{b(a b + a^2 \sec[d + ex])^3 \tan[d + ex]}{4a^2 e}$$

Result (type 3, 590 leaves):

$$\begin{aligned}
& \frac{a b^4 (d + e x)}{e} + \frac{(-19 a^4 b - 56 a^2 b^3 - 8 b^5) \operatorname{Log}\left[\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right]}{8 e} + \\
& \frac{(19 a^4 b + 56 a^2 b^3 + 8 b^5) \operatorname{Log}\left[\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right]}{8 e} + \frac{a^4 b}{16 e \left(\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right)^4} + \\
& \frac{4 a^5 + 57 a^4 b + 16 a^3 b^2 + 72 a^2 b^3}{48 e \left(\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right)^2} - \frac{a^4 b}{16 e \left(\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right)^4} + \\
& \frac{-4 a^5 - 57 a^4 b - 16 a^3 b^2 - 72 a^2 b^3}{48 e \left(\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right)^2} + \frac{a^5 \sin\left[\frac{1}{2}(d + e x)\right] + 4 a^3 b^2 \sin\left[\frac{1}{2}(d + e x)\right]}{6 e \left(\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right)^3} + \\
& \frac{a^5 \sin\left[\frac{1}{2}(d + e x)\right] + 4 a^3 b^2 \sin\left[\frac{1}{2}(d + e x)\right]}{6 e \left(\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right)^3} + \frac{2 \left(a^5 \sin\left[\frac{1}{2}(d + e x)\right] + 13 a^3 b^2 \sin\left[\frac{1}{2}(d + e x)\right] + 6 a b^4 \sin\left[\frac{1}{2}(d + e x)\right]\right)}{3 e \left(\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right)} + \\
& \frac{2 \left(a^5 \sin\left[\frac{1}{2}(d + e x)\right] + 13 a^3 b^2 \sin\left[\frac{1}{2}(d + e x)\right] + 6 a b^4 \sin\left[\frac{1}{2}(d + e x)\right]\right)}{3 e \left(\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right)}
\end{aligned}$$

**Problem 548: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[x] + C \sin[x]}{a + b \cos[x] + i b \sin[x]} dx$$

Optimal (type 3, 92 leaves, 1 step):

$$-\frac{b(B + i C)x}{2 a^2} - \frac{(i b^2(B + i C) + a^2(i B + C)) \operatorname{Log}[a + b \cos[x] + i b \sin[x]]}{2 a^2 b} + \frac{(i B - C)(\cos[x] - i \sin[x])}{2 a}$$

Result (type 3, 195 leaves):

$$\begin{aligned}
& \frac{(a^2 B - b^2 B - i a^2 C - i b^2 C)x}{4 a^2 b} - \frac{(a^2 B + b^2 B - i a^2 C + i b^2 C) \operatorname{ArcTan}\left[\frac{(a-b) \cos\left[\frac{x}{2}\right]}{-a \sin\left[\frac{x}{2}\right] + b \sin\left[\frac{x}{2}\right]}\right]}{2 a^2 b} + \\
& \frac{i(B + i C) \cos[x]}{2 a} - \frac{i(a^2 B + b^2 B - i a^2 C + i b^2 C) \operatorname{Log}[a^2 + b^2 + 2 a b \cos[x]]}{4 a^2 b} + \frac{(B + i C) \sin[x]}{2 a}
\end{aligned}$$

**Problem 549: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \cos[x] + C \sin[x]}{a + b \cos[x] - i b \sin[x]} dx$$

Optimal (type 3, 90 leaves, 1 step):

$$-\frac{b(B - iC)x}{2a^2} + \frac{(ia^2(B + iC) + b^2(iB + C)) \operatorname{Log}[a + b \operatorname{Cos}[x] - ib \operatorname{Sin}[x]]}{2a^2b} - \frac{(iB + C)(\operatorname{Cos}[x] + i \operatorname{Sin}[x])}{2a}$$

Result (type 3, 195 leaves):

$$\frac{(a^2B - b^2B + ia^2C + ib^2C)x}{4a^2b} + \frac{(a^2B + b^2B + ia^2C - ib^2C) \operatorname{ArcTan}\left[\frac{(a+b)\operatorname{Cos}\left[\frac{x}{2}\right]}{a\operatorname{Sin}\left[\frac{x}{2}\right] - b\operatorname{Sin}\left[\frac{x}{2}\right]}\right]}{2a^2b} - \frac{i(B - iC)\operatorname{Cos}[x]}{2a} + \frac{i(a^2B + b^2B + ia^2C - ib^2C) \operatorname{Log}[a^2 + b^2 + 2ab \operatorname{Cos}[x]]}{4a^2b} + \frac{(B - iC)\operatorname{Sin}[x]}{2a}$$

**Problem 556: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Cos}[x] + c \operatorname{Sin}[x])^{5/2} (d + b e \operatorname{Cos}[x] + c e \operatorname{Sin}[x]) dx$$

Optimal (type 4, 390 leaves, 8 steps):

$$\frac{1}{105 \sqrt{\frac{a+b \operatorname{Cos}[x]+c \operatorname{Sin}[x]}{a+\sqrt{b^2+c^2}}}} \left( 2(161a^2d + 63(b^2+c^2)d + 15a^3e + 145a(b^2+c^2)e) \operatorname{EllipticE}\left[\frac{1}{2}(x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a+b \operatorname{Cos}[x]+c \operatorname{Sin}[x]} - \left( 2(a^2 - b^2 - c^2)(56ad + 15a^2e + 25(b^2+c^2)e) \operatorname{EllipticF}\left[\frac{1}{2}(x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b \operatorname{Cos}[x]+c \operatorname{Sin}[x]}{a+\sqrt{b^2+c^2}}}\right) / \right. \\ \left. \left( 105 \sqrt{a+b \operatorname{Cos}[x]+c \operatorname{Sin}[x]} \right) - \frac{2}{7} (a+b \operatorname{Cos}[x]+c \operatorname{Sin}[x])^{5/2} (c e \operatorname{Cos}[x] - b e \operatorname{Sin}[x]) - \frac{2}{35} (a+b \operatorname{Cos}[x]+c \operatorname{Sin}[x])^{3/2} (c(7d+5ae) \operatorname{Cos}[x] - b(7d+5ae) \operatorname{Sin}[x]) - \frac{2}{105} \sqrt{a+b \operatorname{Cos}[x]+c \operatorname{Sin}[x]} (c(56ad+15a^2e+25(b^2+c^2)e) \operatorname{Cos}[x] - b(56ad+15a^2e+25(b^2+c^2)e) \operatorname{Sin}[x]) \right)$$

Result (type 6, 7823 leaves):

$$\sqrt{a+b \operatorname{Cos}[x]+c \operatorname{Sin}[x]} \left( \frac{2b(161a^2d + 63b^2d + 63c^2d + 15a^3e + 145ab^2e + 145ac^2e)}{105c} - \right)$$

$$\begin{aligned}
& \frac{1}{210} c (308 a d + 180 a^2 e + 115 b^2 e + 115 c^2 e) \cos [x] - \frac{2}{35} b c (7 d + 15 a e) \cos [2 x] - \frac{1}{14} c (3 b^2 - c^2) e \cos [3 x] + \\
& \frac{1}{210} b (308 a d + 180 a^2 e + 115 b^2 e + 115 c^2 e) \sin [x] + \frac{1}{35} (b^2 - c^2) (7 d + 15 a e) \sin [2 x] + \frac{1}{14} b (b^2 - 3 c^2) e \sin [3 x] \Big) + \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c} \\
& 2 a^3 d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{15 \sqrt{1 + \frac{b^2}{c^2}} c} \\
& 34 a b^2 d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \\
& \frac{1}{15 \sqrt{1 + \frac{b^2}{c^2}} c} 34 a c d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [x + \operatorname{ArcTan} [\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{7 \sqrt{1 + \frac{b^2}{c^2}} c} \\
18 a^2 b^2 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \\
\frac{1}{21 \sqrt{1 + \frac{b^2}{c^2}} c} 10 b^4 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{7 \sqrt{1 + \frac{b^2}{c^2}}} \\
18 a^2 c e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{21 \sqrt{1 + \frac{b^2}{c^2}}} \\
20 b^2 c \operatorname{e AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \\
\frac{1}{21 \sqrt{1 + \frac{b^2}{c^2}}} 10 c^3 \operatorname{e AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{15 c}
\end{aligned}$$



$$23 a^2 b^2 d \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\ \left. \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}}}{b^2+c^2}}{\sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{5c}$$

$$3 b^4 d \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right.$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{\frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right)}{b^2+c^2} - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right}}} \right) +$$

$$\frac{23}{15} a^2 c d \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}\right] \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) \right) /$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{\frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right)}{b^2+c^2} - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right}}} \right) +$$

$$\frac{6}{5} b^2 c d \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}\right] \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) \right) /$$

$$\begin{aligned}
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) + \\
& \frac{3}{5} c^3 d \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{7c}
\end{aligned}$$

$$a^3 b^2 e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \right) /$$

$$\left( \frac{b\sqrt{1+\frac{c^2}{b^2}}}{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} - \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}}}{b^2+c^2}} \right) + \frac{1}{21c}$$

$$29 a b^4 e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \right) /$$

$$\left( \frac{b\sqrt{1+\frac{c^2}{b^2}}}{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\begin{aligned}
& \left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) + \\
& \frac{1}{7} a^3 c e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \\
& \left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) + \\
& \frac{58}{21} a b^2 c e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) + \\
& \frac{29}{21} a c^3 e \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right)
\end{aligned}$$

Problem 557: Result unnecessarily involves higher level functions and more than twice size of optimal

antiderivative.

$$\int (a + b \cos [x] + c \sin [x])^{3/2} (d + b e \cos [x] + c e \sin [x]) dx$$

Optimal (type 4, 294 leaves, 7 steps):

$$\frac{2 (20 a d + 3 a^2 e + 9 (b^2 + c^2) e) \operatorname{EllipticE}\left[\frac{1}{2} (x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a + b \cos [x] + c \sin [x]}}{15 \sqrt{\frac{a+b \cos [x]+c \sin [x]}{a+\sqrt{b^2+c^2}}}}$$

$$\frac{2 (a^2 - b^2 - c^2) (5 d + 3 a e) \operatorname{EllipticF}\left[\frac{1}{2} (x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b \cos [x]+c \sin [x]}{a+\sqrt{b^2+c^2}}}}{15 \sqrt{a + b \cos [x] + c \sin [x]}}$$

$$\frac{2}{5} (a + b \cos [x] + c \sin [x])^{3/2} (c e \cos [x] - b e \sin [x]) - \frac{2}{15} \sqrt{a + b \cos [x] + c \sin [x]} (c (5 d + 3 a e) \cos [x] - b (5 d + 3 a e) \sin [x])$$

Result (type 6, 5218 leaves):

$$\begin{aligned} & \sqrt{a + b \cos [x] + c \sin [x]} \\ & \left( \frac{2 b (20 a d + 3 a^2 e + 9 b^2 e + 9 c^2 e)}{15 c} - \frac{2}{15} c (5 d + 6 a e) \cos [x] - \frac{2}{5} b c e \cos [2 x] + \frac{2}{15} b (5 d + 6 a e) \sin [x] + \frac{1}{5} (b^2 - c^2) e \sin [2 x] \right) + \\ & \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c} {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\ & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} + \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \end{aligned}$$

$$\frac{1}{3\sqrt{1+\frac{b^2}{c^2}}c} {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}\right] \operatorname{Sec}\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]$$

$$\sqrt{\frac{c\sqrt{\frac{b^2+c^2}{c^2}}-c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a+c\sqrt{\frac{b^2+c^2}{c^2}}}\sqrt{a+c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} + \sqrt{\frac{c\sqrt{\frac{b^2+c^2}{c^2}}+c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a+c\sqrt{\frac{b^2+c^2}{c^2}}}}$$

$$\frac{1}{3\sqrt{1+\frac{b^2}{c^2}}} {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}\right] \operatorname{Sec}\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]$$

$$\sqrt{\frac{c\sqrt{\frac{b^2+c^2}{c^2}}-c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a+c\sqrt{\frac{b^2+c^2}{c^2}}}\sqrt{a+c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} + \sqrt{\frac{c\sqrt{\frac{b^2+c^2}{c^2}}+c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a+c\sqrt{\frac{b^2+c^2}{c^2}}}}$$

$$\frac{1}{5\sqrt{1+\frac{b^2}{c^2}}c} {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}\right] \operatorname{Sec}\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]$$

$$\sqrt{\frac{c\sqrt{\frac{b^2+c^2}{c^2}}-c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a+c\sqrt{\frac{b^2+c^2}{c^2}}}\sqrt{a+c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} + \sqrt{\frac{c\sqrt{\frac{b^2+c^2}{c^2}}+c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a+c\sqrt{\frac{b^2+c^2}{c^2}}}}$$



$$\begin{aligned}
& \frac{1}{5 \sqrt{1 + \frac{b^2}{c^2}}} 8 a c e \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{3c}} \\
& 4 a b^2 d \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \operatorname{Sin} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \\
& \left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \operatorname{Sin} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos} \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) +
\end{aligned}$$

$$\frac{4}{3} a c d \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \left( \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}}}{b^2+c^2}}{\sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) + \frac{1}{5c}$$

$$a^2 b^2 e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{\frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) + \frac{1}{5c}$$

$$3b^4 e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{\frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) +$$

$$\frac{1}{5} a^2 c e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\begin{aligned}
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right}}} \right) + \\
& \frac{6}{5} b^2 c e \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right}}} \right) +
\end{aligned}$$

$$\frac{3}{5} c^3 e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) \left( \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}} \right) \right)$$

**Problem 558: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a+b \cos [x]+c \sin [x]} (d+b e \cos [x]+c e \sin [x]) dx$$

Optimal (type 4, 229 leaves, 6 steps):

$$\frac{2(3d+ae) \operatorname{EllipticE} \left[ \frac{1}{2} (x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}} \right] \sqrt{a+b \cos [x]+c \sin [x]}}{3 \sqrt{\frac{a+b \cos [x]+c \sin [x]}{a+\sqrt{b^2+c^2}}}}$$

$$\frac{2(a^2-b^2-c^2) e \operatorname{EllipticF} \left[ \frac{1}{2} (x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}} \right] \sqrt{\frac{a+b \cos [x]+c \sin [x]}{a+\sqrt{b^2+c^2}}}}{3 \sqrt{a+b \cos [x]+c \sin [x]}} - \frac{2}{3} \sqrt{a+b \cos [x]+c \sin [x]} (c e \cos [x] - b e \sin [x])$$

Result (type 6, 3006 leaves):

$$\sqrt{a + b \cos[x] + c \sin[x]} \left( \frac{2b(3d + ae)}{3c} - \frac{2}{3} c e \cos[x] + \frac{2}{3} b e \sin[x] \right) + \frac{1}{\sqrt{1 + \frac{b^2}{c^2} c}}$$

$$2 a d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} +$$

$$\frac{1}{3 \sqrt{1 + \frac{b^2}{c^2} c}} 2 b^2 e \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} +$$

$$\frac{1}{3 \sqrt{1 + \frac{b^2}{c^2} c}} 2 c e \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]$$

$$\begin{aligned}
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{c}} \\
& b^2 d \left( - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}\right] \operatorname{Sin}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) / \right. \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \right. \\
& \left. \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) - \frac{c \operatorname{Sin}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \right) + } \\
& c d \left( - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \operatorname{Cos}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}\right] \operatorname{Sin}\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) / \right.
\end{aligned}$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right. \\ \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{\frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right)}{b^2 + c^2} - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}} \right) + \frac{1}{3c}$$

$$a b^2 e \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right] \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) /$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right. \\ \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{\frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right)}{b^2 + c^2} - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}} \right) +$$



$$\frac{1}{3} a c e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) \left( \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}}}{\sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right)$$

Problem 559: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{d + b e \cos [x] + c e \sin [x]}{\sqrt{a + b \cos [x] + c \sin [x]}} dx$$

Optimal (type 4, 180 leaves, 5 steps):

$$\frac{2 e \operatorname{EllipticE} \left[ \frac{1}{2} \left( x - \operatorname{ArcTan} [b, c] \right), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}} \right] \sqrt{a + b \cos [x] + c \sin [x]}}{\sqrt{\frac{a+b \cos [x]+c \sin [x]}{a+\sqrt{b^2+c^2}}}} +$$

$$\frac{2 (d - a e) \operatorname{EllipticF} \left[ \frac{1}{2} \left( x - \operatorname{ArcTan} [b, c] \right), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}} \right] \sqrt{\frac{a+b \cos [x]+c \sin [x]}{a+\sqrt{b^2+c^2}}}}{\sqrt{a + b \cos [x] + c \sin [x]}}$$

Result (type 6, 1319 leaves):

$$\begin{aligned}
& \frac{2 b e \sqrt{a+b \cos [x]+c \sin [x]}}{c} + \frac{1}{\sqrt{1+\frac{b^2}{c^2} c}} \\
& 2 d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin \left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin \left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}}-c \sqrt{\frac{b^2+c^2}{c^2}} \sin \left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a+c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a+c \sqrt{\frac{b^2+c^2}{c^2}} \sin \left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}}+c \sqrt{\frac{b^2+c^2}{c^2}} \sin \left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a+c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{c} \\
& b^2 e \left( \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[x-\operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}}\left(1-\frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[x-\operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}}\left(-1-\frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}\right] \sin \left[x-\operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) / \\
& \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}}-b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[x-\operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[x-\operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \right)
\end{aligned}$$

$$\left( \frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} +$$

$$c e \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \right)$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}}$$

Problem 560: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{d + b e \cos [x] + c e \sin [x]}{(a + b \cos [x] + c \sin [x])^{3/2}} dx$$

Optimal (type 4, 250 leaves, 6 steps):

$$\frac{2(d - ae) \operatorname{EllipticE}\left[\frac{1}{2}(x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a+b\cos[x]+c\sin[x]}}{(a^2 - b^2 - c^2) \sqrt{\frac{a+b\cos[x]+c\sin[x]}{a+\sqrt{b^2+c^2}}}} +$$

$$\frac{2e \operatorname{EllipticF}\left[\frac{1}{2}(x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b\cos[x]+c\sin[x]}{a+\sqrt{b^2+c^2}}}}{\sqrt{a+b\cos[x]+c\sin[x]}} + \frac{2(c(d - ae)\cos[x] - b(d - ae)\sin[x])}{(a^2 - b^2 - c^2) \sqrt{a+b\cos[x]+c\sin[x]}}$$

Result (type 6, 3176 leaves):

$$\sqrt{a+b\cos[x]+c\sin[x]} \left( \frac{2(b^2+c^2)(-d+ae)}{bc(-a^2+b^2+c^2)} - \frac{2(-acd+a^2ce-b^2d\sin[x]-c^2d\sin[x]+a^2e\sin[x]+ac^2e\sin[x])}{b(-a^2+b^2+c^2)(a+b\cos[x]+c\sin[x])} \right) -$$

$$\frac{1}{\sqrt{1+\frac{b^2}{c^2}}c(-a^2+b^2+c^2)} {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin[x+\operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin[x+\operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}\right]$$

$$\operatorname{Sec}\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c\sqrt{\frac{b^2+c^2}{c^2}}-c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a+c\sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a+c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\sqrt{\frac{c\sqrt{\frac{b^2+c^2}{c^2}}+c\sqrt{\frac{b^2+c^2}{c^2}}\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a+c\sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{\sqrt{1+\frac{b^2}{c^2}}c(-a^2+b^2+c^2)}$$

$$2b^2e {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}, -\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}}c}\right)c}\right] \operatorname{Sec}\left[x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]$$

$$\frac{\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} +$$

$$\frac{1}{\sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)} {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right]$$

$$\operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}}$$

$$\sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} - \frac{1}{c (-a^2 + b^2 + c^2)}$$

$$b^2 d \left( \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right] \sin\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) /$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \right)$$

$$\left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{\frac{2b\left(a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]\right)}{b^2+c^2} - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}}}}}}}{\sqrt{a + b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}} \right) - \frac{1}{-a^2 + b^2 + c^2}$$

$$c d \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a + b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}\right] \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b\sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right)$$

$$\left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{\frac{2b\left(a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]\right)}{b^2+c^2} - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}}}}}}}{\sqrt{a + b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}} \right) + \frac{1}{c(-a^2 + b^2 + c^2)}$$

$$a b^2 e \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a + b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}\right] \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) \right) /$$

$$\begin{aligned}
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2} \right. \\
& \left. \frac{1}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} - a^2 + b^2 + c^2} \right) \\
& a c e \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{b^2 + c^2} \right. \\
& \left. \frac{1}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} - a^2 + b^2 + c^2} \right)
\end{aligned}$$

Problem 561: Result unnecessarily involves higher level functions and more than twice size of optimal

## antiderivative.

$$\int \frac{d + b e \cos [x] + c e \sin [x]}{(a + b \cos [x] + c \sin [x])^{5/2}} dx$$

Optimal (type 4, 378 leaves, 7 steps):

$$\frac{2(4ad - a^2e - 3(b^2 + c^2)e) \operatorname{EllipticE}\left[\frac{1}{2}(x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a + b \cos [x] + c \sin [x]} + 3(a^2 - b^2 - c^2)^2 \sqrt{\frac{a+b \cos [x] + c \sin [x]}{a+\sqrt{b^2+c^2}}}}{3(a^2 - b^2 - c^2) \sqrt{a + b \cos [x] + c \sin [x]} + \frac{2(d - a e) \operatorname{EllipticF}\left[\frac{1}{2}(x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b \cos [x] + c \sin [x]}{a+\sqrt{b^2+c^2}}}}{3(a^2 - b^2 - c^2) \sqrt{a + b \cos [x] + c \sin [x]}} + \frac{2(c(d - a e) \cos [x] - b(d - a e) \sin [x])}{3(a^2 - b^2 - c^2)(a + b \cos [x] + c \sin [x])^{3/2}} + \frac{2(c(4ad - a^2e - 3(b^2 + c^2)e) \cos [x] - b(4ad - a^2e - 3(b^2 + c^2)e) \sin [x])}{3(a^2 - b^2 - c^2)^2 \sqrt{a + b \cos [x] + c \sin [x]}}$$

Result (type 6, 5554 leaves):

$$\sqrt{a + b \cos [x] + c \sin [x]} \left( -\frac{2(b^2 + c^2)(-4ad + a^2e + 3b^2e + 3c^2e)}{3bc(-a^2 + b^2 + c^2)^2} - \frac{2(-acd + a^2ce - b^2d \sin [x] - c^2d \sin [x] + ab^2e \sin [x] + ac^2e \sin [x])}{3b(-a^2 + b^2 + c^2)(a + b \cos [x] + c \sin [x])^2} + \frac{(2(-3a^2cd - b^2cd - c^3d + 4ab^2ce + 4ac^3e - 4ab^2d \sin [x] - 4ac^2d \sin [x] + a^2b^2e \sin [x] + 3b^4e \sin [x] + a^2c^2e \sin [x] + 6b^2c^2e \sin [x] + 3c^4e \sin [x]))}{(3b(-a^2 + b^2 + c^2)^2(a + b \cos [x] + c \sin [x]))} \right) + \left( 2a^2d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin [x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right)$$



$$\begin{aligned}
& \left( \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \right) \\
& \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^2 \right) + \left[ 2 b^2 d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \right) \\
& \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \left( \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} \right) / \left( 3 \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^2 \right) + \\
& \left[ 2 c d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right)
\end{aligned}$$

$$\left( \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \right) /$$

$$\left( 3 \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)^2 \right) - \left( 8 a b^2 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \right)$$

$$\operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\left( \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \right) / \left( 3 \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^2 \right) -$$

$$\left( 8 a c e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \right)$$

$$\begin{aligned}
& \text{Sec}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \text{Sin}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \left/ \left( 3 \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)^2 \right) + \frac{1}{3c (-a^2 + b^2 + c^2)^2} \right. \\
& \left. 4 a b^2 d \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}\right], -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}\right] \text{Sin}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right/ \right. \\
& \left. \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right) \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) - \frac{c \text{Sin}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}}}}{b^2+c^2}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}} + \frac{1}{3(-a^2 + b^2 + c^2)^2}
\end{aligned}$$

$$4 a c d \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) \left( \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}}}{b^2+c^2} \right) \left( \frac{1}{3c(-a^2+b^2+c^2)^2} \right) \sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}$$

$$a^2 b^2 e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

$$\left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{\frac{2b\left(a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]\right)}{b^2+c^2} - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}}}}}} \right) = \frac{1}{c(-a^2 + b^2 + c^2)^2}$$

$$b^4 e \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}\right] \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) \right)$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right)$$

$$\left( \frac{\sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b\sqrt{\frac{b^2+c^2}{b^2}}}}}{\sqrt{\frac{2b\left(a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]\right)}{b^2+c^2} - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}}}}}} \right) = \frac{1}{3(-a^2 + b^2 + c^2)^2}$$

$$a^2 c e \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)}\right] \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) \right)$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right. \\ \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right}}} - \frac{1}{(-a^2 + b^2 + c^2)^2} \right)$$

$$2 b^2 c e \left( \left( \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}\right] \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) /$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]} \right. \\ \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} - \frac{2b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) - \frac{c \sin\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right}}} - \frac{1}{(-a^2 + b^2 + c^2)^2} \right)$$

$$c^3 e \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}} \right)} \right] \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \right.$$

$$\left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}} \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right.$$

$$\left. \left. \left( \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) \right) \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}}}{\sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right)$$

**Problem 581: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{a+b \cos [x] \sin [x]} dx$$

Optimal (type 4, 225 leaves, 9 steps):

$$-\frac{i x \operatorname{Log} \left[ 1 - \frac{i b e^{2 i x}}{2 a - \sqrt{4 a^2 - b^2}} \right]}{\sqrt{4 a^2 - b^2}} + \frac{i x \operatorname{Log} \left[ 1 - \frac{i b e^{2 i x}}{2 a + \sqrt{4 a^2 - b^2}} \right]}{\sqrt{4 a^2 - b^2}} - \frac{\operatorname{PolyLog} \left[ 2, \frac{i b e^{2 i x}}{2 a - \sqrt{4 a^2 - b^2}} \right]}{2 \sqrt{4 a^2 - b^2}} + \frac{\operatorname{PolyLog} \left[ 2, \frac{i b e^{2 i x}}{2 a + \sqrt{4 a^2 - b^2}} \right]}{2 \sqrt{4 a^2 - b^2}}$$

Result (type 4, 789 leaves):

$$\begin{aligned}
& \frac{1}{2} \left( \frac{\pi \operatorname{ArcTan} \left[ \frac{b+2a \operatorname{Tan}[x]}{\sqrt{4a^2-b^2}} \right]}{\sqrt{4a^2-b^2}} + \frac{1}{\sqrt{-4a^2+b^2}} \left( 2 \operatorname{ArcCos} \left[ -\frac{2a}{b} \right] \operatorname{ArcTanh} \left[ \frac{(2a-b) \operatorname{Cot} \left[ \frac{\pi}{4} + x \right]}{\sqrt{-4a^2+b^2}} \right] + (\pi - 4x) \operatorname{ArcTanh} \left[ \frac{(2a+b) \operatorname{Tan} \left[ \frac{\pi}{4} + x \right]}{\sqrt{-4a^2+b^2}} \right] - \right. \right. \\
& \left. \left( \operatorname{ArcCos} \left[ -\frac{2a}{b} \right] + 2i \operatorname{ArcTanh} \left[ \frac{(2a-b) \operatorname{Cot} \left[ \frac{\pi}{4} + x \right]}{\sqrt{-4a^2+b^2}} \right] \right) \operatorname{Log} \left[ \frac{(2a+b) (-2a+b-i\sqrt{-4a^2+b^2}) (1+i \operatorname{Cot} \left[ \frac{\pi}{4} + x \right])}{b (2a+b+\sqrt{-4a^2+b^2} \operatorname{Cot} \left[ \frac{\pi}{4} + x \right])} \right] - \right. \\
& \left. \left( \operatorname{ArcCos} \left[ -\frac{2a}{b} \right] - 2i \operatorname{ArcTanh} \left[ \frac{(2a-b) \operatorname{Cot} \left[ \frac{\pi}{4} + x \right]}{\sqrt{-4a^2+b^2}} \right] \right) \operatorname{Log} \left[ \frac{(2a+b) (2ia-i b+\sqrt{-4a^2+b^2}) (i+\operatorname{Cot} \left[ \frac{\pi}{4} + x \right])}{b (2a+b+\sqrt{-4a^2+b^2} \operatorname{Cot} \left[ \frac{\pi}{4} + x \right])} \right] + \right. \\
& \left. \left( \operatorname{ArcCos} \left[ -\frac{2a}{b} \right] + 2i \left( \operatorname{ArcTanh} \left[ \frac{(2a-b) \operatorname{Cot} \left[ \frac{\pi}{4} + x \right]}{\sqrt{-4a^2+b^2}} \right] + \operatorname{ArcTanh} \left[ \frac{(2a+b) \operatorname{Tan} \left[ \frac{\pi}{4} + x \right]}{\sqrt{-4a^2+b^2}} \right] \right) \right) \operatorname{Log} \left[ \frac{(-1)^{1/4} \sqrt{-4a^2+b^2} e^{-ix}}{2\sqrt{b} \sqrt{a+b \operatorname{Cos}[x]} \operatorname{Sin}[x]} \right] + \\
& \left. \left( \operatorname{ArcCos} \left[ -\frac{2a}{b} \right] - 2i \operatorname{ArcTanh} \left[ \frac{(2a-b) \operatorname{Cot} \left[ \frac{\pi}{4} + x \right]}{\sqrt{-4a^2+b^2}} \right] - 2i \operatorname{ArcTanh} \left[ \frac{(2a+b) \operatorname{Tan} \left[ \frac{\pi}{4} + x \right]}{\sqrt{-4a^2+b^2}} \right] \right) \operatorname{Log} \left[ \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \sqrt{-4a^2+b^2} e^{ix}}{\sqrt{b} \sqrt{2a+b \operatorname{Sin}[2x]}} \right] + i \left( \operatorname{PolyLog} \left[ 2, \right. \right. \\
& \left. \left. \frac{(2a-i\sqrt{-4a^2+b^2}) (2a+b-\sqrt{-4a^2+b^2} \operatorname{Cot} \left[ \frac{\pi}{4} + x \right])}{b (2a+b+\sqrt{-4a^2+b^2} \operatorname{Cot} \left[ \frac{\pi}{4} + x \right])} \right] - \operatorname{PolyLog} \left[ 2, \frac{(2a+i\sqrt{-4a^2+b^2}) (2a+b-\sqrt{-4a^2+b^2} \operatorname{Cot} \left[ \frac{\pi}{4} + x \right])}{b (2a+b+\sqrt{-4a^2+b^2} \operatorname{Cot} \left[ \frac{\pi}{4} + x \right])} \right] \right) \right) \right)
\end{aligned}$$

**Problem 588: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sin}[ax]^3}{x (ax \operatorname{Cos}[ax] - \operatorname{Sin}[ax])^2} dx$$

Optimal (type 4, 56 leaves, 4 steps):

$$\frac{\operatorname{Cos}[ax]}{ax} + \frac{\operatorname{Sin}[ax]}{a^2 x^2} + \frac{\operatorname{Sin}[ax]^2}{a^2 x^2 (ax \operatorname{Cos}[ax] - \operatorname{Sin}[ax])} + \operatorname{SinIntegral}[ax]$$

Result (type 4, 242 leaves):

$$\begin{aligned}
& \frac{1}{2ax \operatorname{Cos}[ax] - 2 \operatorname{Sin}[ax]} \left( 1 + \operatorname{Cos}[2ax] + i a e^{ix} \operatorname{Cos}[ax] \operatorname{ExpIntegralEi}[-1-i ax] - i a e^{ix} \operatorname{Cos}[ax] \operatorname{ExpIntegralEi}[-1+i ax] - \right. \\
& i e \operatorname{CosIntegral}[i-ax] (ax \operatorname{Cos}[ax] - \operatorname{Sin}[ax]) + i e \operatorname{CosIntegral}[i+ax] (ax \operatorname{Cos}[ax] - \operatorname{Sin}[ax]) - i e \operatorname{ExpIntegralEi}[-1-i ax] \operatorname{Sin}[ax] + \\
& i e \operatorname{ExpIntegralEi}[-1+i ax] \operatorname{Sin}[ax] + 2ax \operatorname{Cos}[ax] \operatorname{SinIntegral}[ax] - 2 \operatorname{Sin}[ax] \operatorname{SinIntegral}[ax] + \\
& a e^{ix} \operatorname{Cos}[ax] \operatorname{SinIntegral}[i-ax] - e \operatorname{Sin}[ax] \operatorname{SinIntegral}[i-ax] - a e^{ix} \operatorname{Cos}[ax] \operatorname{SinIntegral}[i+ax] + e \operatorname{Sin}[ax] \operatorname{SinIntegral}[i+ax] \left. \right)
\end{aligned}$$



**Problem 597: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos [a x]^3}{x (\cos [a x] + a x \sin [a x])^2} dx$$

Optimal (type 4, 56 leaves, 4 steps):

$$\frac{\cos [a x]}{a^2 x^2} + \text{CosIntegral}[a x] - \frac{\sin [a x]}{a x} - \frac{\cos [a x]^2}{a^2 x^2 (\cos [a x] + a x \sin [a x])}$$

Result (type 4, 237 leaves):

$$\frac{1}{2 (\cos [a x] + a x \sin [a x])} (-1 + \cos [2 a x] - e \cos [a x] \text{CosIntegral}[i + a x] + e \cos [a x] \text{ExpIntegralEi}[-1 - i a x] + e \cos [a x] \text{ExpIntegralEi}[-1 + i a x] - a e x \text{CosIntegral}[i + a x] \sin [a x] + a e x \text{ExpIntegralEi}[-1 - i a x] \sin [a x] + a e x \text{ExpIntegralEi}[-1 + i a x] \sin [a x] + 2 \text{CosIntegral}[a x] (\cos [a x] + a x \sin [a x]) - e \text{CosIntegral}[i - a x] (\cos [a x] + a x \sin [a x]) - i e \cos [a x] \text{SinIntegral}[i - a x] - i a e x \sin [a x] \text{SinIntegral}[i - a x] - i e \cos [a x] \text{SinIntegral}[i + a x] - i a e x \sin [a x] \text{SinIntegral}[i + a x])$$

**Problem 623: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{c \tan [a + b x] \tan [2 (a + b x)]}} dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c} \tan [2 a + 2 b x]}{\sqrt{-c + c \sec [2 a + 2 b x]}}\right]}{b \sqrt{c}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{c} \tan [2 a + 2 b x]}{\sqrt{2} \sqrt{-c + c \sec [2 a + 2 b x]}}\right]}{\sqrt{2} b \sqrt{c}}$$

Result (type 6, 170 leaves):

$$\left( 3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot [a + b x]^2, -\cot [a + b x]^2\right] \sin [a + b x]^2 \tan [a + b x] \right) / \left( b \left( 2 \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot [a + b x]^2, -\cot [a + b x]^2\right] + \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot [a + b x]^2, -\cot [a + b x]^2\right] - 3 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot [a + b x]^2, -\cot [a + b x]^2\right] \tan [a + b x]^2 \right) \sqrt{c \tan [a + b x] \tan [2 (a + b x)]} \right)$$

### Problem 624: Result unnecessarily involves higher level functions.

$$\int \frac{\cos[2(a+bx)]}{\sqrt{c \tan[a+bx] \tan[2(a+bx)]}} dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}\right]}{2b\sqrt{c}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{2}\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}\right]}{\sqrt{2}b\sqrt{c}} + \frac{\sin[2a+2bx]}{2b\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}$$

Result (type 6, 226 leaves):

$$\frac{1}{4bc} \left( \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] \cos[2(a+bx)] \tan[a+bx] \right) / \right. \\ \left. \left( 2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] - \right. \right. \\ \left. \left. 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] \tan[a+bx]^2 \right) + \right. \\ \left. \cot[a+bx] \left( 2 \cos[2(a+bx)] + \operatorname{ArcTan}\left[\sqrt{-1+\tan[a+bx]^2}\right] \sqrt{-1+\tan[a+bx]^2} \right) \right) \sqrt{c \tan[a+bx] \tan[2(a+bx)]}$$

### Problem 625: Result unnecessarily involves higher level functions.

$$\int \frac{\cos[2(a+bx)]^2}{\sqrt{c \tan[a+bx] \tan[2(a+bx)]}} dx$$

Optimal (type 3, 182 leaves, 8 steps):

$$\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}\right]}{8b\sqrt{c}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{2}\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}\right]}{\sqrt{2}b\sqrt{c}} + \frac{\sin[2a+2bx]}{8b\sqrt{-c+c \operatorname{Sec}[2a+2bx]}} + \frac{\cos[2a+2bx] \sin[2a+2bx]}{4b\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}$$

Result (type 6, 235 leaves):

$$\left( \left( 42 \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot [a + b x]^2, -\cot [a + b x]^2 \right] \sin [a + b x]^2 \tan [a + b x] \right) / \right. \\ \left( 2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot [a + b x]^2, -\cot [a + b x]^2 \right] + \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot [a + b x]^2, -\cot [a + b x]^2 \right] - \right. \\ \left. 3 \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot [a + b x]^2, -\cot [a + b x]^2 \right] \tan [a + b x]^2 \right) + \\ \left( 2 (1 + \cos [2 (a + b x)]) + \cos [4 (a + b x)] \right) + \operatorname{ArcTan} \left[ \sqrt{-1 + \tan [a + b x]^2} \right] \sqrt{-1 + \tan [a + b x]^2} \right) \tan [2 (a + b x)] \Big/ \\ \left( 16 b \sqrt{c \tan [a + b x] \tan [2 (a + b x)]} \right)$$

**Problem 630: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(c \tan [a + b x] \tan [2 (a + b x)])^{3/2}} dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$-\frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{c} \tan [2 a + 2 b x]}{\sqrt{-c + c \operatorname{Sec} [2 a + 2 b x]}} \right]}{b c^{3/2}} + \frac{5 \operatorname{ArcTanh} \left[ \frac{\sqrt{c} \tan [2 a + 2 b x]}{\sqrt{2} \sqrt{-c + c \operatorname{Sec} [2 a + 2 b x]}} \right]}{4 \sqrt{2} b c^{3/2}} - \frac{\tan [2 a + 2 b x]}{4 b (-c + c \operatorname{Sec} [2 a + 2 b x])^{3/2}}$$

Result (type 6, 226 leaves):

$$\frac{1}{8 b c^2} \left( - \left( \left( 12 \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot [a + b x]^2, -\cot [a + b x]^2 \right] \cos [2 (a + b x)] \tan [a + b x] \right) / \right. \right. \\ \left( 2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot [a + b x]^2, -\cot [a + b x]^2 \right] + \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot [a + b x]^2, -\cot [a + b x]^2 \right] - \right. \\ \left. 3 \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot [a + b x]^2, -\cot [a + b x]^2 \right] \tan [a + b x]^2 \right) \Big) - \\ \cot [a + b x] \left( -2 + \operatorname{Csc} [a + b x]^2 + \operatorname{ArcTan} \left[ \sqrt{-1 + \tan [a + b x]^2} \right] \sqrt{-1 + \tan [a + b x]^2} \right) \Big) \sqrt{c \tan [a + b x] \tan [2 (a + b x)]}$$

**Problem 631: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos [2 (a + b x)]}{(c \tan [a + b x] \tan [2 (a + b x)])^{3/2}} dx$$

Optimal (type 3, 178 leaves, 8 steps):

$$-\frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \operatorname{Tan}[2 a+2 b x]}{\sqrt{-c+c \operatorname{Sec}[2 a+2 b x]}}\right]}{2 b c^{3/2}} + \frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \operatorname{Tan}[2 a+2 b x]}{\sqrt{2} \sqrt{-c+c \operatorname{Sec}[2 a+2 b x]}}\right]}{4 \sqrt{2} b c^{3/2}} - \frac{\operatorname{Sin}[2 a+2 b x]}{4 b(-c+c \operatorname{Sec}[2 a+2 b x])^{3/2}} - \frac{3 \operatorname{Sin}[2 a+2 b x]}{4 b c \sqrt{-c+c \operatorname{Sec}[2 a+2 b x]}}$$

Result (type 6, 249 leaves):

$$\frac{1}{8 b c^2} \left( -2 \operatorname{Cot}[a+b x] - \operatorname{Cot}[a+b x] \operatorname{Csc}[a+b x]^2 + 4 \operatorname{Sin}[2(a+b x)] - 3 \operatorname{ArcTan}\left[\sqrt{-1+\operatorname{Tan}[a+b x]^2}\right] \operatorname{Cot}[a+b x] \sqrt{-1+\operatorname{Tan}[a+b x]^2} - \right. \\ \left. \left( 18 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \operatorname{Cot}[a+b x]^2, -\operatorname{Cot}[a+b x]^2\right] \operatorname{Cos}[2(a+b x)] \operatorname{Tan}[a+b x] \right) / \right. \\ \left. \left( 2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \operatorname{Cot}[a+b x]^2, -\operatorname{Cot}[a+b x]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \operatorname{Cot}[a+b x]^2, -\operatorname{Cot}[a+b x]^2\right] - \right. \right. \\ \left. \left. 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \operatorname{Cot}[a+b x]^2, -\operatorname{Cot}[a+b x]^2\right] \operatorname{Tan}[a+b x]^2 \right) \right) \sqrt{c \operatorname{Tan}[a+b x] \operatorname{Tan}[2(a+b x)]} \right)$$

**Problem 632: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Cos}[2(a+b x)]^2}{(c \operatorname{Tan}[a+b x] \operatorname{Tan}[2(a+b x)])^{3/2}} dx$$

Optimal (type 3, 234 leaves, 9 steps):

$$-\frac{19 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \operatorname{Tan}[2 a+2 b x]}{\sqrt{-c+c \operatorname{Sec}[2 a+2 b x]}}\right]}{8 b c^{3/2}} + \frac{13 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \operatorname{Tan}[2 a+2 b x]}{\sqrt{2} \sqrt{-c+c \operatorname{Sec}[2 a+2 b x]}}\right]}{4 \sqrt{2} b c^{3/2}} - \\ \frac{\operatorname{Cos}[2 a+2 b x] \operatorname{Sin}[2 a+2 b x]}{4 b(-c+c \operatorname{Sec}[2 a+2 b x])^{3/2}} - \frac{7 \operatorname{Sin}[2 a+2 b x]}{8 b c \sqrt{-c+c \operatorname{Sec}[2 a+2 b x]}} - \frac{\operatorname{Cos}[2 a+2 b x] \operatorname{Sin}[2 a+2 b x]}{2 b c \sqrt{-c+c \operatorname{Sec}[2 a+2 b x]}}$$

Result (type 6, 251 leaves):

$$\left( (-9 \operatorname{Cos}[a+b x] + 4 \operatorname{Cos}[3(a+b x)] + \operatorname{Cos}[5(a+b x)]) \operatorname{Csc}[a+b x] - \right. \\ \left. \left( 114 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \operatorname{Cot}[a+b x]^2, -\operatorname{Cot}[a+b x]^2\right] \operatorname{Sin}[a+b x]^2 \operatorname{Tan}[a+b x] \right) / \right. \\ \left. \left( 2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \operatorname{Cot}[a+b x]^2, -\operatorname{Cot}[a+b x]^2\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \operatorname{Cot}[a+b x]^2, -\operatorname{Cot}[a+b x]^2\right] - \right. \right. \\ \left. \left. 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \operatorname{Cot}[a+b x]^2, -\operatorname{Cot}[a+b x]^2\right] \operatorname{Tan}[a+b x]^2 \right) - \right. \\ \left. 7 \operatorname{ArcTan}\left[\sqrt{-1+\operatorname{Tan}[a+b x]^2}\right] \sqrt{-1+\operatorname{Tan}[a+b x]^2} \operatorname{Tan}[2(a+b x)] \right) / \left( 16 b c \sqrt{c \operatorname{Tan}[a+b x] \operatorname{Tan}[2(a+b x)]} \right)$$

**Problem 634: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Csc}[x]^2 \operatorname{Sec}[x]}{\sqrt{\operatorname{Sin}[2x]} (-2 + \operatorname{Tan}[x])} dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$\frac{\operatorname{Cos}[x]}{2\sqrt{\operatorname{Sin}[2x]}} + \frac{\operatorname{Cos}[x] \operatorname{Cot}[x]}{3\sqrt{\operatorname{Sin}[2x]}} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{Tan}[x]}}{\sqrt{2}}\right] \operatorname{Sin}[x]}{2\sqrt{2}\sqrt{\operatorname{Sin}[2x]}\sqrt{\operatorname{Tan}[x]}}$$

Result (type 4, 119 leaves):

$$\frac{1}{4}\sqrt{\operatorname{Sin}[2x]} \left( \left( 1 + \frac{2 \operatorname{Cot}[x]}{3} \right) \operatorname{Csc}[x] + 5 \sqrt{\frac{\operatorname{Cos}[x]}{-2 + 2 \operatorname{Cos}[x]}} \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{2}{-1 + \sqrt{5}}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[\frac{1}{2}(-1 + \sqrt{5}), -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]}}\right], -1\right] \operatorname{Sec}[x] \sqrt{\operatorname{Tan}\left[\frac{x}{2}\right]} \right) \right)$$

**Problem 635: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{Cos}[x]^2 \operatorname{Sin}[x]}{(\operatorname{Sin}[x]^2 - \operatorname{Sin}[2x]) \operatorname{Sin}[2x]^{5/2}} dx$$

Optimal (type 3, 79 leaves, 6 steps):

$$\frac{\operatorname{Cos}[x]^4 \operatorname{Sin}[x]}{3 \operatorname{Sin}[2x]^{5/2}} + \frac{\operatorname{Cos}[x]^3 \operatorname{Sin}[x]^2}{2 \operatorname{Sin}[2x]^{5/2}} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{Tan}[x]}}{\sqrt{2}}\right] \operatorname{Sin}[x]^5}{2\sqrt{2}\operatorname{Sin}[2x]^{5/2}\operatorname{Tan}[x]^{5/2}}$$

Result (type 4, 139 leaves):

$$\begin{aligned}
& -\frac{1}{16(-1+2\cot[x])}\operatorname{Csc}[x](2\cos[x]-\sin[x])\sqrt{\sin[2x]} \\
& \left( -\frac{1}{3}(3+2\cot[x])\operatorname{Csc}[x]-5\sqrt{\frac{\cos[x]}{-2+2\cos[x]}} \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{2}{-1+\sqrt{5}}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] + \right. \right. \\
& \left. \left. \operatorname{EllipticPi}\left[\frac{1}{2}(-1+\sqrt{5}), -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] \operatorname{Sec}[x]\sqrt{\tan\left[\frac{x}{2}\right]} \right) \right)
\end{aligned}$$

**Problem 636: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[x]^3 \cos[2x]}{(\sin[x]^2 - \sin[2x]) \sin[2x]^{5/2}} dx$$

Optimal (type 3, 95 leaves, 6 steps):

$$\frac{\cos[x]^5}{5\sin[2x]^{5/2}} + \frac{\cos[x]^4 \sin[x]}{6\sin[2x]^{5/2}} - \frac{3\cos[x]^3 \sin[x]^2}{4\sin[2x]^{5/2}} + \frac{3\operatorname{ArcTanh}\left[\frac{\sqrt{\tan[x]}}{\sqrt{2}}\right] \sin[x]^5}{4\sqrt{2}\sin[2x]^{5/2}\tan[x]^{5/2}}$$

Result (type 4, 188 leaves):

$$\begin{aligned}
& \frac{1}{960}\operatorname{Sec}[x]\sqrt{\sin[2x]} \left( -114\cot[x] + 20\cot[x]^2 + 24\cot[x]\operatorname{Csc}[x]^2 - 45\sqrt{2}\sqrt{\frac{\cos[x]}{-1+\cos[x]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} - \right. \\
& 45\sqrt{2}\sqrt{\frac{\cos[x]}{-1+\cos[x]}} \operatorname{EllipticPi}\left[-\frac{2}{-1+\sqrt{5}}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} - \\
& \left. 45\sqrt{2}\sqrt{\frac{\cos[x]}{-1+\cos[x]}} \operatorname{EllipticPi}\left[\frac{1}{2}(-1+\sqrt{5}), -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] \sqrt{\tan\left[\frac{x}{2}\right]} \right)
\end{aligned}$$

### Problem 638: Result more than twice size of optimal antiderivative.

$$\int (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx]) dx$$

Optimal (type 3, 26 leaves, 1 step):

$$\frac{(b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^4}{4d}$$

Result (type 3, 938 leaves):

$$\begin{aligned} & \frac{8b^4 \operatorname{Cos}[c + dx] (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])}{d (3a \operatorname{Cos}[c + dx] + a \operatorname{Cos}[3c + 3dx] + 4b \operatorname{Sin}[c + dx]) (2b + a \operatorname{Sin}[2c + 2dx])^3} + \\ & \left( \frac{a^4 \operatorname{Cos}[4c] \operatorname{Cos}[4dx] \operatorname{Cos}[c + dx]^5 (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])}{(d (3a \operatorname{Cos}[c + dx] + a \operatorname{Cos}[3c + 3dx] + 4b \operatorname{Sin}[c + dx]) (2b + a \operatorname{Sin}[2c + 2dx])^3)} + \right. \\ & \left. \frac{16a^2 b^2 \operatorname{Cos}[c + dx]^3 \operatorname{Sec}[c] (3a \operatorname{Cos}[c] + 2b \operatorname{Sin}[c]) (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])}{(d (3a \operatorname{Cos}[c + dx] + a \operatorname{Cos}[3c + 3dx] + 4b \operatorname{Sin}[c + dx]) (2b + a \operatorname{Sin}[2c + 2dx])^3)} - \right. \\ & \left. \frac{4a^3 \operatorname{Cos}[2dx] \operatorname{Cos}[c + dx]^5 (a \operatorname{Cos}[2c] + 4b \operatorname{Sin}[2c]) (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])}{(d (3a \operatorname{Cos}[c + dx] + a \operatorname{Cos}[3c + 3dx] + 4b \operatorname{Sin}[c + dx]) (2b + a \operatorname{Sin}[2c + 2dx])^3)} + \right. \\ & \left. \frac{32a^3 b^3 \operatorname{Cos}[c + dx]^2 \operatorname{Sec}[c] \operatorname{Sin}[dx] (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])}{(d (3a \operatorname{Cos}[c + dx] + a \operatorname{Cos}[3c + 3dx] + 4b \operatorname{Sin}[c + dx]) (2b + a \operatorname{Sin}[2c + 2dx])^3)} + \right. \\ & \left. \frac{32a^3 b \operatorname{Cos}[c + dx]^4 \operatorname{Sec}[c] \operatorname{Sin}[dx] (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])}{(d (3a \operatorname{Cos}[c + dx] + a \operatorname{Cos}[3c + 3dx] + 4b \operatorname{Sin}[c + dx]) (2b + a \operatorname{Sin}[2c + 2dx])^3)} + \right. \\ & \left. \frac{4a^3 \operatorname{Cos}[c + dx]^5 (-4b \operatorname{Cos}[2c] + a \operatorname{Sin}[2c]) \operatorname{Sin}[2dx] (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])}{(d (3a \operatorname{Cos}[c + dx] + a \operatorname{Cos}[3c + 3dx] + 4b \operatorname{Sin}[c + dx]) (2b + a \operatorname{Sin}[2c + 2dx])^3)} - \right. \\ & \left. \frac{a^4 \operatorname{Cos}[c + dx]^5 \operatorname{Sin}[4c] \operatorname{Sin}[4dx] (b \operatorname{Sec}[c + dx] + a \operatorname{Sin}[c + dx])^3 (a \operatorname{Cos}[c + dx] + b \operatorname{Sec}[c + dx] \operatorname{Tan}[c + dx])}{(d (3a \operatorname{Cos}[c + dx] + a \operatorname{Cos}[3c + 3dx] + 4b \operatorname{Sin}[c + dx]) (2b + a \operatorname{Sin}[2c + 2dx])^3)} \right) / \end{aligned}$$

### Problem 654: Result more than twice size of optimal antiderivative.

$$\int \operatorname{Cos}[x]^3 (a + b \operatorname{Cos}[x]^2)^3 \operatorname{Sin}[x] dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$\frac{a (a + b \cos [x]^2)^4}{8 b^2} - \frac{(a + b \cos [x]^2)^5}{10 b^2}$$

Result (type 3, 137 leaves):

$$\frac{1}{32} \left( -12 a^2 b \cos [x]^4 - 8 a b^2 \cos [x]^6 - 2 b^3 \cos [x]^8 - 4 a^3 \cos [2 x] - 4 a^2 b \cos [x]^3 \cos [3 x] - a^3 \cos [4 x] - \frac{1}{32} a b^2 (48 \cos [2 x] + 36 \cos [4 x] + 16 \cos [6 x] + 3 \cos [8 x]) - \frac{1}{320} b^3 (140 \cos [2 x] + 100 \cos [4 x] + 50 \cos [6 x] + 15 \cos [8 x] + 2 \cos [10 x]) \right)$$

**Problem 657:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\cos [x]^2 \sin [x]}{\sqrt{1 - \cos [x]^6}} dx$$

Optimal (type 3, 9 leaves, 3 steps):

$$-\frac{1}{3} \text{ArcSin}[\cos [x]^3]$$

Result (type 4, 162 leaves):

$$-\left( \left( i \cos [x]^2 \text{EllipticPi} \left[ \frac{3}{2} + \frac{i \sqrt{3}}{2}, i \text{ArcSinh} \left[ \sqrt{-\frac{2 i}{-3 i + \sqrt{3}}} \tan [x] \right], \frac{3 i - \sqrt{3}}{3 i + \sqrt{3}} \right] \sin [x] \sqrt{1 - \frac{2 i \tan [x]^2}{-3 i + \sqrt{3}}} \sqrt{1 + \frac{2 i \tan [x]^2}{3 i + \sqrt{3}}} \right) \right) / \left( \sqrt{2} \sqrt{-\frac{i}{-3 i + \sqrt{3}}} \sqrt{1 - \cos [x]^6} \right)$$

**Problem 670:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \cos [x] \sqrt{1 + \csc [x]} dx$$

Optimal (type 3, 21 leaves, 4 steps):

$$\text{ArcTanh}[\sqrt{1 + \csc [x]}] + \sqrt{1 + \csc [x]} \sin [x]$$

Result (type 6, 5067 leaves):









$$\begin{aligned}
& \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + 7 \left( -\frac{3}{14} \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right. \\
& \quad \left. \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \frac{3}{28} \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) + 2 \tan\left[\frac{x}{4}\right]^2 \\
& \left( -\frac{7}{22} \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 2, \frac{15}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \frac{21}{44} \text{AppellF1}\left[\frac{11}{4}, \frac{5}{2}, 1, \frac{15}{4}, \tan\left[\frac{x}{4}\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] - 2 \left( -\frac{7}{11} \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 3, \frac{15}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \frac{7}{44} \text{AppellF1}\left[\frac{11}{4}, \right. \right. \right. \\
& \quad \left. \left. \frac{3}{2}, 2, \frac{15}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) \right) \right) / \left( 7 \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \\
& \left. 2 \left( -2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right)^2 - \\
& \left( 27 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \tan\left[\frac{x}{4}\right] \left( \left( -2 \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + 9 \left( -\frac{5}{18} \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \right. \right. \\
& \quad \left. \left. \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \frac{5}{36} \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) + 2 \tan\left[\frac{x}{4}\right]^2 \right) \\
& \left( -\frac{9}{26} \text{AppellF1}\left[\frac{13}{4}, \frac{3}{2}, 2, \frac{17}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \frac{27}{52} \text{AppellF1}\left[\frac{13}{4}, \frac{5}{2}, 1, \frac{17}{4}, \tan\left[\frac{x}{4}\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] - 2 \left( -\frac{9}{13} \text{AppellF1}\left[\frac{13}{4}, \frac{1}{2}, 3, \frac{17}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \frac{9}{52} \text{AppellF1}\left[\frac{13}{4}, \right. \right. \right. \\
& \quad \left. \left. \frac{3}{2}, 2, \frac{17}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) \right) \right) / \left( 9 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \\
& \left. 2 \left( -2 \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right)^2 \right) \right) \right)
\end{aligned}$$

Problem 673: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[x]}{\sqrt{2 \sin[x] + \sin[x]^2}} dx$$

Optimal (type 3, 19 leaves, 3 steps):

$$2 \text{ArcTanh}\left[\frac{\sin[x]}{\sqrt{2 \sin[x] + \sin[x]^2}}\right]$$

Result (type 3, 40 leaves):

$$\frac{2 \operatorname{ArcSinh}\left[\frac{\sqrt{\sin[x]}}{\sqrt{2}}\right] \sqrt{\sin[x]} \sqrt{2 + \sin[x]}}{\sqrt{\sin[x] (2 + \sin[x])}}$$

**Problem 676: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \operatorname{Sec}[\sin[x]] dx$$

Optimal (type 3, 4 leaves, 2 steps):

$$\operatorname{ArcTanh}[\sin[\sin[x]]]$$

Result (type 3, 37 leaves):

$$-\operatorname{Log}\left[\cos\left[\frac{\sin[x]}{2}\right] - \sin\left[\frac{\sin[x]}{2}\right]\right] + \operatorname{Log}\left[\cos\left[\frac{\sin[x]}{2}\right] + \sin\left[\frac{\sin[x]}{2}\right]\right]$$

**Problem 677: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \sin[x]^3 (a + b \sin[x]^2)^3 dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{a (a + b \sin[x]^2)^4}{8 b^2} + \frac{(a + b \sin[x]^2)^5}{10 b^2}$$

Result (type 3, 128 leaves):

$$\frac{1}{10240} \left( -20 (64 a^3 + 24 a b^2 + 7 b^3) \cos[2x] + 20 (16 a^3 + 18 a b^2 + 5 b^3) \cos[4x] + b (-10 b (16 a + 5 b) \cos[6x] + 15 b (2 a + b) \cos[8x] - 2 b^2 \cos[10x] + 3840 a^2 \sin[x]^4 + 2560 a b \sin[x]^6 + 640 b^2 \sin[x]^8 - 1280 a^2 \sin[x]^3 \sin[3x]) \right)$$

**Problem 691: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[x]^2}{1 - \tan[x]^2} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcTanh}[2 \cos[x] \sin[x]]$$

Result (type 3, 23 leaves):

$$-\frac{1}{2} \operatorname{Log}[\operatorname{Cos}[x] - \operatorname{Sin}[x]] + \frac{1}{2} \operatorname{Log}[\operatorname{Cos}[x] + \operatorname{Sin}[x]]$$

**Problem 705: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{Sec}[x]^2 \operatorname{Tan}[x]^6 (1 + \operatorname{Tan}[x]^2)^3 dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$\frac{\operatorname{Tan}[x]^7}{7} + \frac{\operatorname{Tan}[x]^9}{3} + \frac{3 \operatorname{Tan}[x]^{11}}{11} + \frac{\operatorname{Tan}[x]^{13}}{13}$$

Result (type 3, 67 leaves):

$$-\frac{16 \operatorname{Tan}[x]}{3003} - \frac{8 \operatorname{Sec}[x]^2 \operatorname{Tan}[x]}{3003} - \frac{2 \operatorname{Sec}[x]^4 \operatorname{Tan}[x]}{1001} - \frac{5 \operatorname{Sec}[x]^6 \operatorname{Tan}[x]}{3003} + \frac{53}{429} \operatorname{Sec}[x]^8 \operatorname{Tan}[x] - \frac{27}{143} \operatorname{Sec}[x]^{10} \operatorname{Tan}[x] + \frac{1}{13} \operatorname{Sec}[x]^{12} \operatorname{Tan}[x]$$

**Problem 709: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[x]^2}{\sqrt{4 - \operatorname{Sec}[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\operatorname{ArcSin}\left[\frac{\operatorname{Tan}[x]}{\sqrt{3}}\right]$$

Result (type 3, 43 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{\operatorname{Sin}[x]}{\sqrt{1+2\operatorname{Cos}[2x]}}\right] \sqrt{1+2\operatorname{Cos}[2x]} \operatorname{Sec}[x]}{\sqrt{4 - \operatorname{Sec}[x]^2}}$$

**Problem 710: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Sec}[x]^2}{\sqrt{1 - 4 \operatorname{Tan}[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\frac{1}{2} \text{ArcSin}[2 \text{Tan}[x]]$$

Result (type 3, 52 leaves):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{2}\sin[x]}{\sqrt{-3+5\cos[2x]}}\right] \sqrt{-3+5\cos[2x]} \text{Sec}[x]}{2\sqrt{2-8\text{Tan}[x]^2}}$$

**Problem 711: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sec}[x]^2}{\sqrt{-4+\text{Tan}[x]^2}} dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\text{ArcTanh}\left[\frac{\text{Tan}[x]}{\sqrt{-4+\text{Tan}[x]^2}}\right]$$

Result (type 3, 51 leaves):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2}\sin[x]}{\sqrt{3+5\cos[2x]}}\right] \sqrt{3+5\cos[2x]} \text{Sec}[x]}{\sqrt{2}\sqrt{-4+\text{Tan}[x]^2}}$$

**Problem 712: Result more than twice size of optimal antiderivative.**

$$\int \sqrt{1-\text{Cot}[x]^2} \text{Sec}[x]^2 dx$$

Optimal (type 3, 19 leaves, 3 steps):

$$\text{ArcSin}[\text{Cot}[x]] + \sqrt{1-\text{Cot}[x]^2} \text{Tan}[x]$$

Result (type 3, 52 leaves):

$$\left(-\text{ArcTan}\left[\frac{\cos[x]}{\sqrt{-\cos[2x]}}\right] \cos[x] \sqrt{-\cos[2x]} + \cos[2x]\right) \sqrt{1-\text{Cot}[x]^2} \text{Sec}[2x] \text{Tan}[x]$$

### Problem 728: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[x] \operatorname{Tan}[x]}{\sqrt{4 + \operatorname{Sec}[x]^2}} dx$$

Optimal (type 3, 5 leaves, 3 steps):

$$\operatorname{ArcCsch}[2 \operatorname{Cos}[x]]$$

Result (type 3, 38 leaves):

$$\frac{\operatorname{ArcTanh}\left[\sqrt{3 + 2 \operatorname{Cos}[2x]}\right] \sqrt{3 + 2 \operatorname{Cos}[2x]} \operatorname{Sec}[x]}{\sqrt{4 + \operatorname{Sec}[x]^2}}$$

### Problem 738: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Cot}[6x] \operatorname{Csc}[6x]}{(5 - 11 \operatorname{Csc}[6x]^2)^2} dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\sqrt{\frac{5}{11}} \operatorname{Sin}[6x]\right]}{60 \sqrt{55}} + \frac{\operatorname{Sin}[6x]}{60 (11 - 5 \operatorname{Sin}[6x]^2)}$$

Result (type 3, 97 leaves):

$$\frac{1}{6600 (17 + 5 \operatorname{Cos}[12x])} \left( 17 \sqrt{55} \left( \operatorname{Log}[\sqrt{55} - 5 \operatorname{Sin}[6x]] - \operatorname{Log}[\sqrt{55} + 5 \operatorname{Sin}[6x]] \right) + 5 \sqrt{55} \operatorname{Cos}[12x] \left( \operatorname{Log}[\sqrt{55} - 5 \operatorname{Sin}[6x]] - \operatorname{Log}[\sqrt{55} + 5 \operatorname{Sin}[6x]] \right) + 220 \operatorname{Sin}[6x] \right)$$

### Problem 759: Result more than twice size of optimal antiderivative.

$$\int (\operatorname{Cos}[x]^{12} \operatorname{Sin}[x]^{10} - \operatorname{Cos}[x]^{10} \operatorname{Sin}[x]^{12}) dx$$

Optimal (type 3, 12 leaves, ? steps):

$$\frac{1}{11} \operatorname{Cos}[x]^{11} \operatorname{Sin}[x]^{11}$$

Result (type 3, 49 leaves):



$$\frac{21 \operatorname{Sin}[2 x]}{1048576} - \frac{15 \operatorname{Sin}[6 x]}{1048576} + \frac{15 \operatorname{Sin}[10 x]}{2097152} - \frac{5 \operatorname{Sin}[14 x]}{2097152} + \frac{\operatorname{Sin}[18 x]}{2097152} - \frac{\operatorname{Sin}[22 x]}{23068672}$$

**Problem 779: Result more than twice size of optimal antiderivative.**

$$\int 3 x^2 \operatorname{Cos}[7 + x^3] dx$$

Optimal (type 3, 6 leaves, 3 steps):

$$\operatorname{Sin}[7 + x^3]$$

Result (type 3, 23 leaves):

$$3 \left( \frac{1}{3} \operatorname{Cos}[x^3] \operatorname{Sin}[7] + \frac{1}{3} \operatorname{Cos}[7] \operatorname{Sin}[x^3] \right)$$

**Problem 781: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Sin}[1 + x^2] dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\frac{1}{2} \operatorname{Cos}[1 + x^2]$$

Result (type 3, 21 leaves):

$$-\frac{1}{2} \operatorname{Cos}[1] \operatorname{Cos}[x^2] + \frac{1}{2} \operatorname{Sin}[1] \operatorname{Sin}[x^2]$$

**Problem 782: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Cos}[1 + x^2] dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$\frac{1}{2} \operatorname{Sin}[1 + x^2]$$

Result (type 3, 21 leaves):

$$\frac{1}{2} \operatorname{Cos}[x^2] \operatorname{Sin}[1] + \frac{1}{2} \operatorname{Cos}[1] \operatorname{Sin}[x^2]$$

**Problem 784: Result more than twice size of optimal antiderivative.**

$$\int x^2 \sin[1 + x^3] \, dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\frac{1}{3} \cos[1 + x^3]$$

Result (type 3, 21 leaves):

$$-\frac{1}{3} \cos[1] \cos[x^3] + \frac{1}{3} \sin[1] \sin[x^3]$$

**Problem 802: Result more than twice size of optimal antiderivative.**

$$\int \sec[x] (1 - \sin[x]) \, dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\log[1 + \sin[x]]$$

Result (type 3, 36 leaves):

$$\log[\cos[x]] - \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right]$$

**Problem 803: Result more than twice size of optimal antiderivative.**

$$\int (1 + \cos[x]) \csc[x] \, dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\log[1 - \cos[x]]$$

Result (type 3, 20 leaves):

$$-\log\left[\cos\left[\frac{x}{2}\right]\right] + \log\left[\sin\left[\frac{x}{2}\right]\right] + \log[\sin[x]]$$

**Problem 805: Result more than twice size of optimal antiderivative.**

$$\int \csc[2x] (\cos[x] + \sin[x]) \, dx$$

Optimal (type 3, 15 leaves, 6 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\cos[x]] + \frac{1}{2} \operatorname{ArcTanh}[\sin[x]]$$

Result (type 3, 61 leaves):

$$-\frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] - \frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \frac{1}{2} \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] + \frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right]$$

**Problem 806: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x](-3 + 2\sin[x])}{2 - 3\sin[x] + \sin[x]^2} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\operatorname{Log}[2 - 3\sin[x] + \sin[x]^2]$$

Result (type 3, 26 leaves):

$$2 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}[2 - \sin[x]]$$

**Problem 807: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x]^2 \sin[x]}{5 + \cos[x]^2} dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$\sqrt{5} \operatorname{ArcTan}\left[\frac{\cos[x]}{\sqrt{5}}\right] - \cos[x]$$

Result (type 3, 82 leaves):

$$\frac{1}{20} \left( -\sqrt{5} \operatorname{ArcTan}\left[\frac{\cos[x]}{\sqrt{5}}\right] + 21\sqrt{5} \operatorname{ArcTan}\left[\frac{1}{\sqrt{5}} - \sqrt{\frac{6}{5}} \tan\left[\frac{x}{2}\right]\right] + 21\sqrt{5} \operatorname{ArcTan}\left[\frac{1}{\sqrt{5}} + \sqrt{\frac{6}{5}} \tan\left[\frac{x}{2}\right]\right] - 20 \cos[x] \right)$$

**Problem 825: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Sec}[5 - x^2] dx$$

Optimal (type 3, 13 leaves, 2 steps):

$$-\frac{1}{2} \text{ArcTanh}[\text{Sin}[5 - x^2]]$$

Result (type 3, 63 leaves):

$$\frac{1}{2} \text{Log}\left[\text{Cos}\left[\frac{5}{2} - \frac{x^2}{2}\right] - \text{Sin}\left[\frac{5}{2} - \frac{x^2}{2}\right]\right] - \frac{1}{2} \text{Log}\left[\text{Cos}\left[\frac{5}{2} - \frac{x^2}{2}\right] + \text{Sin}\left[\frac{5}{2} - \frac{x^2}{2}\right]\right]$$

**Problem 826: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Csc}\left[\frac{1}{x}\right]}{x^2} dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\text{ArcTanh}\left[\text{Cos}\left[\frac{1}{x}\right]\right]$$

Result (type 3, 21 leaves):

$$\text{Log}\left[\text{Cos}\left[\frac{1}{2x}\right]\right] - \text{Log}\left[\text{Sin}\left[\frac{1}{2x}\right]\right]$$

**Problem 834: Result more than twice size of optimal antiderivative.**

$$\int 35 \text{Cos}[x]^3 \text{Sin}[x]^4 dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$7 \text{Sin}[x]^5 - 5 \text{Sin}[x]^7$$

Result (type 3, 33 leaves):

$$35 \left( \frac{3 \text{Sin}[x]}{64} - \frac{1}{64} \text{Sin}[3x] - \frac{1}{320} \text{Sin}[5x] + \frac{1}{448} \text{Sin}[7x] \right)$$

**Problem 850: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{Tan}[c + dx]}{\sqrt{a \text{Sin}[c + dx]^2}} dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a \sin[c+dx]^2}}{\sqrt{a}}\right]}{\sqrt{a} d}$$

Result (type 3, 73 leaves):

$$\frac{\left(-\text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right] - \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c+dx)\right] + \text{Sin}\left[\frac{1}{2}(c+dx)\right]\right]\right) \text{Sin}[c+dx]}{d \sqrt{a \sin[c+dx]^2}}$$

**Problem 861: Result more than twice size of optimal antiderivative.**

$$\int \text{Sec}[x] \sqrt{\text{Sec}[x] + \text{Tan}[x]} dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$2 \sqrt{\text{Sec}[x] (1 + \text{Sin}[x])}$$

Result (type 3, 37 leaves):

$$2 \sqrt{\frac{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]}}$$

**Problem 885: Result more than twice size of optimal antiderivative.**

$$\int (-\text{Cos}[x] + \text{Sin}[x]) (\text{Cos}[x] + \text{Sin}[x])^5 dx$$

Optimal (type 3, 11 leaves, 1 step):

$$-\frac{1}{6} (\text{Cos}[x] + \text{Sin}[x])^6$$

Result (type 3, 25 leaves):

$$\frac{1}{4} \text{Cos}[4x] - \frac{5}{8} \text{Sin}[2x] + \frac{1}{24} \text{Sin}[6x]$$

**Problem 894: Result more than twice size of optimal antiderivative.**

$$\int \text{Sin}[x] \text{Tan}[x]^5 dx$$

Optimal (type 3, 34 leaves, 5 steps):

$$\frac{15}{8} \text{ArcTanh}[\text{Sin}[x]] - \frac{15 \text{Sin}[x]}{8} - \frac{5}{8} \text{Sin}[x] \text{Tan}[x]^2 + \frac{1}{4} \text{Sin}[x] \text{Tan}[x]^4$$

Result (type 3, 113 leaves):

$$\frac{1}{16} \left( -30 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + 30 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \frac{1}{\left(\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right)^4} - \frac{9}{\left(\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right)^2} - \frac{1}{\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^4} + \frac{9}{\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^2} - 16 \text{Sin}[x] \right)$$

**Problem 904: Result more than twice size of optimal antiderivative.**

$$\int x \text{Sec}[1+x] \text{Tan}[1+x] dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$-\text{ArcTanh}[\text{Sin}[1+x]] + x \text{Sec}[1+x]$$

Result (type 3, 47 leaves):

$$\text{Log}\left[\text{Cos}\left[\frac{1+x}{2}\right] - \text{Sin}\left[\frac{1+x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{1+x}{2}\right] + \text{Sin}\left[\frac{1+x}{2}\right]\right] + x \text{Sec}[1+x]$$

**Problem 906: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\text{Sin}[2x]}{\sqrt{9 - \text{Cos}[x]^4}} dx$$

Optimal (type 3, 11 leaves, 5 steps):

$$-\text{ArcSin}\left[\frac{\text{Cos}[x]^2}{3}\right]$$

Result (type 3, 26 leaves):

$$i \text{Log}\left[i \text{Cos}[x]^2 + \sqrt{9 - \text{Cos}[x]^4}\right]$$

**Problem 910: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{-1 + \text{Sec}[x]}{1 - \text{Tan}[x]} dx$$

Optimal (type 3, 37 leaves, 6 steps):

$$-\frac{x}{2} + \frac{\text{ArcTanh}\left[\frac{\cos[x](1+\tan[x])}{\sqrt{2}}\right]}{\sqrt{2}} + \frac{1}{2} \text{Log}[\cos[x] - \sin[x]]$$

Result (type 3, 40 leaves):

$$\frac{1}{2} \left( -x + (2 - 2i) (-1)^{1/4} \text{ArcTanh}\left[\frac{1 + \tan\left[\frac{x}{2}\right]}{\sqrt{2}}\right] + \text{Log}[\cos[x] - \sin[x]] \right)$$

**Problem 912: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[x] + \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x]}} dx$$

Optimal (type 3, 57 leaves, ? steps):

$$-\sqrt{2} \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right] + \sqrt{2} \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]$$

Result (type 5, 68 leaves):

$$-\frac{1}{3 (\sin[x]^2)^{3/4}} \\ 2 \sqrt{\cos[x]} \sqrt{\sin[x]} \left( 3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[x]^2\right] \sin[x] + \cos[x] \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[x]^2\right] \sqrt{\sin[x]^2} \right)$$

**Problem 927: Result more than twice size of optimal antiderivative.**

$$\int x^5 \sec[a + b x^3]^7 \tan[a + b x^3] dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$-\frac{5 \text{ArcTanh}[\sin[a + b x^3]]}{336 b^2} + \frac{x^3 \sec[a + b x^3]^7}{21 b} - \frac{5 \sec[a + b x^3] \tan[a + b x^3]}{336 b^2} - \frac{5 \sec[a + b x^3]^3 \tan[a + b x^3]}{504 b^2} - \frac{\sec[a + b x^3]^5 \tan[a + b x^3]}{126 b^2}$$

Result (type 3, 352 leaves):

$$\frac{1}{64512b^2} \operatorname{Sec}[a + bx^3]^7$$

$$\left( 3072bx^3 + 105 \operatorname{Cos}[5(a + bx^3)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx^3)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx^3)\right]\right] + 15 \operatorname{Cos}[7(a + bx^3)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx^3)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx^3)\right]\right] \right) +$$

$$525 \operatorname{Cos}[a + bx^3] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx^3)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx^3)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx^3)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx^3)\right]\right] \right) +$$

$$315 \operatorname{Cos}[3(a + bx^3)] \left( \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx^3)\right] - \operatorname{Sin}\left[\frac{1}{2}(a + bx^3)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx^3)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx^3)\right]\right] \right) -$$

$$105 \operatorname{Cos}[5(a + bx^3)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx^3)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx^3)\right]\right] - 15 \operatorname{Cos}[7(a + bx^3)] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}(a + bx^3)\right] + \operatorname{Sin}\left[\frac{1}{2}(a + bx^3)\right]\right] -$$

$$566 \operatorname{Sin}[2(a + bx^3)] - 200 \operatorname{Sin}[4(a + bx^3)] - 30 \operatorname{Sin}[6(a + bx^3)] \Big)$$

**Problem 943: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{Cos}[a + bx]^4 - \operatorname{Sin}[a + bx]^4}{\operatorname{Cos}[a + bx]^4 + \operatorname{Sin}[a + bx]^4} dx$$

Optimal (type 3, 72 leaves, 4 steps):

$$\frac{\operatorname{Log}[1 - \sqrt{2} \operatorname{Tan}[a + bx] + \operatorname{Tan}[a + bx]^2]}{2\sqrt{2}b} + \frac{\operatorname{Log}[1 + \sqrt{2} \operatorname{Tan}[a + bx] + \operatorname{Tan}[a + bx]^2]}{2\sqrt{2}b}$$

Result (type 3, 102 leaves):

$$\frac{i(-2i + 5\sqrt{2}) \left( \operatorname{Log}[-1 - 2i\sqrt{2}e^{2i(a+bx)} + e^{4i(a+bx)}] - \operatorname{Log}[-1 + 2i\sqrt{2}e^{2i(a+bx)} + e^{4i(a+bx)}] \right)}{4(5i + \sqrt{2})b}$$

**Problem 945: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{Cos}[a + bx]^2 - \operatorname{Sin}[a + bx]^2}{\operatorname{Cos}[a + bx]^2 + \operatorname{Sin}[a + bx]^2} dx$$

Optimal (type 3, 16 leaves, 6 steps):

$$\frac{\operatorname{Cos}[a + bx] \operatorname{Sin}[a + bx]}{b}$$

Result (type 3, 33 leaves):

$$\frac{\operatorname{Cos}[2bx] \operatorname{Sin}[2a]}{2b} + \frac{\operatorname{Cos}[2a] \operatorname{Sin}[2bx]}{2b}$$



Problem 950: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{-\operatorname{Csc}[a + b x]^4 + \operatorname{Sec}[a + b x]^4}{\operatorname{Csc}[a + b x]^4 + \operatorname{Sec}[a + b x]^4} dx$$

Optimal (type 3, 72 leaves, 4 steps):

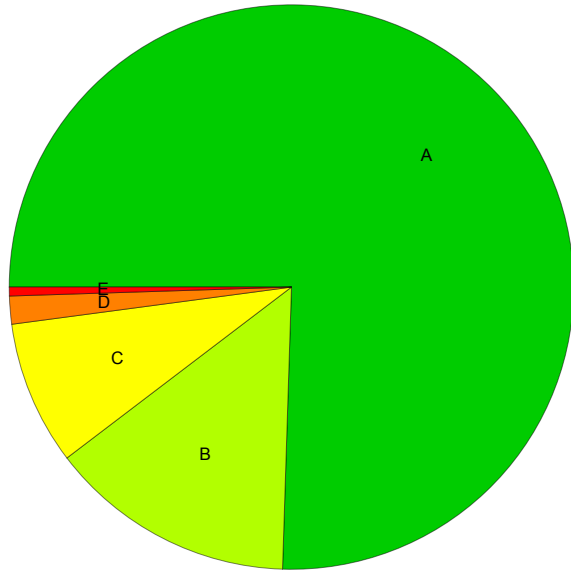
$$\frac{\operatorname{Log}\left[1 - \sqrt{2} \operatorname{Tan}[a + b x] + \operatorname{Tan}[a + b x]^2\right]}{2 \sqrt{2} b} - \frac{\operatorname{Log}\left[1 + \sqrt{2} \operatorname{Tan}[a + b x] + \operatorname{Tan}[a + b x]^2\right]}{2 \sqrt{2} b}$$

Result (type 3, 102 leaves):

$$\frac{i \left(-2 i + 5 \sqrt{2}\right) \left(\operatorname{Log}\left[-1 - 2 i \sqrt{2} e^{2 i (a+b x)} + e^{4 i (a+b x)}\right] - \operatorname{Log}\left[-1 + 2 i \sqrt{2} e^{2 i (a+b x)} + e^{4 i (a+b x)}\right]\right)}{4 \left(5 i + \sqrt{2}\right) b}$$

## Summary of Integration Test Results

2376 integration problems



A - 1794 optimal antiderivatives

B - 336 more than twice size of optimal antiderivatives

C - 196 unnecessarily complex antiderivatives

D - 38 unable to integrate problems

E - 12 integration timeouts